ST. DAVID'S MARIST INANDA

ADVANCED PROGRAMME MATHEMATICS

PRELIMINARY EXAMINATION
PAPER 1: CALCULUS and ALGEBRA

GRADE 12

1 SEPTEMBER 2021

EXAMINER: MRS S RICHARD	**MARKS: 200**
MODERATOR: MRS C KENNEDY	**TIME: 2 hours**

NAME:_____

Please put a cross next to your teacher's name:

Mrs Kennedy	Mrs Richard

INSTRUCTIONS:

- ✓ This paper consists of 27 pages and a separate 4-page formula sheet. Please check that your paper is complete.
- ✓ Please answer all questions on the Question Paper.
- ✓ You may use an approved non-programmable, non-graphics calculator unless otherwise stated. PLEASE ENSURE YOUR CALCULATOR IS IN **RADIAN** MODE.
- ✓ Round answers to 2 decimal places, unless stated otherwise.
- ✓ It is in your interest to show all your working details.
- ✓ Work neatly. Do NOT answer in pencil.
- ✓ Diagrams are not drawn to scale.

QUESTION	Q1 [42]	Q2 [14]	Q3 [11]	Q4 [22]	Q5 [12]	Q6 [27]	Q7 [7]	Q8 [7]	Q9 [14]	Q10 [6]	Q11 [28]	Q12 [10]	TOTAL [200]
LEARNER'S MARKS													

QUESTION 1

a) Solve for x, without using a calculator and showing all working:

i) $\left|\frac{3}{x-1}\right| = 12$ (4)

ii) $\log(2x+1) - \log(x-1) = 1$ State restrictions where necessary. (5)

iii) $\ln(e^{2x} - 20) = x$ (give your answer in terms of *ln a*, given *a* is a constant) (6)

b) The following formula models the number of years (t), from now, in terms of the number of people (P) that stay in a town at time t:

$$t = 100 \ln\left(\frac{4}{3} - \frac{P}{60000}\right)$$

i) Determine how many people initially live in the town when t = 0.
(Without the use of a calculator and showing all working out.) (5)

ii) As a result of migration to the cities, the town's population is decreasing. Calculate after how many years (to the nearest year) there will be no residents left in the town. (3)

iii) Change the subject of the formula to P, hence write the formula as P = . . (5)

iv) Hence, or otherwise, determine the initial **rate** at which the population decreases (that is when t = 0 years). (5)

c) Given $f(x) = (x-1)\ln(x-1)$ for $x > 1$ and $g(x) = e^x + 1$

i) Show that $f \circ g(x) = x.e^x$ (4)

ii) Hence solve for x if $f \circ g(x) = 2x$ (5)

[42]

QUESTION 2 Note $i = \sqrt{-1}$

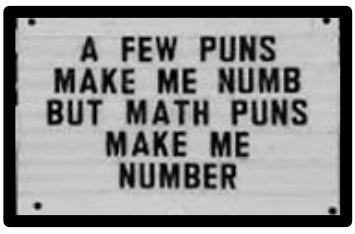
a) Determine an equation in the form: $x^4 + ax^3 + bx^2 + cx + d = 0$ given that $x = 1 - \sqrt{2}$ and $x = 3 - i$ are roots of the equation. (8)

b) Determine the values of *a* and *b*, where *a* and *b* are real numbers that satisfy the equation: $\frac{a+2i}{1-3i} \times bi = -7 - i$ (6)

[14]

QUESTION 3

Prove by Mathematical induction that $8^n - 7n + 6$ is divisible by 7 for all $n \in \mathbb{N}$ (11)



[11]

