

ST STITHIANS GIRLS COLLEGE

MATHEMATICS

AP MATHS: GRADE 12

DATE: 13 July 2012

TIME: 2 hours

EXAMINER: Mrs V van Rooy

MARKS: 200

MODERATOR: Mr M Ancillotti

NAME: _____ TEACHER: _____

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 7 pages, including the front cover. A datasheet is attached at the back which you may remove.
2. Read the questions carefully.
3. Answer all the questions.
4. Number your answers exactly as the questions are numbered.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Unless otherwise stated, round answers to **two** decimal places where necessary.
7. All the necessary working details must be clearly shown.
8. It is in your own interest to write legibly and to present your work neatly.

CALCULUS and ALGEBRA**Question 1****[10 marks]**

Use Mathematical induction and prove that $3^n + 3^{n+1} + 3^{n+2}$ is divisible by 13, for all $n \in \mathbb{N}$.

Question 2**[42 marks]**

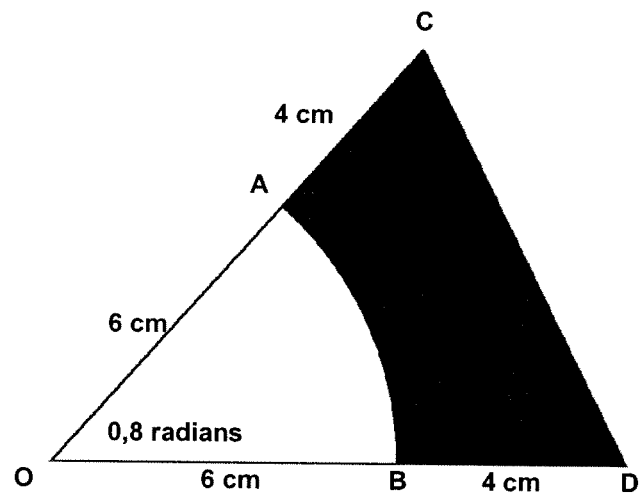
- a) Solve for x , where $x \in \mathbb{C}$: $x^3 + 2x^2 + 4x + 8 = 0$ (5)
- b) Determine a , b and c if $xy = 5 + i$ and $x + y = 3 + 2i$, given that $x = a + bi$ and $y = c - i$. (8)
- c) Determine the following limits:
- i) $\lim_{x \rightarrow 0} \frac{5 \sin 2x}{\sin 4x}$ (6)
- ii) $\lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2 - x}}$ (8)
- d) Given the functions: $f(x) = |x - 1| - 4$ and $g(x) = 2x + 3$
- i) Determine:
- 1) the coordinates of the x -intercepts of f . (3)
 - 2) write down the coordinates of the salient point of f . (1)
 - 3) draw the graphs of f and g on the same system of axes (4)
- ii) Calculate the point of intersection between f and g . (5)
- iii) Use your graph and your answer in (ii) to give the values where $|x - 1| > 2x + 7$ (2)

Question 3**[18 marks]**

- a) If $\ln 3 = 1,10$ and $\ln 5 = 1,161$, find $\ln 45$ without the use of a calculator. (5)
- b) Rewrite the following as a single logarithm: $4\ln x - \frac{1}{2}\ln y + \ln z$ (2)
- c) Solve for k , rounded to two decimal places: $34 = e^{10k}$ (3)
- d) Sketch the function $f(x) = -e^{x+1} + 2$ and its inverse function $f^{-1}(x)$ on the same system of axes, clearly indicating intercepts and asymptotes. (8)

Question 4**[11 marks]**

In the diagram OCD is an isosceles triangle with $OC = OD = 10$ cm and $\hat{C}OD = 0,8$ radians. The points A and B on OC and OD respectively, are joined by an arc of a circle with centre O and radius 6 cm.



Find:

- a) The area of the shaded region. (5)
- b) The perimeter of the shaded region. (6)

Question 5**[5 marks]**

Prove the identity:

$$\frac{1}{1+\sin x} + \frac{1}{1-\sin x} = 2\sec^2 x$$

Question 6**[19 marks]**a) Differentiate with respect to x . DO NOT simplify your answers.

i) $y = (3x^2 + 1)^4$ (3)

ii) $f(x) = x^3 \sin x - 5\cos 2x$ (3)

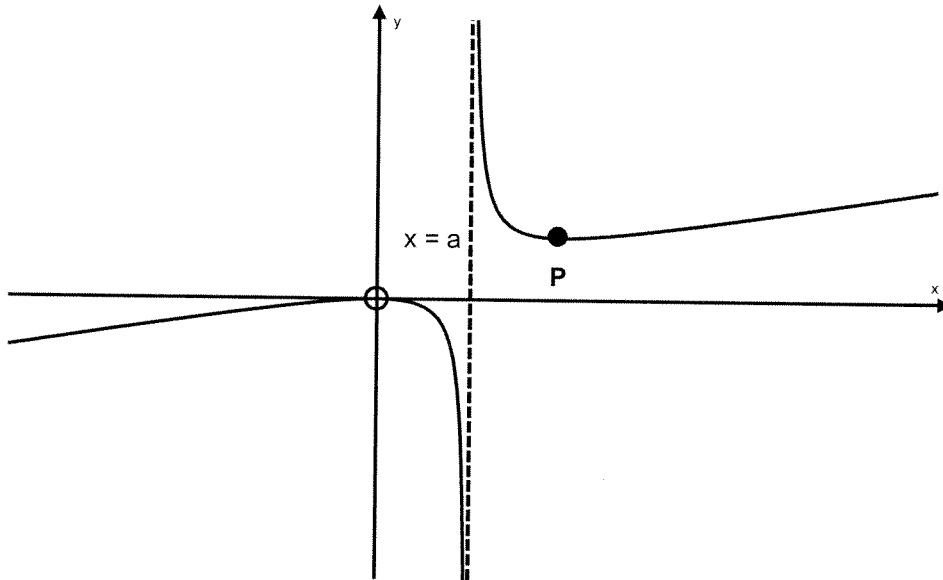
iii) $y = \frac{\sec 2x}{x^2}$ (3)

