basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

EXEMPLAR 2014

MEMORANDUM

MARKS: 150

This memorandum consists of 13 pages.

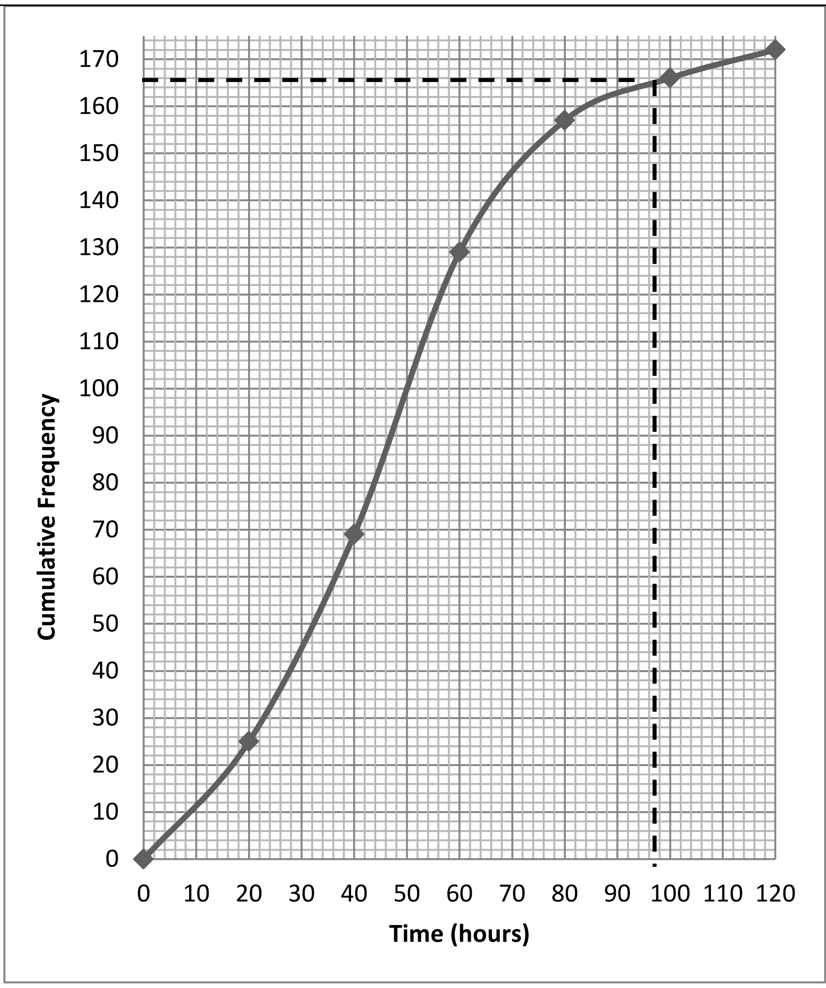
NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum.
- Assuming answers/values in order to solve a problem is NOT acceptable.

QUESTION 1

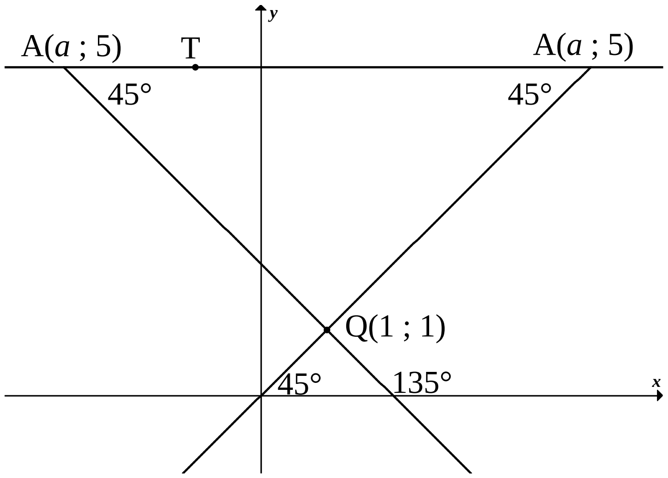
1.1	As the number of days that an athlete trained increased, the time taken to run the 100m event decreased. **OR** The fewer number of days an athlete trained, the longer the time he took to complete the 100m sprint. **OR** The greater number of days an athlete trained, the shorter the time he ran the 100m sprint.	✓ explanation (1)
1.2	(60 ; 18,1)	✓ (1)
1.3	$a = 17{,}81931464\ldots$ $b = -\,0{,}070685358...$ $\therefore\ \hat{y} = -\,0{,}07x + 17{,}82$	✓✓ a ✓ b ✓ equation (4)
1.4	$\therefore\ \hat{y} \approx -\,0{,}07(45) + 17{,}82$ $\approx 14{,}67$ seconds	✓ substitution ✓ answer (2)
1.5	$r = -\,0{,}74\ (-0{,}740772594...)$	✓✓ r (2)
1.6	There is a moderately strong relationship between the variables.	✓ moderately strong (1) **[11]**