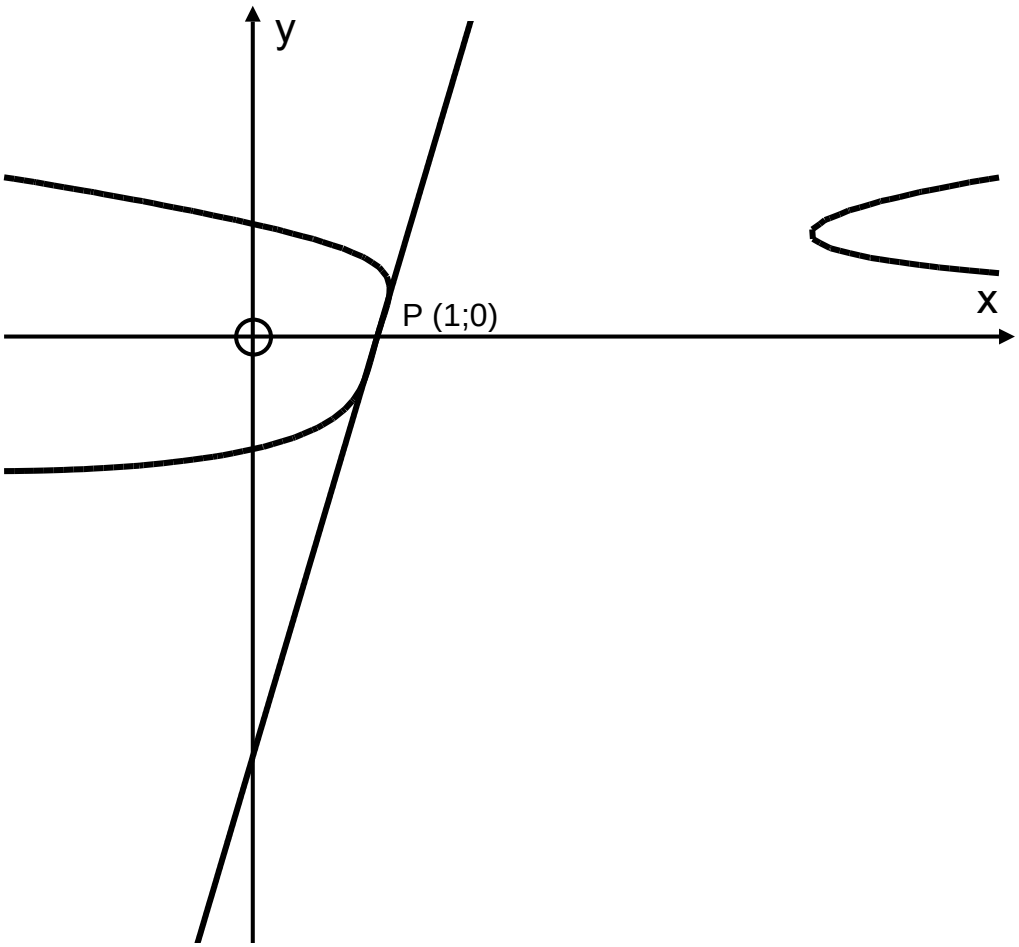
QUESTION 4

a) Determine the derivative of $f(x) = \frac{1}{\sqrt{3x-2}}$ by first principles. (8)

b) Determine: $D_x\left[\left(\frac{3x-1}{2x+5}\right)^5\right]$ (6)

c) Determine the gradient of the tangent to the curve $3y^4 + 4x - x^2 \sin y - 4 = 0$

at the point P (1; 0) (8)



[22]

QUESTION 5

The function f(x) is defined as follows:

$$f(x) = \begin{cases} \dfrac{a}{x} & \text{if } x \geq 2 \\ b - 2x & \text{if } x < 2 \end{cases}$$

Determine the values of a and b if f(x) is **differentiable** at x = 2.

Justify your answer, using correct notation of limits. (12)

[12]

