GRADE 10 IEB STANDARDISATION PROJECT
NOVEMBER 2012

MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 2 hours | 100 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

SECTION A

QUESTION 1

The mathematics marks obtained by the 93 pupils from a certain school are represented below:

Interval	Frequency
$0 \le x < 10$	0
$10 \le x < 20$	0
$20 \le x < 30$	4
$30 \le x < 40$	5
$40 \le x < 50$	8
$50 \le x < 60$	12
$60 \le x < 70$	23
$70 \le x < 80$	16
$80 \le x < 90$	10
$90 \le x < 100$	15

(a) What is the modal interval?

$60 \le x < 70$ ✓ (1)

(b) In which interval does the median lie?

$60 \le x < 70$ ✓ (1)

(c) In which interval does the lower quartile lie?

$50 \le x < 60$ ✓ (1)

(d) Give an estimate for the mean mark. You should show working in order to demonstrate your understanding of the process.
Express your answer correct to one decimal place.

$$mean = \frac{25\times4+35\times5+45\times8+55\times12+65\times23+75\times16+85\times10+95\times15 \checkmark}{93 \checkmark}$$

$$= \frac{6\ 265}{93} \checkmark = 67{,}4 \checkmark$$

(4)

(e) The actual mean of the data is known to be 66.8% correct to one decimal place. Explain why your estimate in (d) differs from the exact answer.

With the grouped data we don't have the actual values so we are using the mid-points of the intervals as our best estimate. ✓ (1)

[8]

QUESTION 2

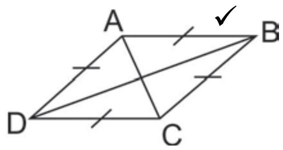
(a) Indicate whether each of the following statements is TRUE or FALSE. If false, give a neat sketch of a counter example.

(1) All rectangles are parallelograms

TRUE ✓ (1)

(2) The diagonals of a rhombus are equal in length.

FALSE ✓

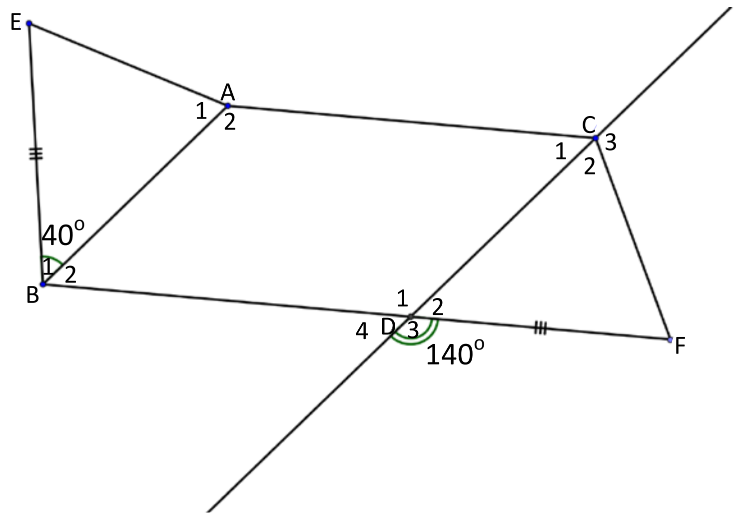


$AC \neq BD$ (2)

(3) If the corresponding sides of two triangles are in the same proportion then the triangles must be equiangular.

TRUE ✓ (1)

(b) In the diagram below ABDC is a parallelogram. $BE = DF$, $\hat{B}_1 = 40°$, $\hat{D}_3 = 140°$. Prove that $AE = CF$



