NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2011

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours 150 marks

These marking guidelines were used as the basis for the official IEB marking session. They were prepared for use by examiners and sub-examiners, all of whom were required to attend a rigorous standardisation meeting to ensure that the guidelines were consistently and fairly interpreted and applied in the marking of candidates' scripts.

At standardisation meetings, decisions are taken regarding the allocation of marks in the interests of fairness to all candidates in the context of an entirely summative assessment.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines, and different interpretations of the application thereof. Hence, the specific mark allocations have been omitted.

SECTION A

QUESTION 1

(a)(1) $3x^2 = 2(x + 5)$

$3x^2 - 2x - 10 = 0$ **A**

$$x = \frac{2 \pm \sqrt{4 + 120}}{6}$$ **M** Sub. into Quadratic formula

$= 2{,}2$ **A** or $-1{,}5$ **A**

(4)

(2) $\frac{3}{x - 4} + \frac{x - 3}{x} = 2 \qquad x \neq 0, x \neq 4$

$$\frac{3x + (x - 4)(x - 3)}{x(x - 4)} = \frac{2x(x - 4)}{x(x - 4)}$$ **M** Writing with LCD

$3x + x^2 - 7x + 12 = 2x^2 - 8x$ **A**

$x^2 - 4x - 12 = 0$

$(x - 6)(x + 2) = 0$ **A**

$x = 6$ or $x = -2$ **CA**

(4)

(3) $125^{3x - 2} = 25^{4x + 10}$

$\left(5^3\right)^{3x - 2} = \left(5^2\right)^{4x + 10}$ **M** Prime bases

$5^{9x - 6} = 5^{8x + 20}$

$9x - 6 = 8x + 20$ **A** **M** Equating exponents

$x = 26$ **A**

(4)

(b) $\dfrac{25^n.36^{n+1}}{81.30^{2n}}$

$= \dfrac{\left(5^2\right)^n\left(2^2.3^2\right)^{n+1}}{3^4.\left(2.3.5\right)^{2n}}$ **M** Prime bases

$= \dfrac{5^{2n.}.2^{2n+2}.3^{2n+2}}{3^4.2^{2n}.3^{2n}.5^{2n}}$ **A**

$= 2^{2n+2-2n}.3^{2n+2-4-2n}$ **M** Simplifying

$= \dfrac{2^2}{3^2}$

$= \dfrac{4}{9}$ **A**

(4)

(c) $T_n = \dfrac{4n}{4n+1}$ **A**

(1)

(d) $\text{T}_\text{n} = a + (n - 1)d = 163$

$T_n = -5 + (n-1).7 = 163$ **A** **M** Tn of AP = 163

$7(n-1) = 168$

$n - 1 = 24$ **M** Solving

$n = 25$ **A**

i.e. T_{25}

(4)

(e) $\sum_{k=1}^{n}(3 + 2k)$

$= 5 + 7 + 9 + \ldots.. + (3 + 2n)$ **M** Expanding

$S_n = \dfrac{n}{2}\left[2 \times 5 + (n - 1).2\right] = 896$ **A** correct sub **M** Sn of AP = 896

$n\left[5 + n - 1\right] = 896$

$n^2 + 4n - 896 = 0$ **M** Simplifying

$(n + 32)(n - 28) = 0$ **A**

$n = -32 \quad or \quad n = 28$

✘N.V.•**M** **A**

(7)

[28]

QUESTION 2

(a) $\lim_{x \to 6} \frac{x^2 - 36}{x^2 - 6x}$ — P1N

$= \lim_{x \to 6} \frac{(x - 6)(x + 6)}{x(x - 6)}$ **M** — Factorising

$= \lim_{x \to 6} \frac{x + 6}{x}$ **A**

$= 2$ **A**

(3)

