



NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2011

MATHEMATICS: PAPER I

Time: 3 hours

150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 10 pages and an Information Sheet of 2 pages (i – ii). Please check that your paper is complete.
 2. Read the questions carefully.
 3. Answer all the questions.
 4. Number your answers exactly as the questions are numbered.
 5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
 6. Round off your answers to one decimal digit where necessary.
 7. All the necessary working details must be clearly shown.
 8. It is in your own interest to write legibly and to present your work neatly.
-

SECTION A

QUESTION 1

(a) Solve for x :

(1) $3x^2 = 2(x+5)$, giving your answer correct to one decimal digit. (4)

(2) $\frac{3}{x-4} + \frac{x-3}{x} = 2$ (4)

(3) $125^{3x-2} = 25^{4x+10}$ (4)

(b) Evaluate:

$$\frac{25^n \cdot 36^{n+1}}{81 \cdot 30^{2n}} \quad (4)$$

(c) Write down an expression for the n^{th} term of the sequence:

$$\frac{4}{5}; \frac{8}{9}; \frac{12}{13}; \frac{16}{17} \quad (1)$$

(d) Determine which term of the arithmetic sequence:

$$-5; 2; 9; 16; 23; \dots \text{ equals } 163. \quad (4)$$

(e) Determine the value of n such that $\sum_{k=1}^n (3 + 2k) = 896$. (7)

[28]

QUESTION 2

(a) Evaluate: $\lim_{x \rightarrow 6} \left(\frac{x^2 - 36}{x^2 - 6x} \right)$ (3)

(b) Find $\frac{dy}{dx}$ if: (1) $y = 5x^2(2x - 1)$ (4)

(2) $y = \frac{4x^3 - x^2 - 3}{x}$ (4)

leaving your answer with positive exponents. (4)

(c) Given $f(x) = \frac{3x^2}{2} - 24\sqrt{x}$, calculate $f'(9)$. (5)

[16]

QUESTION 3

- (a) A point A has coordinates (3 ; 5).

Write down the coordinates of the image of A under the transformation that:

- (1) reflects it across the y -axis. (1)
- (2) moves it vertically upwards by two units. (1)
- (3) shifts it horizontally one unit to the left. (1)
- (4) reflects it across the line having equation $y = x$. (1)
- (5) 'enlarges' it about the origin by a factor of 3. (1)

- (b) Given:
- $g(x) = 3x - 2$

Determine each of the following:

- (1) $g^{-1}(x)$ (2)
- (2) $\frac{1}{g(x)}$ (1)
- (3) $g\left(\frac{1}{x}\right)$ (1)

- (c) The price (
- p
-) of each item in a consignment of
- q
- items,

is given by $p = \log\left(10 + \frac{q}{2}\right)$

where q is the number of items supplied at a price p (in Rands) per unit.

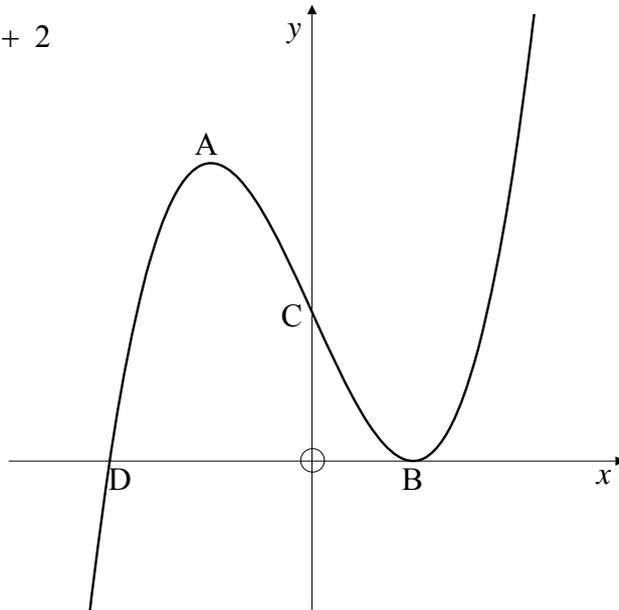
- (1) Calculate p and the total price of the consignment, when the consignment has 1980 items. (3)
- (2) Determine the number of items in the consignment when the price of each item is R2. (3)

[15]

QUESTION 4

(a) Refer to the figure showing the graph of:

$$f(x) = x^3 - 3x + 2$$



- (1) Determine the coordinates of the turning points A and B. (5)
- (2) Determine the coordinates of C and D, intercepts of the curve with the axes. (5)
- (3) Calculate the average gradient of f between A and B. (2)
- (4) Using the graph, state the solution to $f'(x) > 0$. (2)

(b) Given: $f(x) = x^3 - 3x^2 + 3x - 1$

- (1) Show that f is never decreasing. (4)
- (2) Show that the graph of $y = f(x)$ has a point of inflection when $x = 1$. (2)

[20]

79 marks

SECTION B

QUESTION 5

- (a) Sapna bought a new computer. It depreciated in value from R12 000 to R7 500 over a period of 3 years.