QUESTION 2

2.1		✓grounding at 0 ✓plotting at upper limits ✓ smooth shape of curve (3)
2.2	$40 \le t < 60$	✓ class (1)
2.3	(96 ; 164) $\therefore$ 172 – 164 = 8 learners	✓164 ✓ 8 (2)
2.4	Frequency: 25; 44; 60; 28; 9; 6 Mean = $\dfrac{25\times10+44\times30+60\times50+28\times70+9\times90+6\times110}{172}$ $= \dfrac{8000}{172}$ = 46,51 hours	✓frequency ✓ midpoints ✓ $\dfrac{8000}{172}$ ✓ answer (4) **[10]**

QUESTION 3

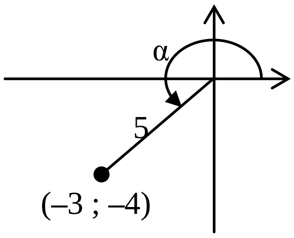
3.1	K(7 ; 0)	✓ answer (1)
3.2	$1=\frac{x_M+7}{2}$ *and* $1=\frac{y_M+3}{2}$ $\therefore$M(– 5 ; – 1)	✓ x ✓ y (2)
3.3	$m_{PM}=\frac{3-1}{7-1}$ $=\frac{1}{3}$	✓ substitution ✓answer (2)
3.4	$\tan\,\mathrm{P\hat{S}K}=m_{PM}=\frac{1}{3}$ $\mathrm{P\hat{S}K}=\tan^{-1}\left(\frac{1}{3}\right)=18{,}43°$ $\therefore\ \theta$ = 180° – 90° – 18,43° = 71,57°	✓ $\tan\mathrm{P\hat{S}K}=m_{PM}$ ✓ $\mathrm{P\hat{S}K}$ ✓ θ (3)
3.5	cos 71,57° = $\frac{\mathrm{PK}}{\mathrm{PS}}=\frac{3}{\mathrm{PS}}$ PS = $\frac{3}{\cos 71{,}57°}$ = 9,49 units **OR** sin 18,43° = $\frac{\mathrm{PK}}{\mathrm{PS}}=\frac{3}{\mathrm{PS}}$ PS = $\frac{3}{\sin 18{,}43°}$ = 9,49 units	✓ correct ratio ✓ PS as subject ✓ answer (3) ✓ correct ratio ✓ PS as subject ✓ answer (3)
3.6	N(x ; –2x + 17) $m_{TN}=m_{PM}$ (TN | | PM) $\frac{-2x+17-5}{x-(-1)}=\frac{1}{3}$ $-6x+36=x+1$ $-7x=-35$ $x=5$ $\therefore\ y$ = –2(5) + 17 = 7 $\therefore$N(5 ; 7) **OR**	✓ N in terms of x ✓ equal gradients ✓ substitution ✓ x -value ✓ y -value (5)