QUESTION 6

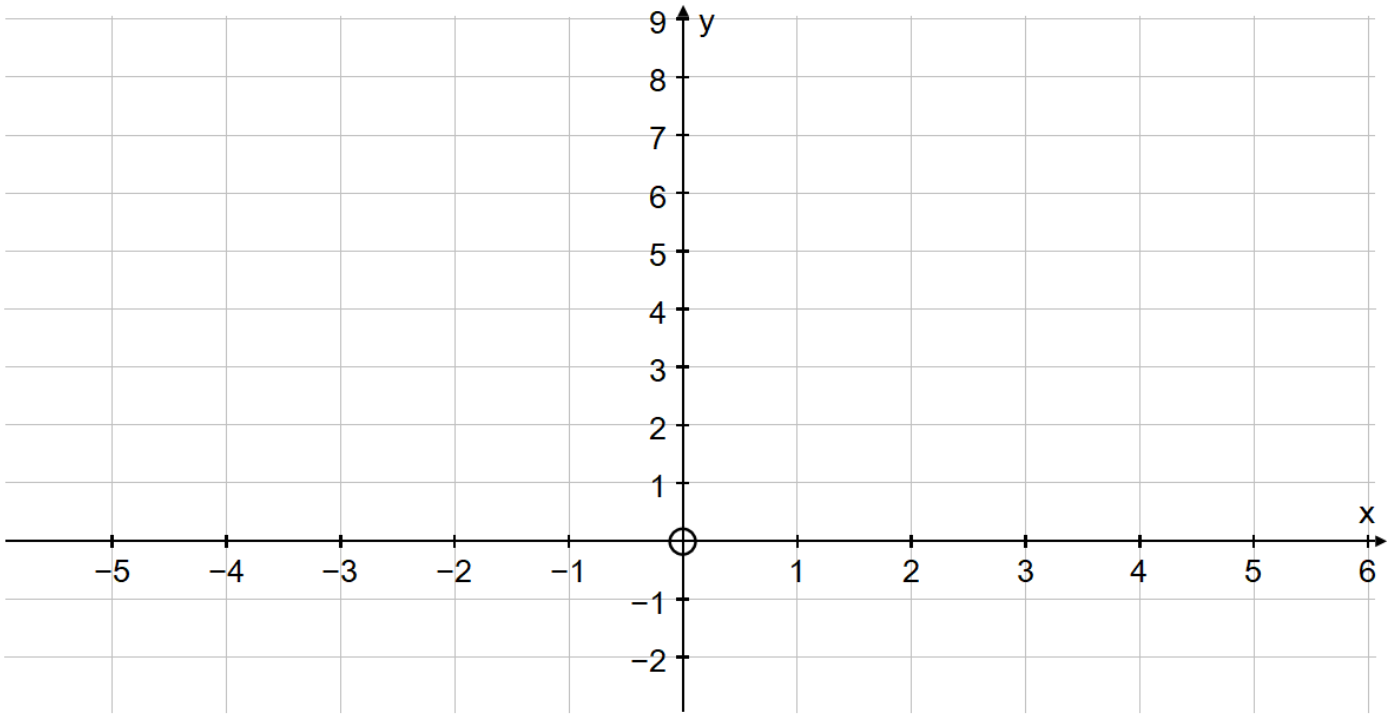
Given the function $f(x) = \dfrac{x^3 + 4x^2 + x - 6}{x^2 - 1}$

a) Determine the coordinates of the stationary points. (9)

b) Determine the intercepts with the axes. (4)

c) Determine the equations of any asymptotes. (4)

d) Sketch the graph of f(x) on the given axes. (10)

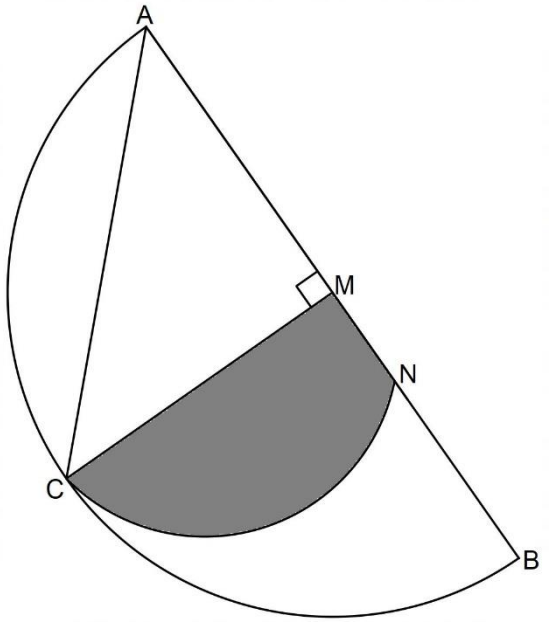


[27]

QUESTION 7

ABC is a semi-circle with centre M.
ANC is a sector with centre A and corresponding arc NC.

