

MEMO - STATS PAPER
PRELIM - 2021

QUESTION 1:

a) $6C_2 + 6C_4 = 30$ (3)

or $6C_2 \times 4C_4 \times 2 = 30$

b. Mark, Jon and Sarah in taxi p:

$(5C_1 \times 2! \times 2!) \times 4P_4$
 $= 480$ (4)

QUESTION 2

a) $P(X) = P(\text{exactly 2 balls have same number})$

$P(2:2:2) = \frac{1}{4} \times 1 \times \frac{1}{7} = \frac{1}{28}$

$P(8:8:N8) = \frac{1}{4} \times \frac{2}{5} \times \frac{3}{7} = \frac{3}{70}$

$P(8:N8:8) = \frac{1}{4} \times \frac{3}{5} \times \frac{4}{7} = \frac{3}{35}$

$P(N8:8:8) = \frac{3}{4} \times \frac{2}{5} \times \frac{4}{7} = \frac{6}{35}$

$\therefore P(X) = \frac{47}{140} (0.3357)$ (6)

b) $P(X \cap Y) = P(4:8:8)$

$= \frac{1}{4} \times \frac{2}{5} \times \frac{4}{7}$

$= \frac{2}{35}$ ✓

$P(Y) = \frac{1}{4}$ ✓

$\therefore \frac{2}{35} \neq \frac{1}{4} \times \frac{47}{140}$

not independent.

c) $P(2:2 | \text{same})$

$= (\frac{1}{4} \times \frac{1}{7}) \div \frac{47}{140}$ ✓ (3)

$= \frac{5}{47} (0.106)$ ✓

QUESTION 3

a) $2k + 4k + 6k + 6k = 1$ ✓

$k = \frac{1}{18}$ ✓ (5)

b) $E(X) = 2 \times 2k + 4 \times 4k + 6 \times 6k$
 $+ 8 \times 6k = 104k$ ✓

$= 104 \times \frac{1}{18} = \frac{52}{9}$ ✓ (5.7778)

$E(X^2) = 4 \times 2k + 16 \times 4k + 36 \times 6k + 64 \times 6k$
 $= \frac{112}{3}$ ✓ (37.3333) (7)

c) $Var(X) = \frac{112}{3} - \left(\frac{52}{9}\right)^2 = \frac{320}{81}$ ✓ (3.95) (2)

$$\begin{aligned}
 4). P(X > 1) \\
 &= 1 - P(X = 0) \\
 &= 1 - \frac{{}^3C_0 {}^{17}C_5}{{}^{20}C_5} \\
 &= \frac{137}{228} (0,6009) \\
 &\quad (6)
 \end{aligned}$$

$$\begin{aligned}
 b1. P(X > 3900) \\
 &= P(Z > \frac{3900 - 4520}{560}) \\
 &= P(Z > -1,107) = 0,8665 \\
 \therefore \text{Number of days} &= 365 \times 0,8665 \\
 &= 316 \\
 &\quad (6)
 \end{aligned}$$

$$\begin{aligned}
 2. \quad \text{Normal distribution curve with mean } 0,378 \text{ and standard deviation } 0,122. \\
 z = 1,165 \text{ (or } 1,16 \text{ or } 1,17) \\
 1,165 = \frac{8000 - m}{560} \\
 m = 7347,6 \\
 &\quad (4)
 \end{aligned}$$

$$\begin{aligned}
 3. P(X = 0) + P(X = 1) \\
 &= {}^6C_0 (0,122)^0 (0,878)^6 \\
 &\quad + {}^6C_1 (0,122)^1 (0,878)^5 \\
 &= 0,8400 \\
 &\quad (5)
 \end{aligned}$$

QUESTION 5

$$\begin{aligned}
 a) i. \quad \frac{20}{100} \pm 1,88 \sqrt{\frac{0,2 \times 0,8}{100}} \\
 [0,1248; 0,2752] \\
 = [12,48\%; 27,52\%] \\
 &\quad (6)
 \end{aligned}$$

2. $\frac{1}{6}$ is within this range
so no evidence of bias concerning 2.

$$\begin{aligned}
 b. \quad \bar{x} \pm z \cdot \frac{\sigma}{\sqrt{n}} \\
 = 2,4 \pm 2,33 \cdot \frac{1,3}{\sqrt{20}} \\
 = [1,7226; 3,0773] \\
 \therefore \text{margin of error} = 0,6773 \\
 &\quad (6)
 \end{aligned}$$

$$561. \quad P(SLL) = 0,3 \times 0,55 \times 0,55 \\ = 0,09075 \left(\frac{363}{4000} \right)$$

$$P(SRR) = 0,3 \times 0,15 \times 0,15 \\ = 0,00675 \left(\frac{27}{4000} \right)$$

$$\therefore Total = 3C_1(0,09075) + 3C_1(0,00675) \\ = 0,2925 \left(\frac{117}{400} \right) \quad (5)$$

$$2. \quad P(S | \text{same direction}) \\ = 0,3 \times 0,3 \times 0,3 \\ \frac{(0,15)^3 + (0,55)^3 + (0,3)^3}{0,1372 \left(\frac{108}{787} \right)} \quad (5)$$

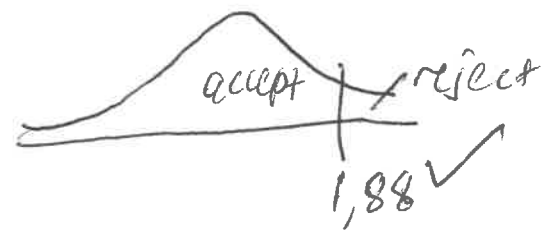
$$3. \quad P = 0,55 \\ E(X) = 49,5 \quad Var(X) = 22,275$$

$$P(42 < X < 54) \\ = P\left(\frac{42,5 - 49,5}{\sqrt{22,275}} < Z < \frac{53,5 - 49,5}{\sqrt{22,275}} \right)$$

$$= P(-1,48 < Z < 1,907) \\ = 0,4306 + 0,4719 \\ = 0,9025 \quad (7)$$

QUESTION 6:

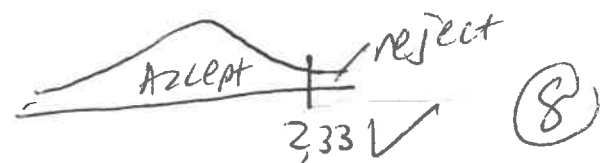
$$a) \quad H_0: \mu = 140 \\ H_1: \mu > 140 \quad (7)$$



$$Z = \frac{146 - 140}{\frac{27}{\sqrt{151}}} = 2,78$$

$\therefore$ We reject H_0 and conclude
The mean systolic blood
pressure for African men
is greater than 140

$$b) \quad H_0: \mu_S \leq \mu_u \\ H_1: \mu_S > \mu_u$$



$$Z = \frac{5,5 - 5,3}{\sqrt{\frac{0,4^2}{50} + \frac{0,3^2}{100}}} = 3,123$$

Reject H_0 and conclude
that the ultrasound method
is faster.

