ST. DAVID'S MARIST INANDA

ADVANCED PROGRAMME MATHEMATICS

PRELIMINARY EXAMINATION
PAPER 2: OPTIONAL MODULE
STATISTICS

GRADE 12

1 SEPTEMBER 2021

EXAMINER: MRS S RICHARD MODERATOR: MR L VICENTE	MARKS: 100 TIME: 1 hour

NAME: Memo

INSTRUCTIONS:

- ✓ This paper consists of 12 pages and a separate 4-page formula sheet. Please check that your paper is complete.
- ✓ Please answer all questions on the Question Paper.
- ✓ You may use an approved non-programmable, non-graphics calculator unless otherwise stated.
- ✓ Round answers to 2 decimal places, unless stated otherwise.
- ✓ It is in your interest to show all your working details.
- ✓ Work neatly. Do NOT answer in pencil.
- ✓ Diagrams are not drawn to scale.

QUESTION	Q1 [9]	Q2 [20]	Q3 [13]	Q4 [12]	Q5 [16]	Q6 [12]	Q7 [7]	Q8 [11]	TOTAL [100]
LEARNER'S MARKS									

QUESTION 1 (Answers to 4 decimal places, where applicable)

A small packet of Jelly-tots contains 5 pink, 7 yellow and 3 green sweets. Only these 3 colours are present on the packet. If I remove 3 Jelly-tots one at a time, without replacement, calculate:

a) The probability that all three are pink. (4)

$$P(\text{all pink}) = \frac{\binom{5}{3}}{\binom{15}{3}} \checkmark\checkmark = 0{,}02198\ldots \checkmark$$

$$\frac{60}{2730} = \frac{10}{455} = \frac{2}{91}$$

$$= 0{,}0220 \checkmark \text{ rounding}$$

or

$$\frac{5}{15} \times \frac{4}{14} \times \frac{3}{13} \checkmark\checkmark\checkmark = \frac{2}{91} = 0{,}0220 \checkmark$$

b) The probability that I will get one of each colour. (5)

$$P(\text{one of each}) = \frac{5 \times 7 \times 3}{\binom{15}{3}} \checkmark\checkmark\checkmark$$

$$= 0{,}2308 \checkmark\checkmark$$

or

$$6\left(\frac{105}{\cancel{455}\ 2730}\right) = 6\left(\frac{5}{15} \times \frac{7}{14} \times \frac{3}{13}\right) \checkmark\checkmark\checkmark = \frac{1}{26} \times 6$$

$$\frac{630}{2730}$$

$$6\,(0{,}038\ldots) = \frac{3}{13}$$

↑ only 2/5

$$= 0{,}2308 \checkmark\checkmark$$

[9]

QUESTION 2 (Answers to 4 decimal places, where applicable)

a) A standard pack of cards have 13 hearts and 39 other cards. The pack of cards is shuffled, and ten cards are drawn and not replaced back not the pack. What is the probability that:

i) Four of the drawn cards will be hearts. (5)

$$\frac{\binom{13}{4}\binom{39}{6}}{\binom{52}{10}} = 0{,}1475$$

ii) At most one of the drawn cards will be hearts. (7)

$$P(X = 0;1) = \frac{\binom{13}{0}\binom{39}{10} + \binom{13}{1}\binom{39}{9}}{\binom{52}{10}}$$

$$= 0{,}040186.. + 0{,}1741...$$

$$= 0{,}2143$$

b) A group of friends are all playing the same computer game. The probability that one of them will beat the computer is 0,4. How many players are needed to play the game to ensure that the probability of at least one of them beating the computer is 98%. (8)

$P(X \geq 1) = 1 - P(X = 0)$ ✓ ✓

$1 - 0,6^n > 0,98$ ✓ ✓

$0,6^n < 0,02$ ✓ ✓

$n > 7,7$ ✓

$\therefore n = 8$ ✓

[20]

QUESTION 3 (Answers to 3 decimal places, where applicable)

The time taken by a garage to replace worn-out brake pads follows a normal distribution with mean 90 minutes and standard deviation 5,8 minutes.

a) Determine the probability that the garage takes longer than 105 minutes. (4)

$$z = \frac{x-\mu}{\sigma} = \frac{105-90}{5,8} = 2,586 \checkmark$$

$$P(1-(0,5+0,495)) = 0,005 \checkmark$$

b) Determine the probability that the garage takes less than 85 minutes. (5)

$$z = \frac{x-\mu}{\sigma} = \frac{90-8,5}{5,8} = 0,862 \checkmark$$

$$P(\text{time} < z) = P(\text{time} > z)$$

$$= 1-(0,5+0,305)$$

$$= 0,195 \checkmark$$

c) The garage claims to complete the repairs in "a to b" minutes. If this claim is to be correct for 90% of the repairs, find a and b, based on a symmetrical interval centred on the mean. (4)

$$90 \pm 1,65 \times 5,8 = 90 \pm 9,57$$

$$= 90 \pm 10$$

$$\therefore [80; 100]$$

$$a = 80 \quad b = 100$$

[13]