(b) (1) $y = 5x^2(2x - 1)$

$y = 10x^3 - 5x^2$ **A** **M** — Simplifying

$\frac{dy}{dx} = 30x^2 - 10x$ **A** **M** — Finding derivative

(2) $y = \frac{4x^3 - x^2 - 3}{x}$ — P1N

$= 4x^2 - x - 3x^{-1}$ **A**

(4)

$\frac{dy}{dx} = 8x - 1 + 3x^{-2}$ **M** — Finding derivative

$= 8x - 1 + \frac{3}{x^2}$ **CA** **CA**

(4)

(c) $f(x) = \frac{3x^2}{2} - 24\sqrt{x}$

$= \frac{3x^2}{2} - 24x^{\frac{1}{2}}$ **A** — PIN

$f'(x) = 3x - 24 \times \frac{1}{2}x^{-\frac{1}{2}}$ **M** — Finding derivative

$= 3x - \frac{12}{\sqrt{x}}$ **A**

$f'(9) = 3 \times 9 - \frac{12}{\sqrt{9}}$ **M** — Sub. 9 into derivative

$= 27 - \frac{12}{3}$

$= 23$ **CA**

(5)

[16]

QUESTION 3

(a)(1) (– 3 ; 5) **A**

(2) (3 ; 7) **A**

(3) (2 ; 5) **A**

(4) (5 ; 3) **A**

(5) (9 ; 15) **A** (5)

(b) $g(x) = 3x - 2$

(1) $g^{-1}: x = 3y - 2$ **M** $x \leftrightarrow y$

$3y = x + 2$

$y = \dfrac{x+2}{3}$ **A** (2)

(2) $\dfrac{1}{g(x)}$

$= \dfrac{1}{3x - 2}$ **A** (1)

(3) $g\left(\dfrac{1}{x}\right)$

$= \dfrac{3}{x} - 2$ **A** (1)

(c)(1) $p = \log\left(10 + \dfrac{1980}{2}\right)$ **M** Sub. for q

$= 3$ **A**

Total price $= 3 \times 1\,980$

$= \text{R}5\,940$ **CA** (3)

(2) $2 = \log\left(10 + \dfrac{q}{2}\right)$ **M** Setting $p = 2$

$10 + \dfrac{q}{2} = 10^2$ **A**

$\dfrac{q}{2} = 90$

$q = 180$ **A** (3)

[15]

QUESTION 4

(a)(1) $f(x) = x^3 - 3x + 2$

At A & B, $f'(x) = 0$ **M** — Derivative = 0

$3x^2 - 3 = 0$ **A**

$x^2 = 1$

$x = \pm 1$ **A**

$f(1) = 1 - 3 + 2$ **M** — Sub. each x into f

$= 0$

$f(-1) = -1 + 3 + 2$

$= 4$

A(– 1 ; 4) , B(1 ;0) **CA**

(5)

(2) C(0 ; 2) **A**

At D : $f(x) = 0$ **A**

$f(x) = (x - 1)(x^2 + x - 2)$ **M** — Factorising

$= (x - 1)(x - 1)(x + 2)$

$= (x - 1)^2(x + 2)$ **A**

$\therefore$ D(– 2 ; 0) **CA**

(5)

(3) Average gradient

$= \dfrac{4 - 0}{-1 - 1}$ **M** — Sub. in coord. for A + B found in (1)

$= -2$ **CA**

(2)

(4) $f'(x) > 0$ for $x < -1$ or $x > 1$

A **A**

(2)

(b) $f(x) = x^3 - 3x^2 + 3x - 1$

(1) f is decreasing when $f'(x) < 0$ **A**

$f'(x) = 3x^2 - 6x + 3$ **A**

$= 3(x^2 - 2x + 1)$

$= 3(x - 1)^2$ **A**

$(x - 1)^2 \geq 0, \quad x \varepsilon R$

$3(x - 1)^2 \geq 0$ **M** — Proofing

$\therefore$ $f'(x)$ is NEVER < 0

So f is never decreasing.