$\hat{D}_2 = 40°$ ($\angle$'s on str. line) ✓

$\therefore \hat{B}_1 = \hat{D}_2$

$AB = CD$ (opp. sides of parallelogram.) ✓

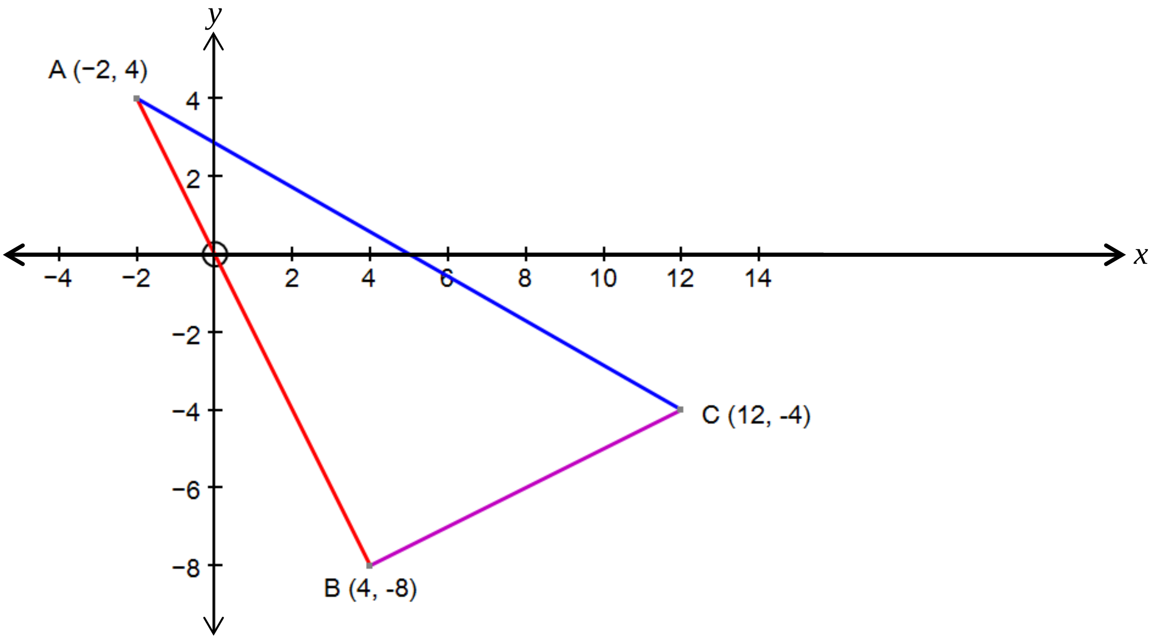
$BE = DF$ (given) ✓

$\therefore \Delta ABE \equiv \Delta CDF$ (SAS) ✓ (5)

$\therefore AE = CF$ (congruency) ✓ **[9]**

QUESTION 3

Consider the diagram below:



(a) Show that ΔABC is right-angled at B.

$mAB = \frac{-12}{6} = -2$ ✓

$mBC = \frac{4}{8} = \frac{1}{2}$ ✓

$As \quad mAB \times mBC = -1, \quad AB \perp BC$ ✓

(3)

(b) Determine the coordinates of P and Q, the mid-points of AB and AC respectively.

$P(1;-2)$ ✓ and $Q(5;0)$ ✓

(2)

(c) Use analytical methods to show that the line joining P and Q is parallel to BC.

$P(1;-2) \quad and \quad Q(5;0)$

$\therefore mPQ = \frac{-2-0}{1-5} = \frac{1}{2}$ ✓ ✓

$since \quad mPQ = mBC,$ ✓ $PQ \parallel BC$ ✓

(4)

(d) Use analytical methods to prove that $PQ = \frac{1}{2}BC$.

$PQ = \sqrt{(5-1)^2 + (0-(-2))^2} = \sqrt{20} = 2\sqrt{5}$ ✓ ✓

$BC = \sqrt{(12-4)^2 + (-4-(-8))^2} = \sqrt{80} = 4\sqrt{5}$ ✓

$PQ = \frac{1}{2}BC$ ✓

(4)

(e) Determine the coordinates of D if $ABCD$ is a rectangle.

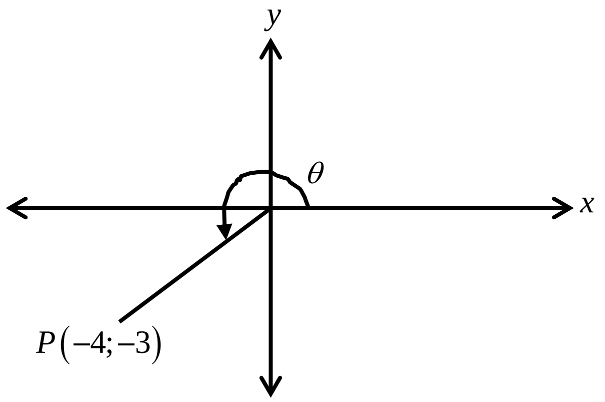
$D(6;8)$ ✓✓

(2)

[15]

QUESTION 4

(a) Consider the diagram below:



Without the use of a calculator,

(1) determine $\cot\theta$, expressing your answer as a fraction.