QUESTION 4 (Answers to 4 decimal places, where applicable)

In a sample of 400 shops it was discovered that 136 of them sold television sets below the list prices that had been recommended by the manufacturers.

a) Calculate the 95% confidence interval limits for this estimate. (6)

$$p = \frac{136}{400} = 0{,}34 \checkmark$$

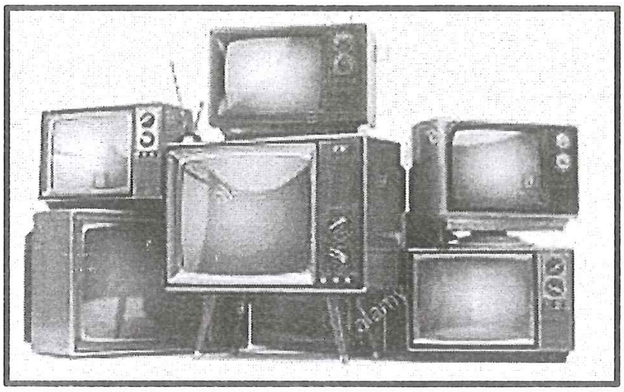
$$p \pm 1{,}96\sqrt{\frac{p(1-p)}{n}}$$

$$= 0{,}34 \checkmark \pm 1{,}96 \checkmark \sqrt{\frac{0{,}34(1-0{,}34)}{400}} \checkmark$$

$$= 0{,}34 \pm 0{,}04642\ldots$$

$$(29{,}36\% \checkmark ;\ 38{,}64\% \checkmark)$$

or $(0{,}2936 ;\ 0{,}3864)$



b) What size sample would have to be taken in order to estimate the percentage to within ± 2%? (6)

$$1,96\sqrt{\frac{p(1-p)}{n}} \leq 0,02$$

$$1,96\sqrt{\frac{0,34(1-0,34)}{n}} \leq 0,02$$

$$\sqrt{\frac{0,34\ (0,66)}{n}} \leq 0,0\ 10204$$

$$n \geqslant 2155,1721..$$

sample size 2156

[12]

QUESTION 5 (Answers to 4 decimal places, where applicable)

A machine is supposed to produce bolts with a mean diameter of 20 mm and a standard deviation of 0,2 mm. A random sample of 40 bolts has a mean of 20,05 mm. You want to make a decision whether or not the machine needs adjustment, using a 5% level of significance.

a) Write down the null hypothesis. (1)

$H_0 : \mu = 20$ ✓

b) Write down the alternate hypothesis. (1)

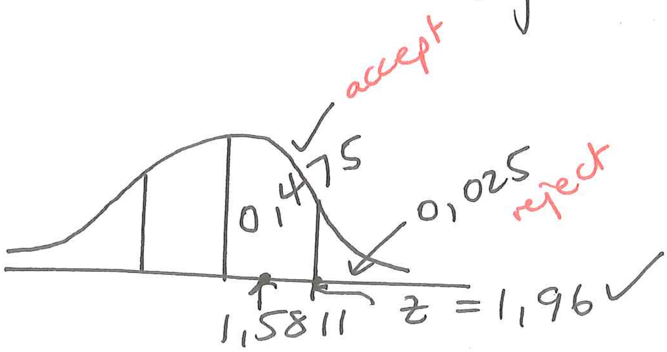
$H_1 : \mu \neq 20$ ✓

c) Test the hypothesis using a 5% level of significance. (8)

Test score $\dfrac{20{,}05 - 20}{\frac{0{,}2}{\sqrt{40}}} = 1{,}5811$ ✓✓ ✓

Not enough evidence to reject null hypothesis at 5% significance ✓

z-value does not lie in the reject region. ✓



d) What mean value(s) would cause management to adjust the machine? (6)