(4)

(2) $f'(1) = 0$ **A**

$f''(x) = 6x - 6$

$f''(1) = 0$

A

So $f''(x) = 0$ and $f'(x)$ does not change sign when $x = 1$ $\therefore$ There is a point of inflection.

(2)

[20]

QUESTION 5

(a)

$$12\,000(1-i)^3 = 7\,500$$ A

$$(1-i)^3 = \frac{5}{8}$$

$$1-i = \sqrt[3]{\frac{5}{8}}$$ M Cube Rooting

$$= 0{,}8549879...$$

$$i = 0{,}145012...$$ A

$$\approx 14{,}5\%$$ CA

(4)

(b)

$$110\,400 = \frac{x\left[1-\left(1+\frac{0{,}1}{12}\right)^{-60}\right]}{\frac{0{,}1}{12}}$$ M Loan = Pv of Annuity

$\frac{0{,}1}{12}$ A

$$x\{47{,}065369...\}$$ M Solving

$$x = \text{R } 2\,345{,}67$$ A

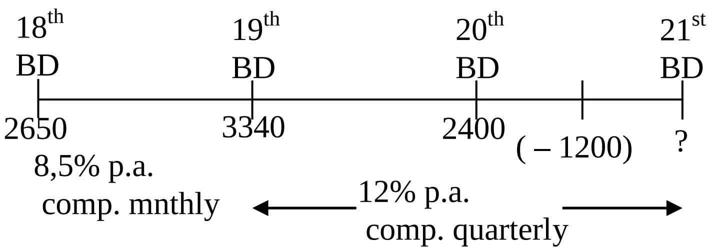
(4)

(c)(1) Deposit = 8% of 120 000

= R9 600 A

(1)

(2)



M Time frames

A

(2)

(3)

$$2650\left(1+\frac{0{,}085}{12}\right)^{12}\left(1+\frac{0{,}12}{4}\right)^{8}$$ A

$$+\ 3340\left(1+\frac{0{,}12}{4}\right)^{8} + 2400\left(1+\frac{0{,}12}{4}\right)^{4}$$ A

$$-\ 1200\left(1+\frac{0{,}12}{4}\right)^{2}$$ A

$$= 9\,312{,}816...$$ A

CA

$\therefore$ Ayanda did NOT have enough.

(5)

[16]

QUESTION 6

(a)(1) $3x + y \geq 18$

$\Rightarrow \quad y \geq -3x + 18$ M — Manipulating constraints

$5x + 4y \leq 60$

$\Rightarrow \quad y \leq \dfrac{-5x}{4} + 15$

$2x + 3y \geq 30$

$\Rightarrow \quad y \geq \dfrac{-2x}{3} + 10$

$\therefore$ Region D A (2)

(2) $x \geq 0; \quad y \geq 0$ A A (2)

(3) $x = 8$ A (1)

(b)(1) $P = 2x + y$

$y = -2x + P$ M — Making y Subject of formula

$m_{CD} = \dfrac{7 - 2}{3 - 6}$

$= \dfrac{5}{-3}$ A

Max. at D A

$\therefore \quad P = 2 \times 6 + 6$

$= 14$ CA (4)

(2) $Q = 3 \times 1 + 5$ M — Sub cords of B into Q

$= 8$ A (2)

(3) $m_{BA} = \dfrac{3 - 5}{2 - 1}$

$= -2$ A

$m_{AD} = \dfrac{3 - 2}{2 - 6}$

$= -\dfrac{1}{4}$ A

$\mathrm{R} = mx + y$

$-2 \leq m \leq -\dfrac{1}{4}$ A A

(4)

[15]

QUESTION 7

(a) $h(x) = \dfrac{2}{x+3} - 1$

(1) Domain: $x \varepsilon R, x \neq -3$ A
Range: $y \varepsilon R, y \neq -1$ A (2)

(2) $y = x + 3 - 1$ M — Transforming $y = x$
$= x + 2$ A
and
$y = -(x+3) - 1$ M — Transforming $y = -x$
$= -x - 4$ A (4)