	$m_{TM} = \frac{1}{3}$ (TN | | PM) equation of TM: $y - y_1 = \frac{1}{3}(x - x_1)$ $y - 5 = \frac{1}{3}(x - (-1))$ $y - 5 = \frac{1}{3}x + \frac{1}{3}$ $y = \frac{1}{3}x + 5\frac{1}{3}$ **OR** $y = \frac{1}{3}x + c$ $5 = \frac{1}{3}(-1) + c$ $5\frac{1}{3} = c$ $y = \frac{1}{3}x + 5\frac{1}{3}$ $-2x + 17 = \frac{1}{3}x + 5\frac{1}{3}$ $-2\frac{1}{3}x = -11\frac{2}{3}$ $x = 5$ $\therefore\ y = -2(5) + 17 = 7$ $\therefore$ N(5 ; 7)	✓ m_{TM} ✓ equation of TM ✓ equating ✓ x -value ✓ y -value (5)
3.7.1	$y = 5$	✓ equation (1)
3.7.2	 gradient of AQ = tan 45° or tan 135° = 1 or –1 $m_{AQ} = \frac{5-1}{a-1} = \pm 1$ $\therefore\ a - 1 = 4$ or -4 $\therefore\ a = 5$ or -3	✓ $m_{AQ} = 1$ or ✓ $m_{AQ} = -1$ ✓ substitution into gradient formula ✓ x-value ✓ y-value (5) **[22]**

QUESTION 4

4.1	M(–1 ; –1)	✓ answer (1)
4.2	$m_{NT} = \frac{2-1}{3-4} = -1$ $\therefore\ m_{AT} = 1$ (radius ⊥ tangent) $y-1 = 1(x-4)$ $y = x-3$	✓ m_{NT} ✓ m_{AT} ✓ reason ✓ substitution of m and (4 ; 1) ✓ equation (5)
4.3	MR ⊥ AB (line from centre to midpt of chord) $MB^2 = MR^2 + RB^2$ (Theorem of Pythagoras) $9 = (\frac{\sqrt{10}}{2})^2 + RB^2$ $RB^2 = \frac{13}{2}$ $RB = \sqrt{\frac{13}{2}}$ $AB = 2\left(\sqrt{\frac{13}{2}}\right) = \sqrt{26}$ *units*	✓ MR ⊥ AB ✓MB = 3 ✓ substitution into Theorem of Pythagoras ✓ AB in surd form (4)
4.4	$MN^2 = (-1-3)^2 + (-1-2)^2$ $= 16 + 9$ $= 25$ MN = 5 units	✓ substitution into distance formula ✓ answer (2)
4.5	$r = 5 - 3 = 2$ units $\therefore\ (x-3)^2 + (y-2)^2 = 4$ $\therefore\ x^2 + y^2 - 6x - 4y + 9 = 0$	✓r ✓substitution into circle equation ✓equation (3) **[15]**

QUESTION 5

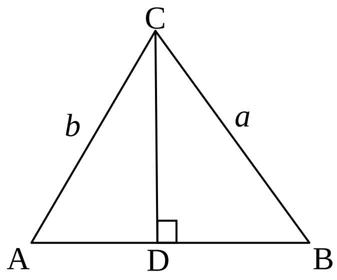
5.1.1	$-\sin\alpha$ $= -(-\frac{4}{5}) = \frac{4}{5}$	✓ reduction ✓ answer (2)
5.1.2	$(-4)^2 + b^2 = 5^2$ $b^2 = 25 - 16 = 9$ $b = -3$ $\cos\alpha = \frac{-3}{5}$  	✓ $b = -3$ ✓answer (2)
5.1.3	$\sin(\alpha - 45°)$ $= \sin\alpha\cos 45° - \cos\alpha\sin 45°$ $= -\frac{4}{5}\cdot\frac{1}{\sqrt{2}} - (-\frac{3}{5})\cdot\frac{1}{\sqrt{2}}$ $= -\frac{1}{5\sqrt{2}}$ **OR** $\sin(\alpha - 45°)$ $= \sin\alpha\cos 45° - \cos\alpha\sin 45°$ $= -\frac{4}{5}\cdot\frac{\sqrt{2}}{2} - (-\frac{3}{5})\cdot\frac{\sqrt{2}}{2}$ $= -\frac{\sqrt{2}}{10}$	✓ expansion ✓ $\frac{1}{\sqrt{2}}$ ✓ answer in simplest form (3) ✓ expansion ✓ $\frac{\sqrt{2}}{2}$ ✓ answer in simplest form (3)
5.2.1	$LHS = \frac{8\sin x.\cos x}{\sin^2 x - \cos^2 x}$ $= \frac{4(2\sin x.\cos x)}{\sin^2 x - \cos^2 x}$ $= \frac{4\sin 2x}{-(\cos^2 x - \sin^2 x)}$ $= \frac{4\sin 2x}{-\cos 2x}$ $= -4\tan 2x$	✓ sin x ✓ cos x ✓ $\cos^2 x$ ✓ 4 sin 2x ✓ factorise ✓ – cos 2x (6)
5.2.2	Undefined when cos 2x = 0 or tan 2x = ∞: x = 45° and x = 135°	✓ 45° ✓ 135° (2)