To reject test score

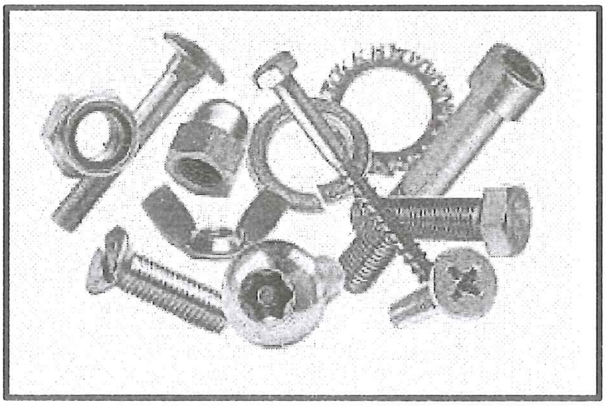
$z < -1,96$ ✓ or $z > 1,96$ ✓

$1,96 < \dfrac{\bar{x} - 20}{\frac{0,2}{\sqrt{40}}}$ ✓ or $-1,96 > \dfrac{\bar{x} - 20}{\frac{0,2}{\sqrt{40}}}$ ✓

$\therefore \bar{x} > 20,06$ ✓ or $\bar{x} < 19,94$ ✓

machine would need to be adjusted.

[16]



QUESTION 6 (Answers to 4 decimal places, where applicable)

A botanist finds that 6% of a certain type of plant seed fails to germinate. 500 seeds are planted.

Let X represent the number of seeds that fail to germinate.

a) Show that the conditions for the normal approximation to the binomial distribution hold. (4)

$$np = 0,06 \times 500$$
$$= 30 > 5$$

$$n(1-p) = 0,94 \times 500$$
$$= 470 > 5$$

$\therefore$ conditions hold

b) Find the probability that at most 25 plant seeds will fail to germinate. (8)

$$x \leq 25 \quad \therefore \quad P(X \leq 25)$$

$$\longrightarrow \quad P(X < 25,5)$$

$$\mu = np = 30$$
$$\sigma^2 = np(1-p) = 500(0,06)(0,94)$$
$$= 28,2$$

$$Z = \frac{X - \mu}{\sigma}$$
$$= \frac{25,5 - 30}{\sqrt{28,2}}$$
$$= -0,8474$$
$$\approx -0,85$$

$$H(z) = 0,3023$$
$$\therefore 0,5 - 0,3023$$
$$= 0,1977$$
$$\therefore P(X \leq 25)$$
$$= 0,1977$$

[12]

QUESTION 7

The probability density function for the lifespan of a certain species is given by:

$$f(x) = \begin{cases} -\frac{3}{16}x^2 + \frac{3}{4}; & 0 \le x \le m \\ 0 & \textit{elsewhere} \end{cases}$$ where x is the age of the insect in years.

Determine m, the maximum lifespan of these insects. (7)

$$\int_0^m -\frac{3}{16}x^2 + \frac{3}{4}\,dx = 1 \checkmark$$

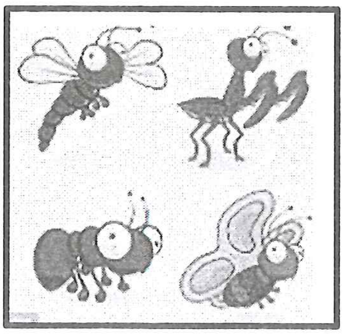
$$\left[-\frac{3}{16} \cdot \frac{x^3}{3} + \frac{3}{4}x\right]_0^m = 1 \checkmark\checkmark$$

$$-\frac{m^3}{16} + \frac{3m}{4} - 0 = 1 \checkmark$$

$$-m^3 + 12m = 16$$

$$m^3 - 12m + 16 = 0 \checkmark$$

$$m \neq -4 \checkmark \quad \text{or} \quad m = 2 \checkmark$$



[7]

QUESTION 8

The probability distribution of the discrete random variable X is shown in the table below:

x	–3	–1	0	4
P(X = x)	a	b	0,15	0,4

Given that E[X] = 0,75

a) Determine the values of a and b. (7)

$a + b + 0,15 + 0,4 = 1$ ✓

$a + b = 0,45 \quad \ldots (1)$ ✓

$E[X] = -3a - b + 4(0,4) = 0,75$ ✓

$-3a - b = -0,85$ ✓

$3a + b = 0,85 \quad \ldots (2)$ ✓

$a + b = 0,45 \quad \ldots (1)$

$2a = 0,4$

$\therefore a = 0,2$ ✓

$b = 0,25$ ✓

b) Hence determine Var[X] (to 2 decimal places). (4)

$Var[X] = (-3)^2\left(\frac{1}{5}\right) + (-1)^2\left(\frac{1}{4}\right) + 0$

$+ (4)^2(0,4) - (0,75)^2$ ✓✓

$= 7,89$ ✓✓

[11]

[TOTAL: 100 MARKS]