Calculate the rate of depreciation per year, using depreciation on a reducing balance. (4)

- (b) Mr Kekana takes a bank loan of R110 400 to be paid back over 60 months at an interest rate of 10% p.a. compounded monthly. Calculate the value of his monthly payments. (4)

- (c) Ayanda's father agreed to buy her a car costing R120 000 for her 21st birthday.



He had one condition: she had to supply the 8% deposit required.

- (1) Determine the amount that Ayanda had to supply as a deposit. (1)

Ayanda decided to invest every year's birthday money in a savings account, starting on her 18th birthday. At that stage, the bank gave an interest rate of 8,5% p.a. compounded monthly.

On her 18th birthday, she received R2 650.

On her 19th birthday, she received R3 340, but the interest rate then changed to 12% p.a. compounded quarterly.

On her 20th birthday she received R2 400.

Six months after her 20th birthday, she withdrew R1 200 to pay towards a holiday.

- (2) Summarise this information on a time line. (2)

- (3) Determine whether Ayanda was able to supply the deposit. (5)

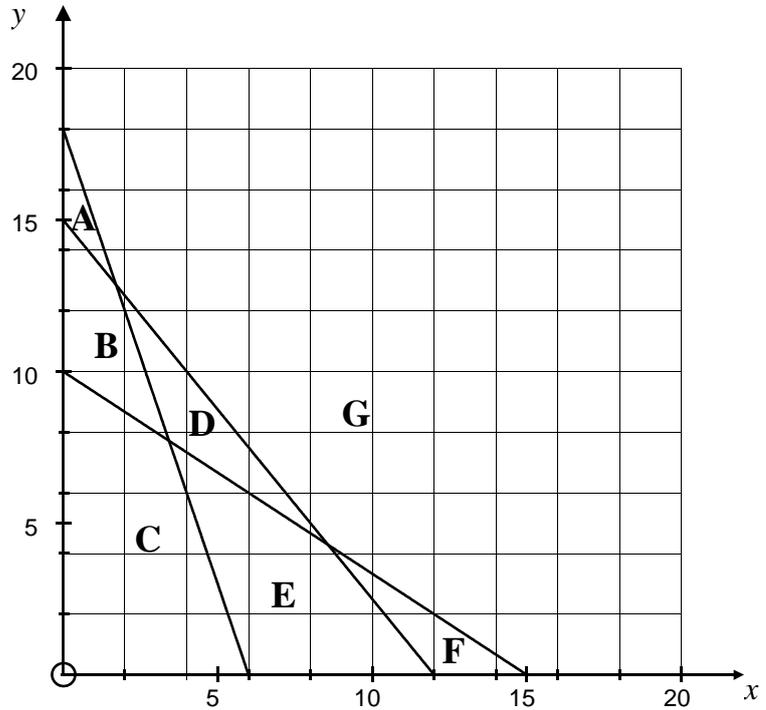
[16]

QUESTION 6

(a) A feasible region is described by the following set of constraints:

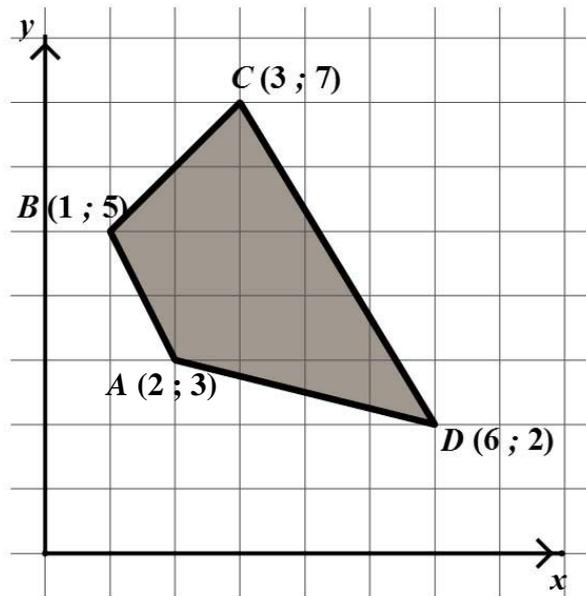
$$\begin{aligned}
 x &\geq 0, \quad y \geq 0, \\
 3x + y &\geq 18 \\
 5x + 4y &\leq 60 \\
 2x + 3y &\geq 30
 \end{aligned}$$

Refer to the diagram alongside.



- (1) Determine which of the seven regions is the feasible region. (2)
- (2) State which constraints are not needed for this feasible region. (2)
- (3) If it is further given that $x \in N$, determine the largest possible value of x that satisfies all the constraints. (1)

- (b) The quadrilateral below is a feasible region of a linear programming problem with vertices $A(2 ; 3)$, $B(1 ; 5)$, $C(3 ; 7)$ and $D(6 ; 2)$.