5.3	$1-2\sin^2\theta+4\sin^2\theta-5\sin\theta-4=0$ $2\sin^2\theta-5\sin\theta-3=0$ $(2\sin\theta+1)(\sin\theta-3)=0$ $\therefore\ \sin\theta=-\frac{1}{2}$ or $\sin\theta=3$ (no solution) $\therefore\theta=210°+360°k$ or $\theta=330°+360°k$; $k\in Z$ **OR** $\therefore\theta=210°+360°k$ of $\theta=30°+360°k$; $k\in Z$	✓ $1-2\sin^2\theta$ ✓ standard form ✓ factors ✓ no solution ✓ 210° ✓ 330° ✓ + 360°k ; $k\in Z$ (7) **[22]**

QUESTION 6

6.1	$b=\frac{1}{2}$	✓ value of b (1)
6.2	A(30° ; 1)	✓ 30° ✓ 1 (2)
6.3	x = 160°	✓ x =160° (1)
6.4	$h(x)=2\cos(x-30°)+1$ $y\in[-1;3]$ **OR** $-1\leq y\leq 3$	✓ critical values ✓ notation (2) **[6]**

QUESTION 7

7.1	Draw CD $\perp$ AB In ΔACD: $\sin A = \dfrac{CD}{b}$ $\quad \therefore$ CD = b. sin A In ΔCBD: $\sin B = \dfrac{CD}{a}$ $\quad \therefore$ CD = a. sin B $\therefore$ b . sin A = a. sin B $\therefore \dfrac{\sin A}{a} = \dfrac{\sin B}{b}$  	✓ construction ✓ sin A ✓ making CD the subject ✓ sin B ✓ b. sin A = a. sin B (5)
7.2.1	$S\hat{P}Q = 180° - 2x$ (opp $\angle$s of cyclic quad) $P\hat{S}Q + P\hat{Q}S = 2x$ (sum of $\angle$s in Δ) $P\hat{S}Q = P\hat{Q}S = x$ ($\angle$s opp equal sides)	✓ $S\hat{P}Q = 180° - 2x$ (S/R) ✓ reason (2)
7.2.2	$\dfrac{\sin S\hat{P}Q}{SQ} = \dfrac{\sin P\hat{S}Q}{PQ}$ $\dfrac{\sin(180° - 2x)}{SQ} = \dfrac{\sin x}{k}$ $SQ = \dfrac{k \sin 2x}{\sin x}$ $SQ = \dfrac{k(2\sin x.\cos x)}{\sin x} = 2k\cos x$ **OR** $SQ^2 = PQ^2 + PS^2 - 2PQ.PS.\cos\ S\hat{P}Q$ $= k^2 + k^2 - 2.k.k.\cos(180° - 2x)$ $= 2k^2 + 2k^2\cos 2x$ $= 2k^2 + 2k^2(2\cos^2 x - 1)$ $= 4k^2\cos^2 x$ $SQ = 2k\cos x$	✓ substitution into correct formula ✓ sin $2x$ ✓ SQ subject ✓ $2\sin x.\cos x$ (4) ✓ substitution into correct formula ✓ $-\cos 2x$ ✓ $2\cos^2 x - 1$ ✓ simplification (4)
7.2.3	$\tan y = \dfrac{3}{k}$ $k = \dfrac{3}{\tan y}$ $\therefore$ $SQ = 2\cos x\left(\dfrac{3}{\tan y}\right)$ $= \dfrac{6\cos x}{\tan y}$	✓ tan ratio ✓ k subject and substitution (2) **[13]**

