

QUESTION 1

1.1 Solve for $x \in R$ if $|x^2 - x| = 42$ (5)

1.2 Given: $e^x + 12e^{-x} = 8$ and give your answer in the form:
 $x = \ln a$ and $x = \ln b$ (7)

1.3 If $\frac{m}{2-4i} = \frac{1}{3} + ni$, calculate the values of m and n (6)

1.4 $f(x) = x^4 + x^3 - 2x^2 + 2x + 4$

Solve for $f(x) = 0$ if $x \in \mathbb{C}$ and it is given that $f(1 - i) = 0$. (10)

1.5 $f(x) = 1 - 2e^{-2x}$

Determine $f^{-1}(x)$ in the form $y = \dots$ and provide the domain. (6)

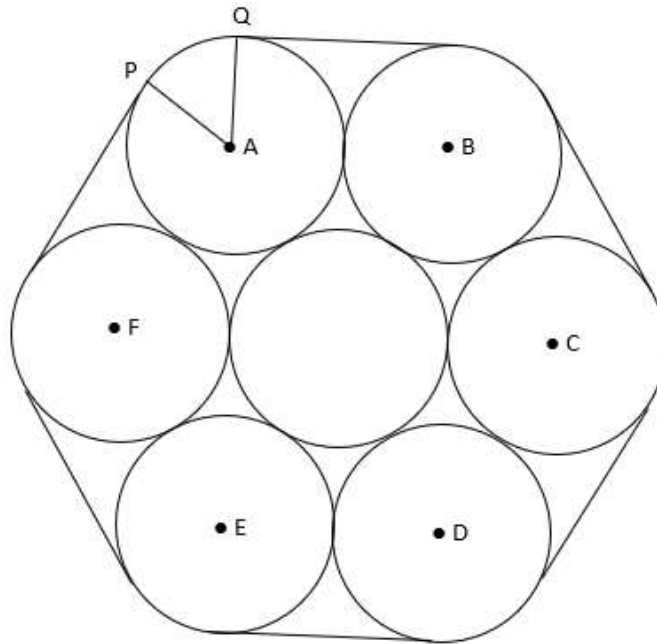
QUESTION 2

Prove by Mathematical Induction that, for all $n \in \mathbb{N}$,

$$\sum_{r=1}^n \frac{5-4r}{5^r} = \frac{n}{5^n}$$

QUESTION 3

The diagram shows a cross-section of seven cylindrical pipes, each of radius 20cm , held together by a thin rope which is wrapped tightly around the pipes. The centres of the six outer pipes are A, B, C, D, E and F . Points P and Q are situated where straight sections of the rope meet the pipe with centre A .



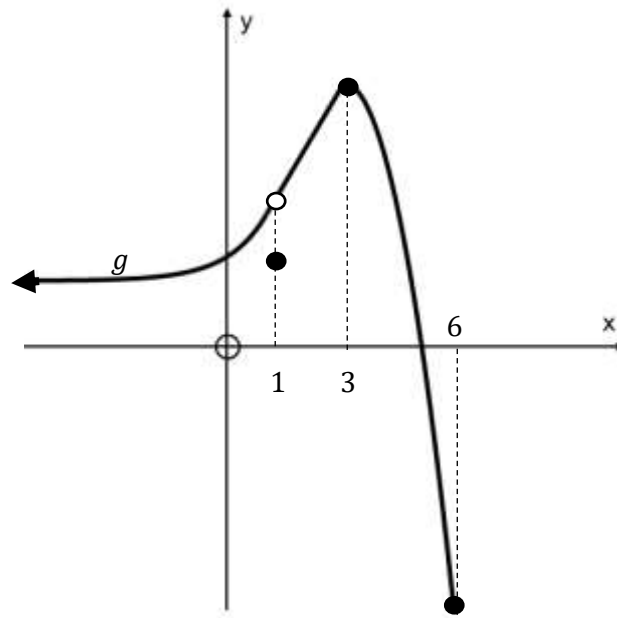
- 3.1 Given that $\angle PAQ = \frac{1}{3}\pi$ radians, find arc PQ , and hence the length of the rope in terms of π . (4)