$\cot\theta = \frac{4}{3}$ ✓

(1)

(2) determine $\sin\theta$, expressing your answer as a fraction.

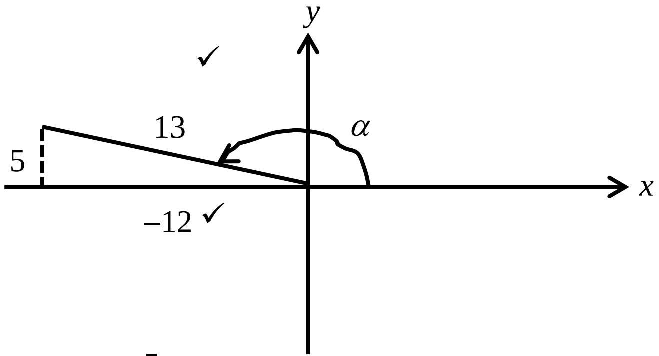
$r = \sqrt{3^2 + 4^2} = 5$ ✓

$\therefore \sin\theta = \frac{-3}{5}$ ✓

(2)

(b) Calculate $\tan\alpha$, if it is given $\sin\alpha = \frac{5}{13}$ *and* $\cos\alpha < 0$

Hint: use a sketch.



$\tan\alpha = \frac{5}{-12}$ ✓✓

(4)

(c) Calculate $\sin 60° + \text{cosec}30° + \tan 45°$ without a calculator. Express your answer as a single fraction.

$$\sin 60° + \text{cosec}30° + \tan 45°$$
$$= \frac{\sqrt{3}}{2} + 2 + 1 \checkmark \checkmark$$
$$= \frac{\sqrt{3}+6}{2} \checkmark$$

(3)

(d) Calculate $\theta \in [0°;90°]$ without a calculator if $\sin\theta = \cos 0° + \sin 180° - \cos 60°$

$$\sin\theta = \cos 0° + \sin 180° - \cos 60°$$
$$\therefore \sin\theta = 1 + 0 - \frac{1}{2} = \frac{1}{2} \checkmark \checkmark$$
$$\therefore \theta = 30° \checkmark$$

(3)

(e) Solve the following equations for $\theta \in [0°;90°]$. Give your answers to one decimal place:

(1) $\sin\theta = 0,234$

$\therefore \theta = 13,5° \checkmark$ (1)

(2) $\cot\theta = \tan 53° + \sin 233°$

$\therefore \cot\theta = 0,528... \checkmark \checkmark$

$\therefore \tan\theta = 1,892... \checkmark$

$\therefore \theta = 62,1° \checkmark$

(4)

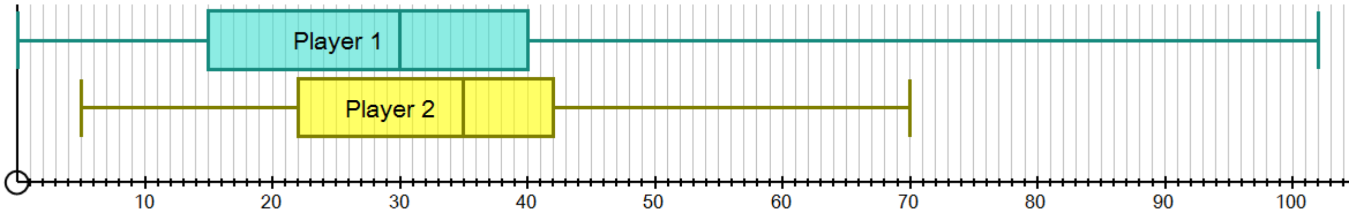
[18]

50 marks

SECTION B

QUESTION 5

The box and whisker plots for the scores made by two cricketers during the past season are as follows:



You do not need any cricketing knowledge for this question other than to know that a high score is better than a low score!

(a) Determine the inter-quartile range for player 1.