Alternatively:
Axes of symmetry pass through (–3; –1) A
And have gradients 1 and –1. A

$y - (-1) = x - (-3)$
$y + 1 = x + 3$
$y = x + 2$ A

And

$y + 1 = -(x + 3)$
$y = -x - 4$ A

(b) $y = \dfrac{x^2}{10} + 3 = \dfrac{2x}{15} + \dfrac{7}{2}$ M — Equating equations

$3x^2 + 90 = 4x + 105$
$3x^2 - 4x - 15 = 0$ A
$(3x + 5)(x - 3) = 0$
$x = -\dfrac{5}{3}$ or $x = 3$ A

$y = \dfrac{2}{15}\left(-\dfrac{5}{3}\right) + \dfrac{7}{2}$ M — Sub into linear expression
$= \dfrac{59}{18}$ $(3{,}27)$ A

$y = \dfrac{2}{15} \times 3 + \dfrac{7}{2}$
$= \dfrac{39}{10}$ $(3{,}9)$ A

Diff. = 0,6222 m
≈ 62 cm CA (7)

[13]

QUESTION 8

(a) $5 - 10x + 20x^2 - 40x^3 + \ldots$

(1) $r = -2x$ **A**

For convergence : $-1 < -2x < 1$ **M** Interval for ratio

$\therefore \ -\frac{1}{2} < x < \frac{1}{2}$ **A**

(3)

(2) $S_\infty = \frac{5}{1-(-2x)} = 100$ — $S_\infty = \frac{5}{1+2x}$ **A**

$100(1 + 2x) = 5$ **M** Solving

$2x = \frac{1}{20} - 1$

$= -\frac{19}{20}$

$x = -\frac{19}{40}$ **A**

(3)

(b)(1) Str. Line : $y = -2x + 6$ ✓**M** Finding equation of str. line

$QR = -2k + 6$ **A**

(2)

(2) Area = OR.QR

$\therefore \ = x(-2x + 6)$

$= -2x^2 + 6x$ **A**

Max. Area when $\frac{dA}{dx} = 0$

$-4x + 6 = 0$ **M** Derivative = 0

$-4x = -6$

$x = \frac{3}{2}$ **A**

$y = -2 \times \frac{3}{2} + 6$ **M** Sub x into QR expression

$= 3$

$\therefore$ Q(1,5 ; 3) **CA**

(5)

Alternatively:

Area = $k(-2k + 6)$ **A**

Roots at $k = 0$ or $k = 3$

Max. Area halfway between roots

$\therefore k = \frac{0 + 3}{2}$ **M**

$= \frac{3}{2}$

$\therefore x_Q = \frac{3}{2}$ **A**		
$y_Q = -2 \times \frac{3}{2} + 6$ **M**		
$= 3$ **CA**		
(3) Max. Area $= \frac{3}{2} \times 3$ **M**	Sub into Area expression	
$= \frac{9}{2}$ (4,5) **CA**		(2)
		[20]

QUESTION 9

(a)(1) $T_5 + T_6$

$= 5^2 - 1 + 22 - 3 \times 6$ **M** **A**

$= 24 + 4$

$= 28$ **CA**

(3)

(2) $n^2 - 1 \geq -1$

For $T_k = -2$, k must be even **A**

$T_k = 22 - 3k = -2$ **M** Even formula = -2

$-3k = -24$

$k = 8$ **A**

(3)

(b)(1) $T_n = a + (n-1)d$ **M** Sub into T*n* of AP

$= 30 + (n-1)(-3)$

$= 30 - 3n + 3$

$= 33 - 3n$ **A**

(2)

(2) $T_p + T_q = 0$ **M** Sum of expressions from (1) = 0

$33 - 3p + 33 - 3q = 0$

$-3(p + q) = -66$ **M** Finding p i.t.o. q

$p + q = 22$

$p = 22 - q$ **A**

$1 \leq q \leq 21, \quad q \varepsilon N$ **A**

(4)

[12]