- (1) For a point $(x; y)$ within the feasible region, determine the maximum value of $P = 2x + y$. (4)

- (2) The minimum value of an objective function $Q = 3x + y$ is obtained at B. Calculate this minimum. (2)

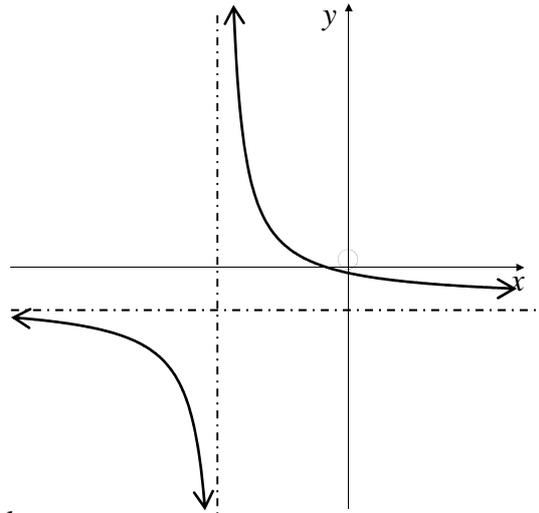
- (3) Suppose the vertex A to be a minimum point for an objective function $R = mx + y$. Determine all the possible values of m . (4)

[15]

QUESTION 7

- (a) Refer to the figure, showing the graph of a hyperbola:

$$h(x) = \frac{2}{x + 3} - 1$$

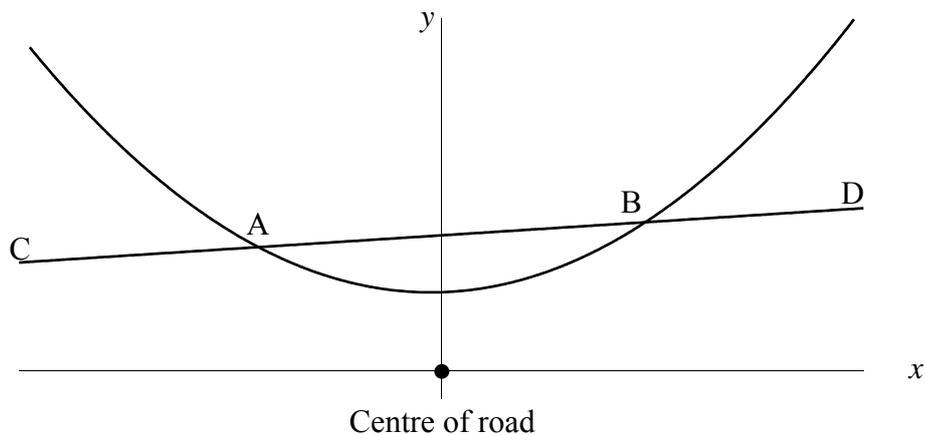


- (1) Write down the domain and range of h . (2)
- (2) Determine the equations of the axes of symmetry of the graph. (4)
- (b) A Metro council installed Christmas decorations in Main Street. A garland of lights was suspended in the shape of a parabola with equation:

$$y = \frac{x^2}{10} + 3$$

where y is the height of the garland (in metres) above the road and x is the horizontal distance (in metres) from the centre of the road.

A rope CD holding up other decorations is connected to the garland at A and B and is described by the function $y = \frac{2x}{15} + \frac{7}{2}$.



- Determine the difference in height above the road of the two points A and B. Give your answer to the nearest centimetre. (7)

[13]

QUESTION 8

(a) Consider the geometric series: $5 - 10x + 20x^2 - 40x^3 + \dots$

(1) Determine the values of x that will ensure that the sum converges. (3)

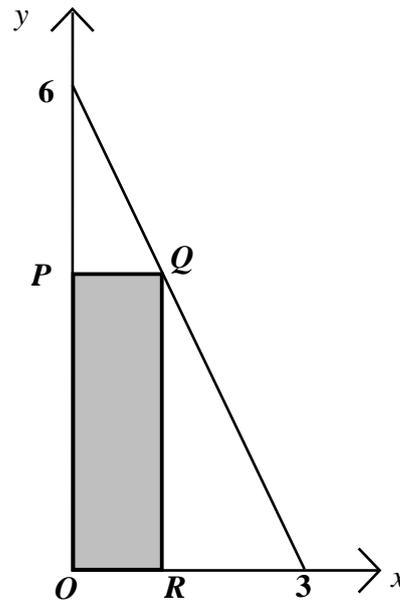
(2) Calculate the value of x for which $S_{\infty} = 100$. (3)

(b) Refer to the figure.

A point Q lies on the line joining $(0, 6)$ and $(3, 0)$.

$OPQR$ is a rectangle, where P and R lie on the axes.

Let $OR = k$.



(1) Write an expression for QR in terms of k . (2)

(2) Find the coordinates of Q for which the rectangle has a maximum area. (5)

(3) Hence determine this maximum. (2)

[15]

QUESTION 9

- (a) The n^{th} term T_n of a sequence is $n^2 - 1$ if n is odd, and $22 - 3n$ if n is even.
- (1) Calculate $T_5 + T_6$. (3)
- (2) Find k such that $T_k = -2$. (3)
- (b) Given an arithmetic sequence with first term equal to 30 and a constant difference of -3 .
- (1) Determine a simplified expression for the n^{th} term of the sequence. (2)
- (2) Suppose $T_p + T_q = 0$.
- Determine p in terms of q , stating the values of q . (4)

[12]

71 marks

Total: 150 marks