$IQR = 40 - 15 = 25$ ✓ ✓ (2)

(b) Player 2 makes the following claim:
"I score more than x in half of my matches"
What is the value of *x*?

$x = 35$ ✓ (2)

(c) Assuming that both players have played a similar number of matches, which player would you select for your team? Justify your answer by referring to statistical concepts you have learnt.

I would select player 2. ✓ He is more consistent, smaller inter-quartile range. ✓ Has a higher median number of runs. (2)

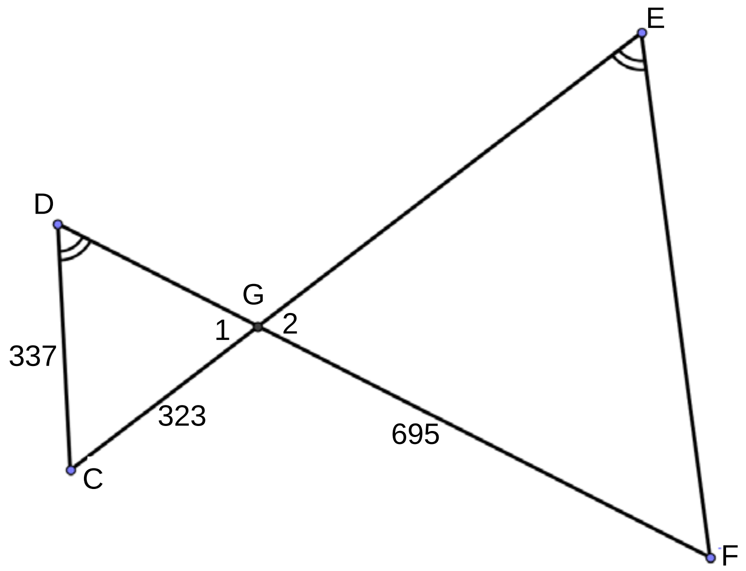
[6]

QUESTION 6

Determine the length of EF to the nearest meter if the following lengths are given.

CG is 323 m
CD is 337 m
GF is 695 m

DF and CE are straight lines.
You are also given that $\hat{D} = \hat{E}$.



You should show all calculations.

$In \quad \Delta's \quad DGC \quad and \quad EGF$

$\hat{D} = \hat{E} \quad (given)$

$\hat{G}_1 = \hat{G}_2 \quad (vert. \quad opp. \quad \angle's)$ ✓

$\hat{C} = \hat{F} \quad (3^{rd} \quad \angle \quad of \quad \Delta)$ ✓

$\therefore \Delta DGC \,|||\, \Delta EGF \quad (AAA)$ ✓

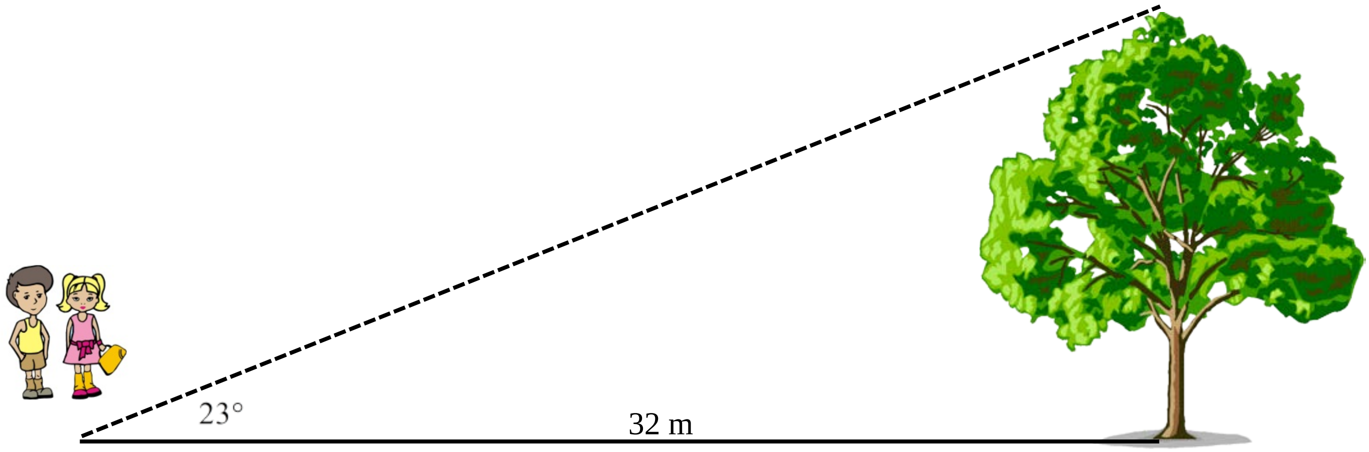
$\therefore \frac{EF}{DC} = \frac{GF}{GC} \quad (||| \quad \Delta's)$ ✓