AM = 15cm, $AMC = \frac{\pi}{2}$ radians and $MAC = \beta$



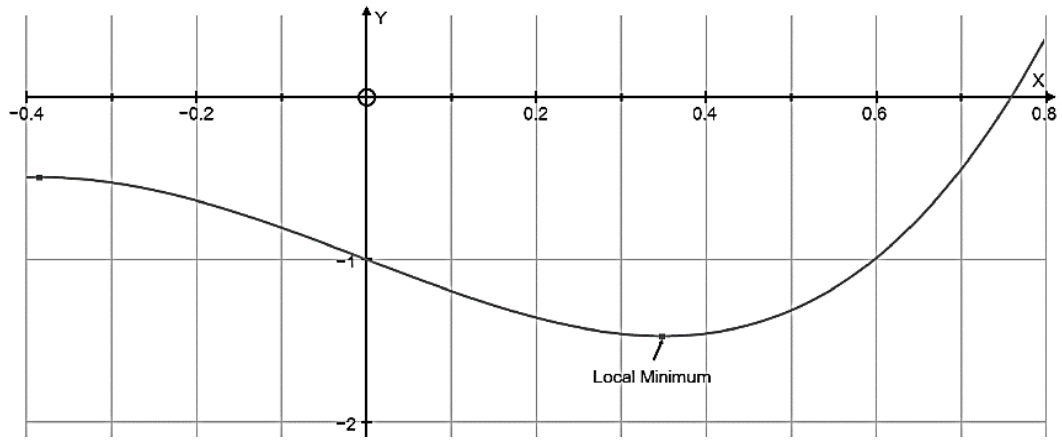
a) Give a reason why $\beta = \frac{\pi}{4}$ radians. (1)

b) Determine the area of the shaded region MNC. (6)

[7]

QUESTION 8

A portion of the graph of $f(x) = x^4 + 5x^3 - 2x - 1$ is shown below:



Use the **Newton-Raphson Method** with an initial approximation of 1 to determine the **x-coordinate of the local minimum** shown above. Give your answer to 4 decimal places. (7)

[7]

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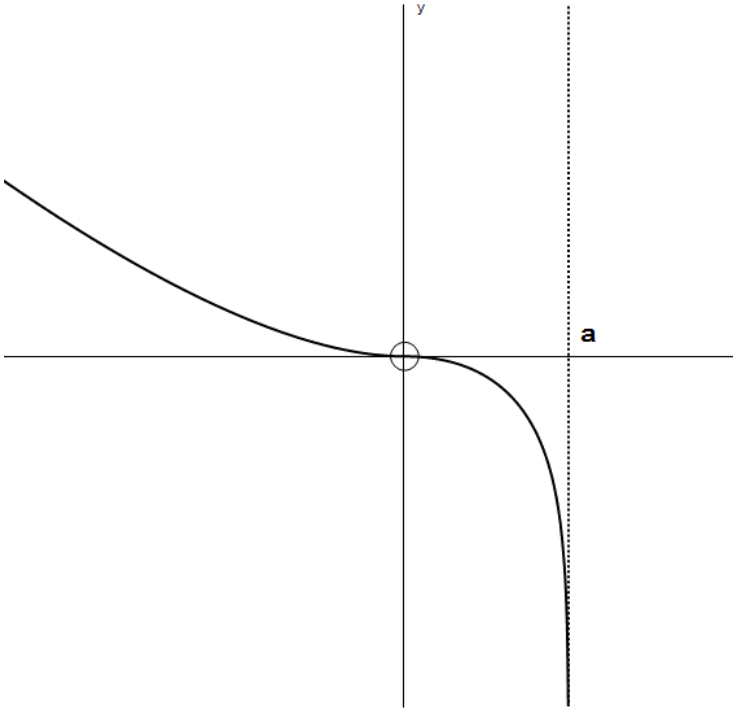
QUESTION 9

The graph of the function $f(x) = |x|\ln(1-x)$, $x < a$ is shown below.



a) Write down the value of a. (2)

b) Sketch the following graphs.