b)
$$f(x) = \begin{cases} 3x + 7 & \text{if } x < -2 \\ 0 & \text{if } x = -2 \\ x^2 - 3 & \text{if } x > -2 \end{cases}$$

i) Determine $\lim_{x \rightarrow -2} f(x)$, if it exists. (5)ii) Is $f(x)$ continuous at $x = -2$? Justify your answer. If not, classify the type of discontinuity. (3)iii) Is $f(x)$ differentiable at $x = -2$? Justify your answer. (2)

Question 7**[22 marks]**

The sketch shows part of the curve defined by $f(x) = \frac{x^2}{3x-1}$



- i) Write down the value of a . (1)

- ii) If P is a turning point of $f(x)$, show that the x -coordinate of P is $\frac{2}{3}$. (8)

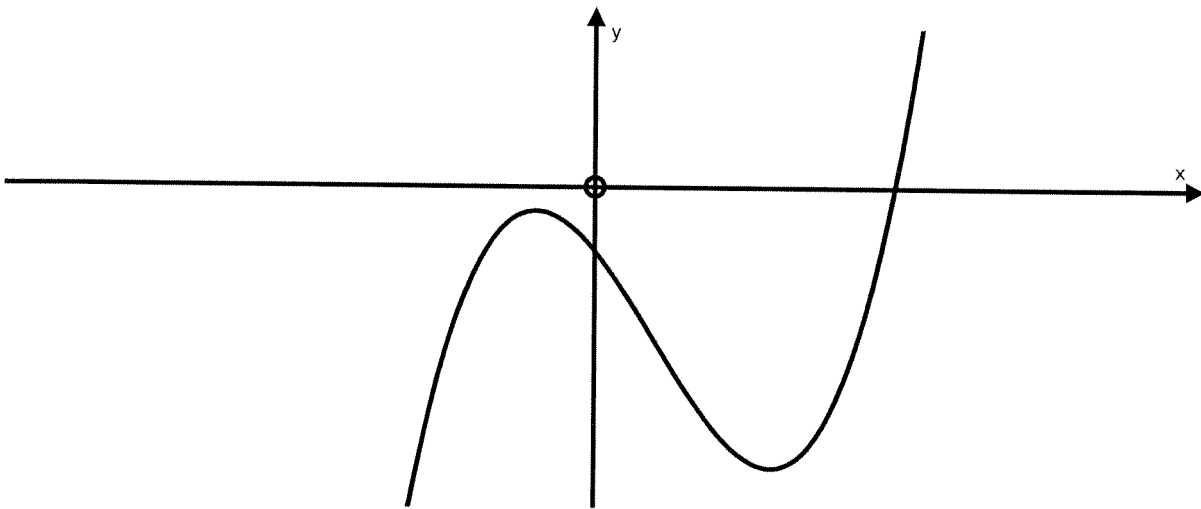
- iii) Write down the slope of the tangent line when $x = 0$. (1)

- iv) If $f(x) - k = 0$ has no real solutions, write down the possible values of k . (4)

- v) Find the equation of the oblique asymptote of f . (8)

Question 8**[4 marks]**

The sketch shows part of the curve defined by $f(x) = x^3 - 3x^2 - 9x - 8$



Use the Newton-Rhapson formula to find the x-intercept, correct to 4 decimal places. (4)

Question 9**[15 marks]**

- a) Find the gradient of the tangent of the curve defined by $7y^4 + x^3y + x = 4$ at the point $(-2; 1)$ (8)
- b) If $y = \frac{1}{1+2x}$, determine a formula for the n^{th} derivative. (7)

Question 10**[29 marks]**

Integrate the following:

- a) $\int \frac{1}{\sqrt{2-3x}} dx$ (6)
- b) $\int \sin x \cdot \cos^2 x dx$ (6)
- c) $\int (\sec x + \tan x)^2 dx$ (5)
- d) Determine $\int_1^3 (10 - x^2) dx$ by using Riemann sums (12)

Question 11**[25 marks]**

The graph of $f(x) = 2x\sqrt{9-x^2}$ is shown below. The shaded area is the area bounded by $f(x)$ and the positive x-axis.

- a) Find the area of the shaded region. (8)
- b) Show that the turning point A, has coordinates $\left(\frac{3}{\sqrt{2}}; 9\right)$ (8)
- c) If the graph of $f(x)$ on the interval $x \in \left[0; \frac{3}{\sqrt{2}}\right]$ is rotated about the x-axis, determine the volume of the revolution. (9)