$\therefore EF = \frac{GF \times DC}{GC} = \frac{695 \times 337}{323} = 725\,m$ ✓ ✓

[6]

QUESTION 7

(a) Sipho and Mary are trying to work out the height of a tree. They are standing 32 m away from the base of the tree.



[Images taken from <http://www.proprofs.com/flashcards/story.php?title=onebuntwoshoethreetree-mnemonic> and <http://www.ignyte.com/portfolio-graphics-illustrations-2.html>]

Sipho has measured the angle of inclination of the top of the tree to be $23°$.

(1) Sipho says that he can use trigonometry with the given information to calculate the height of the tree.
Show the method Sipho might use and calculate the height of the tree in meters to one decimal place.

$$\tan 23° = \frac{ht \quad of \quad tree}{32} \checkmark \checkmark$$

$$\therefore ht \quad of \quad tree = 32\tan 23° = 13,6 \ m \checkmark$$

(3)

(2) Mary says that she has not started trigonometry yet but that she can calculate the height of tree using similar triangles. She measures the length of the tree's shadow as 6,27 m. Then she measures her shadow as 77 cm. Mary uses her height of 1,67 m with the other information to correctly calculate the height of the tree. Show the method Mary might use and calculate the height of the tree in meters to one decimal place.

$$\frac{tree \quad height}{tree \quad shadow} = \frac{Mary's \quad height}{Mary's \quad shadow} \checkmark$$

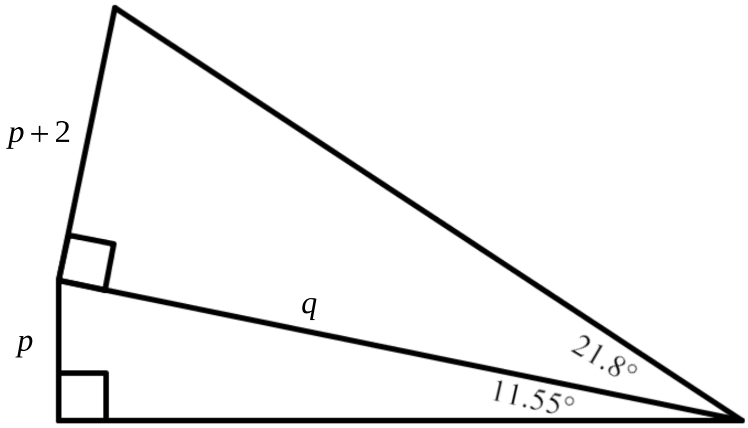
$$\therefore tree \quad height = \frac{Mary's \quad height \times tree \quad shadow}{Mary's \quad shadow} \checkmark$$

$$= \frac{1,67 \times 6,27}{0,77} \checkmark$$

$$= 13,6 \quad m \checkmark$$

(4)

(b) Solve for q to one decimal place in the diagram below:



$$\sin 11.55° = \frac{p}{q}$$
$$\therefore\ p = q\sin 11.55° \quad \checkmark$$
$$\tan 21.8° = \frac{p+2}{q} \quad \checkmark$$
$$\therefore\ \tan 21.8° = \frac{q\sin 11.55° + 2}{q} \quad \checkmark$$
$$\therefore\ q\tan 21.8° = q\sin 11.55° + 2$$
$$\therefore\ q\tan 21.8° - q\sin 11.55° = 2 \quad \checkmark$$
$$\therefore\ q\left(\tan 21.8° - \sin 11.55°\right) = 2 \quad \checkmark$$
$$\therefore\ q = \frac{2}{\tan 21.8° - \sin 11.55°} = 10 \quad units \quad \checkmark$$

(6)

[13]

QUESTION 8

A bug crawls a distance of $\sqrt{40}$ units on a straight line in the Cartesian Plane. If the gradient of the line is $-\frac{1}{3}$ and the bug ends up at the point $\mathrm{T}(2;-5)$, determine the co-ordinates of the point where it started?