You do not need to work out any values – simply show how the shape changes.

i) $y = |f(x)|$ (4)

ii) $y = f(|x|)$ (4)

iii) $y = \dfrac{1}{f(x)}$ (4)

[14]

QUESTION 10

Consider $\int_a^b f(x)dx = \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \left[\left(2 + \frac{3i}{n} \right)^2 - 2\left(2 + \frac{3i}{n} \right) + 2 \right]$

a) Determine the values of a and b. (2)

b) Write down the function $f(x)$ (2)

c) Calculate the area enclosed by the graph of f, the x-axis and the lines x = a and x = b. (2)

[6]

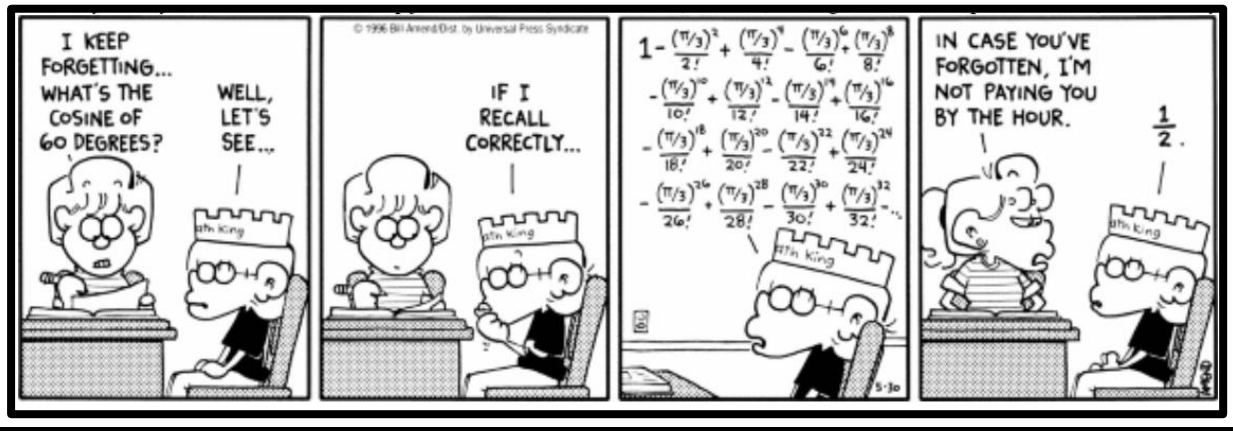
QUESTION 11

a) Determine the following integrals:

i) $\int x^2\sqrt{5x^3-13}\,dx$ (6)

ii) $\int x\cos 3x\,dx$ (using integration by parts) (8)

iii) $\int \cot^3 x.\text{cosec}^2 x \, dx$ (6)



b) Given $f(x) = \dfrac{-x-11}{x^2+x-2}$

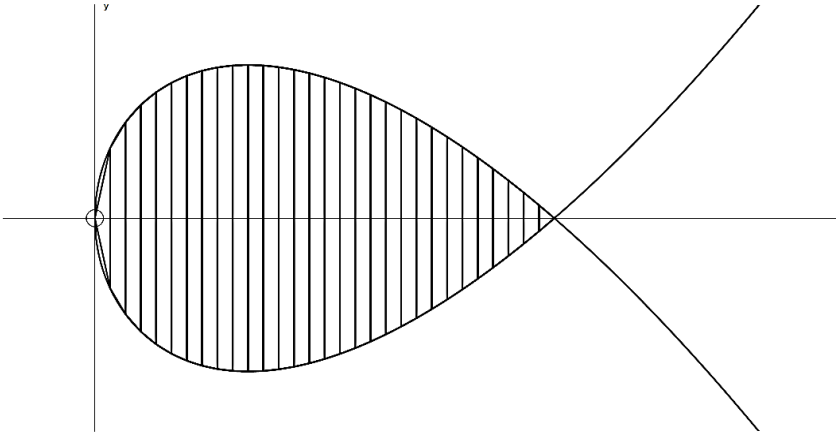
i) Decompose f(x) into partial fractions. (4)

ii) Hence determine $\int \dfrac{-x-11}{x^2+x-2}dx$ (4)

[28]

QUESTION 12

The loop $y^2 = x(a-x)^2$ is shown.



The shaded region is rotated about the x-axis.

Determine the volume of the solid formed by this rotation, in terms of a. (10)

[10]

[Total: 200 marks]

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