3.2 Prove that the area of the hexagon $ABCDEF$ is $2400\sqrt{3} \text{ cm}^2$. (5)

3.3 Find the area of sector PAQ , and hence the area of the complete region enclosed by the rope. Give your final answer to two decimal places. (7)

QUESTION 4

Given: $g(x) = \begin{cases} e^{x-1} + a & \text{if } x < 1 \\ 1,5 & \text{if } x = 1 \\ x + 1 & \text{if } 1 < x \leq 3 \\ -x^2 + 6x - 5 & \text{if } 3 < x \leq 6 \end{cases}$



4.1 Determine the value of a such that the limit at $x = 1$ exists.

(4)

- 4.2 Using the diagram, identify the type of discontinuity that exists at $x = 1$.
Give a reason.

(2)

- 4.3 Given that the graph of g is continuous at $x = 3$, determine whether g is also differentiable at $x = 3$. Show all working.

(4)

[10]

QUESTION 5

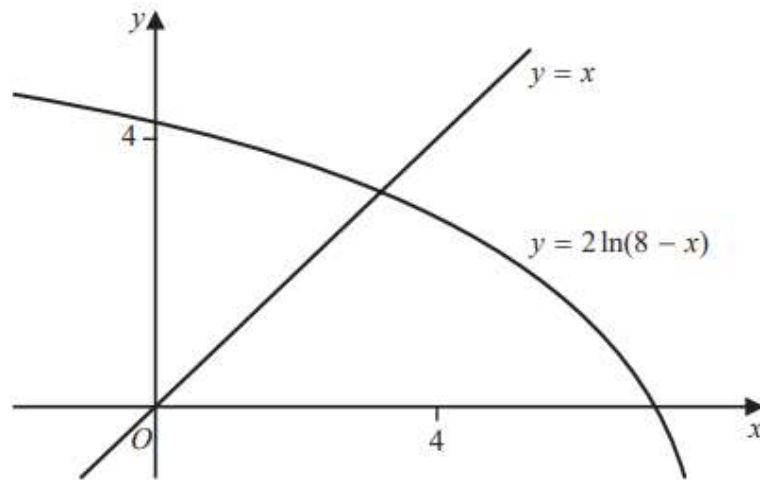
5.1 Determine $f'(x)$ by first principles if $f(x) = \sqrt{x+3}$ (7)

5.2 Find $\frac{dy}{dx}$ if $y = \frac{\sec 2x}{x^2}$, without simplifying. (5)

5.3 Determine $D_x \left[\ln \left(\frac{x-4}{4x-1} \right) \right]$ and simplify your answer. (5)

5.4 Given: $f(x) = \cos x$ and $g(x) = \frac{2}{e^{2x}}$, determine $\frac{d}{dx} (f(g(x)))$. (5)

5.5 The curve with equation $y = 2\ln(8 - x)$ meets the line $y = x$ at a single point, $x = \alpha$



(a) Show that $3 < \alpha < 4$

(4)

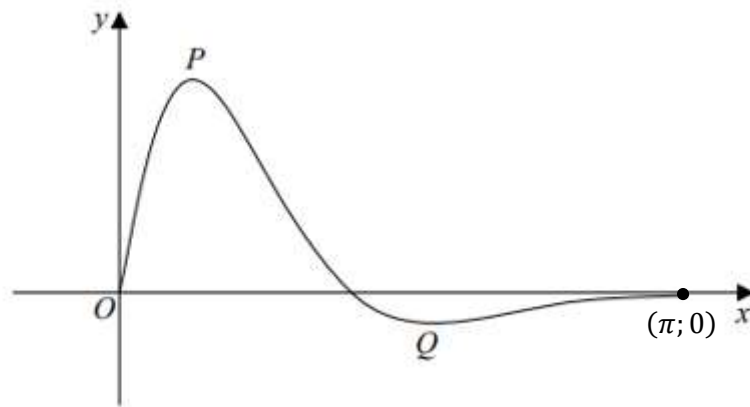
(b) Use Newton's method to determine the x – coordinate of this point of intersection to 4 decimal places.