Let the horizontal movement be *a* and the vertical movement be *b* then:

$\frac{b}{a} = -\frac{1}{3}$ ✓

$\therefore a = -3b$

$but \quad \sqrt{a^2+b^2} = \sqrt{40}$ ✓

$\therefore (-3b)^2 + b^2 = 40$ ✓

$\therefore 9b^2 + b^2 = 40$

$\therefore b^2 = 4$

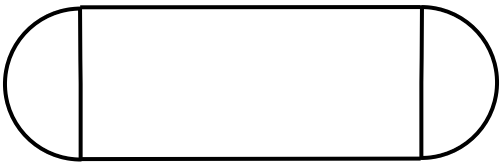
$\therefore b = \pm 2 \quad and \quad a = \pm 6$ ✓

so, bug crawled down 2 *and across* 6

so, the bug started at $(-4;-3)$ ✓ *or* $(8;-7)$ ✓

[6]

QUESTION 9



Given: $V = \frac{4}{3}\pi r^3 \quad V = \pi r^2 h$

A large three-dimensional capsule is made up of two hemispheres and a cylinder. The radius of the hemispheres is 4,22 m.

(a) If the overall length of the capsule is *l*, give an expression for the volume of the capsule in terms of *l*.

$V = \frac{4}{3}\pi \times 4.22^3$ ✓ $+ (l - 2\times 4.22)(\pi \times 4.22^2)$ ✓

(2)

(b) If the volume of the capsule is 1 521 m^3 then determine the value of *l* to the nearest meter.

$1521 = \frac{4}{3}\pi \times 4.22^3 + (l - 2\times 4.22)(\pi \times 4.22^2)$ ✓

$\therefore l = 2\times 4.22 + \frac{1521 - \frac{4}{3}\pi \times 4.22^3}{(\pi \times 4.22^2)}$ ✓✓

$\therefore l = 30m$ ✓

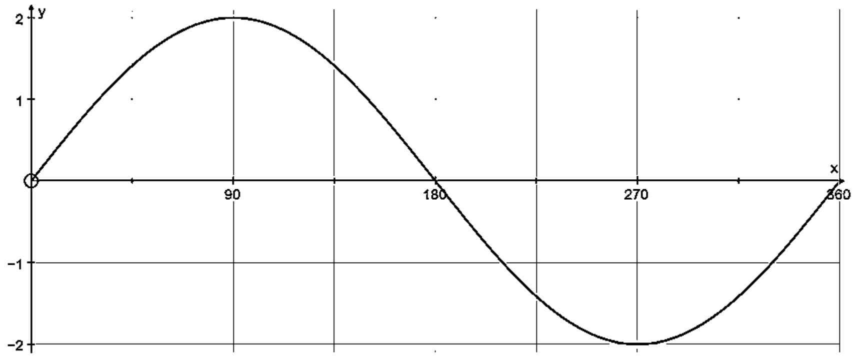
(4)

[6]

QUESTION 10

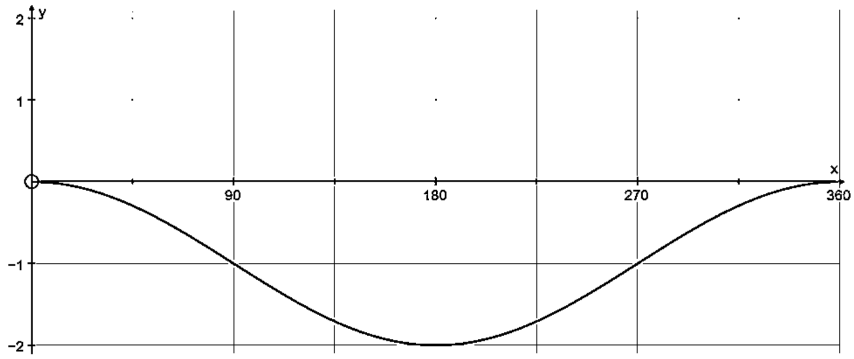
(a) Give the equations of each of the following graphs:

(1)



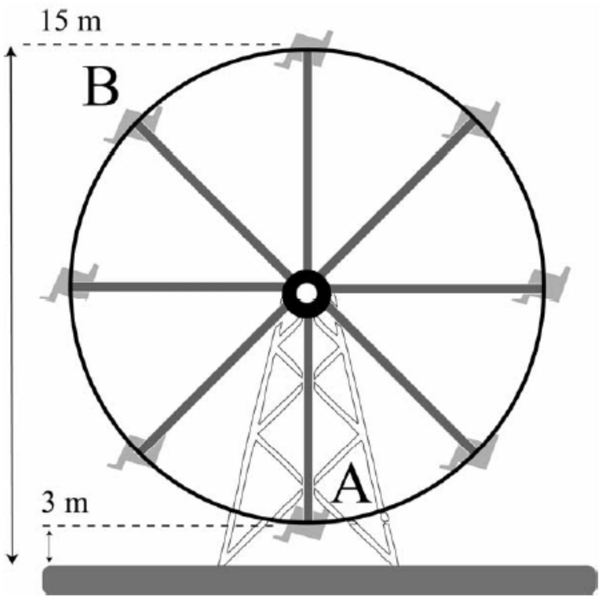
✓ ✓
$y = 2\sin x$ (2)

(2)



✓ ✓
$y = \cos x - 1$ (2)

(b) A Ferris Wheel at an amusement park has riders get on at position A, which is 3 metres above the ground.
The highest point of the ride is 15 metres above the ground.
The ride takes 40 seconds for one complete revolution.



The height of a rider above the ground can be modeled by the formula: $h(\theta) = a\cos\theta + b$ for $\theta \in [0°; 360°]$

(1) Show that $a + b = 3$

When the Ferris wheel is on the ground:

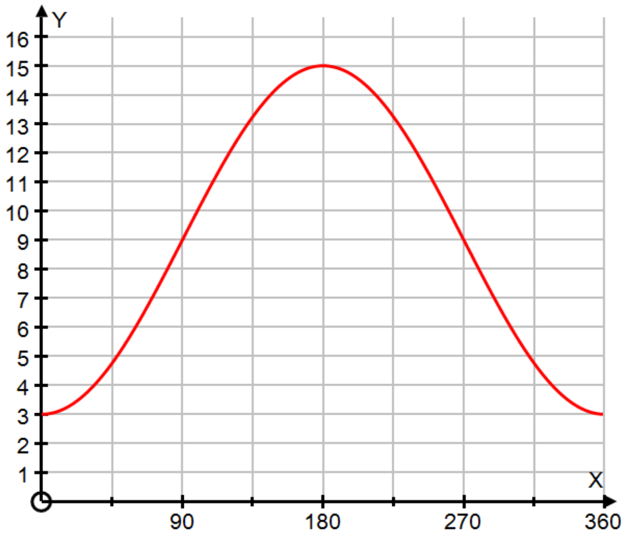
$\theta = 0°$ *and* $h = 3$ ✓
so $3 = a\cos 0° + b$ ✓
$\therefore a + b = 3$ (2)

(2) Show that $-a + b = 15$

When the Ferris wheel is at the highest point:

$\theta = 180°$ *and* $h = 15$ ✓
so $18 = a\cos 180° + b$ ✓
$\therefore -a + b = 15$ (2)

(3) If a rider makes 3 complete revolutions, determine the amount of time (to the nearest second) spent above 13 metres above the ground.
Assume that $h(\theta) = -6\cos\theta + 9$ for $\theta \in [0°;360°]$



$-6\cos\theta + 9 = 13$ ✓

$\therefore -6\cos\theta = 4$

$\therefore \cos\theta = -\frac{2}{3}$ ✓

$\therefore key = \cos^{-1}\left(\frac{2}{3}\right) = 48,2°$

so $\theta = 131.8°$ ✓ *or* $228,2°$ ✓

so, per revolution the rider spends

$\frac{228,2° - 131,8°}{360°} \times 40 = 10.7$ ✓ *seconds above* 13 *m*

in whole ride, rider spends 32 ✓ *seconds above* 13 *m*

(5)
[13]

50 marks

Total: 100 marks