(6)
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QUESTION 6

6.1 Below is the sketch of the curve with equation $y = f(x)$, where

$$f(x) = \frac{4\sin 2x}{e^{\sqrt{2}x-1}}; 0 \leq x \leq \pi$$



The curve has a maximum turning point at P and a minimum turning point at Q .
Show that the x –coordinate of P and Q can be found by solving the equation:

$$\tan 2x = \sqrt{2} \quad (\text{Do not solve this equation})$$

6.2 Below are the graphs of four rational functions.

Below are five equations representing rational functions:

$$f(x) = \frac{x^2+x-2}{x^2-x-6} = \frac{(x-1)(x+2)}{(x+2)(x-3)}$$

$$g(x) = \frac{x^2+2x+1}{x-1}$$

$$k(x) = \frac{x^2-x-2}{2-x}$$

$$m(x) = \frac{x^2+x-2}{x^2-2x-3}$$

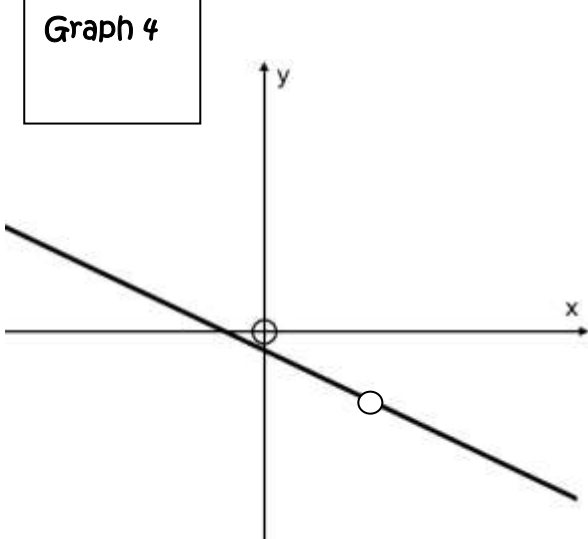
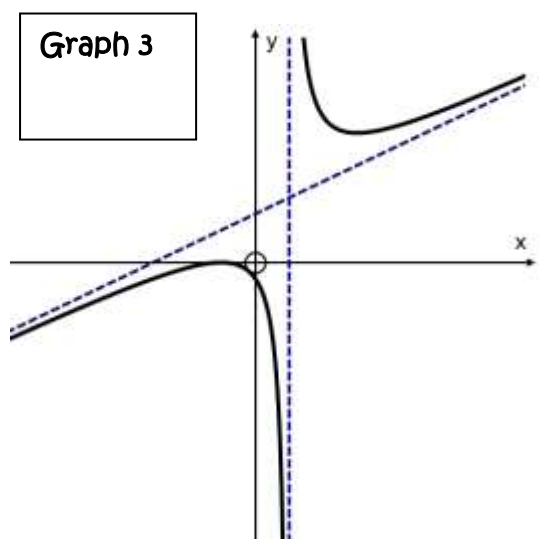
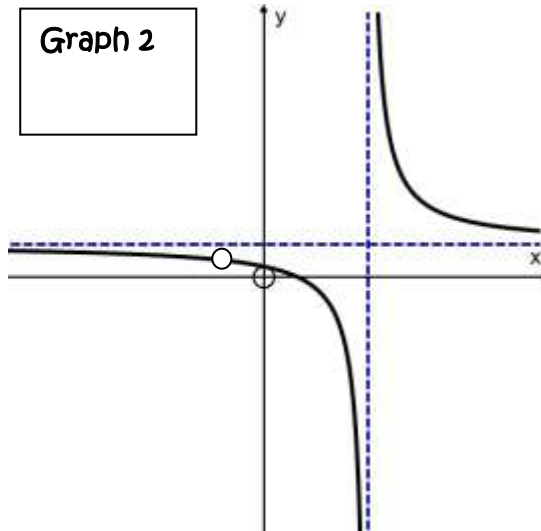
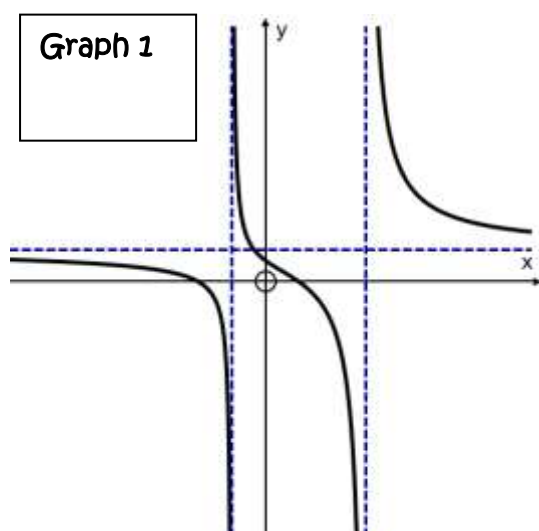
$$n(x) = \frac{x^3+4x^2+x-6}{x^2-1}$$

(a) Fill in the name of the correct function in the boxes:

For example:

Graph 6
 $p(x)$

(10)



(b) Show algebraically that **Graph 2** is always decreasing. (8)

[28]

QUESTION 7

7.1 The area under the curve of $f(x)$ from $x = 2$ to $x = b$ can be calculated by constructing n rectangles of equal width and then using a Riemann sum as follows:

$$\begin{aligned} \text{Area} &= \frac{3}{n} \sum_{i=1}^n \left[-\left(2 + \frac{3i}{n}\right)^2 + 5\left(2 + \frac{3i}{n}\right) \right] \\ &= \frac{27}{2} - \frac{9}{n} - \frac{9}{2n^2} \end{aligned}$$

(a) What is the function $f(x)$? (2)

(b) Determine the value of b . (2)

(c) Calculate the approximate area if the region is divided into 3 rectangles. (2)

(d) Is the estimated area in (c) an over-estimation or an under-estimation of the exact area? Explain. (3)

(e) Determine $\int_2^b f(x)dx$, using the Riemann sum given above. (2)

(f) Calculate the value of $\int_2^b [-f(x)]dx$ and explain your answer. (2)

7.2 Determine the following without a calculator and showing all working:

(a) $\int 3x(x^2 + 4)^5 dx$ (do not use substitution) (5)

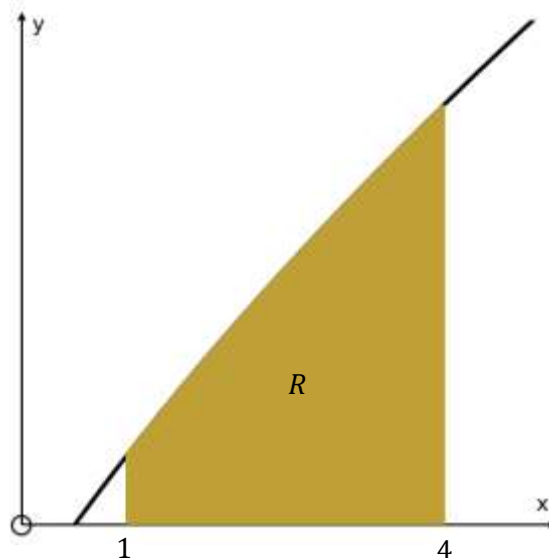
(b) $\int 2x\sqrt{x+2} \, dx$ (use a u –substitution) (9)

(c) (i) Decompose $\frac{3x+3}{x^2+x-2}$ into its partial fractions. (7)

(ii) Hence determine: $\int \frac{3x+3}{x^2+x-2} dx$ (4)

(d) $\int \frac{1}{1+\cot^2 x} dx$ (6)

- 7.3 The sketch below shows a part of the curve with equation $y = x^{\frac{1}{2}} \ln 2x$.
The finite region R is bounded by the curve, the x -axis and the lines $x = 1$ and $x = 4$.



- (a) Khanyisile started off correctly working out the indefinite integral

$$\int x^{\frac{1}{2}} \ln 2x \, dx \text{ using integration by parts.}$$

Complete her work:

$$\int x^{\frac{1}{2}} \ln 2x \, dx$$

$$\text{Let } f(x) = \ln 2x \quad \therefore f'(x) =$$

$$g'(x) = \quad \therefore g(x) =$$

$$\therefore \int x^{\frac{1}{2}} \ln 2x \, dx =$$

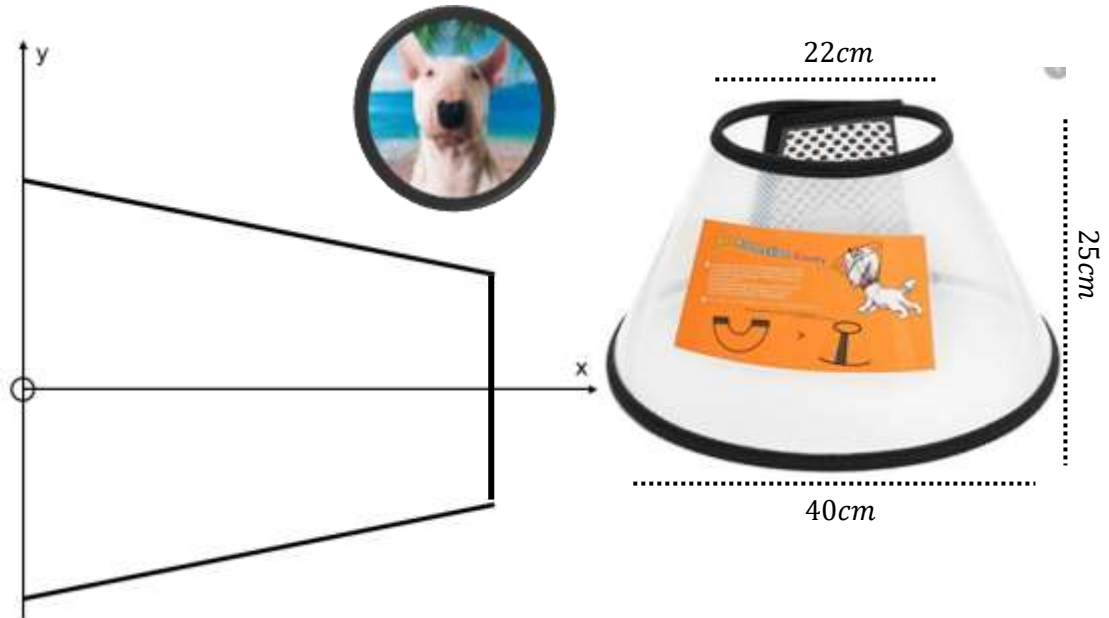
(b) Using your integral in (a), determine the exact area of R , giving your answer in the form:

$a \ln 2 + b$, where a and b are exact constants. (8)

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QUESTION 8

Mrs Vermeulen's bull terrier had a minor operation. He had to wear a head piece, shaped as a truncated cone, so that he would not lick his stitches. She measures the diameter of the top and bottom circles and finds them to be 22cm and 40cm respectively. The vertical height of the head piece is 25cm .



Using the diagram on the cartesian plane above as a guide, Mrs Vermeulen wants you to determine the integral for the **volume** of her dog's head piece in terms of π . Do not work out the value of the integral.

(8)

[8]

Total: 200 marks

Extra working space