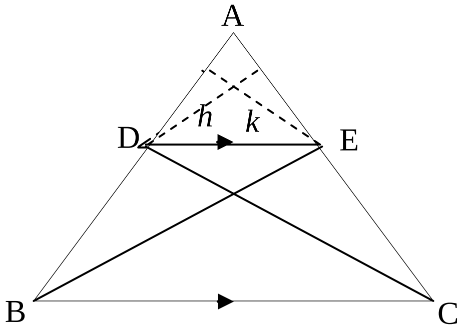
QUESTION 8

8.1	the angle subtended by the chord in the alternate segment	✓correct theorem (1)
8.2.1	$\hat{B}_1 = \hat{E}_1 = 68°$ (tan chord theorem)	✓ $\hat{E}_1 = 68°$ ✓ reason (2)
8.2.2	$\hat{E}_1 = \hat{B}_3 = 68°$ (alt $\angle$s; AE | | BC)	✓ $\hat{B}_3 = 68°$ (S/R) (1)
8.2.3	$\hat{D}_1 = \hat{B}_3 = 68°$ (ext $\angle$ of cyclic quad)	✓ $\hat{D}_1 = 68°$ ✓ reason (2)
8.2.4	$\hat{E}_2 = 20° + 68°$ (ext $\angle$ of Δ) $= 88°$	✓ $\hat{E}_2 = 88°$ (S/R) (1)
8.2.5	$\hat{C} = 180° - 88°$ (opp $\angle$s of cyclic quad) $= 92°$	✓ $\hat{C} = 92°$ ✓ reason (2) **[9]**

QUESTION 9

9.1	$\hat{D}_4 = \hat{A} = x$ (tan chord theorem) $\hat{A} = \hat{D}_2 = x$ ($\angle$s opp equal sides)	✓ $\hat{A} = x$ ✓ reason ✓ $\hat{A} = \hat{D}_2 = x$ (S/R) (3)
9.2	$\hat{M}_1 = 2x$ (ext $\angle$ of Δ) or ($\angle$ at centre = 2$\angle$ at circum) $M\hat{D}E = 90°$ (radius $\perp$ tan) $\hat{M}_2 = 90° - 2x$ $\therefore\ \hat{E} = 180° - (90° + 90° - 2x)$ (sum of $\angle$s in ΔMDE) $= 2x$ $\therefore$ CM is a tangent (converse tan chord theorem)	✓ $\hat{M}_1 = 2x$ (S/R) ✓ $M\hat{D}E = 90°$ (S/R) ✓ $\hat{E} = 2x$ ✓ reason (4)
9.3	$\hat{M}_3 = 90°$ (EM $\perp$ AC) $A\hat{D}B = 90°$ ($\angle$ in semi-circle) $\therefore$ FMBD a cyclic quad (ext $\angle$ of quad = int opp $\angle$) **OR** $E\hat{M}C = 90°$ (EM $\perp$ AC) $A\hat{D}B = 90°$ ($\angle$ in semi-circle) $\therefore$ FMBD a cyclic quad (opp $\angle$s of quad supp)	✓ $\hat{M}_3 = 90°$ ✓ $A\hat{D}B = 90°$ (S/R) ✓ reason (3) ✓ $E\hat{M}C = 90°$ ✓ $A\hat{D}B = 90°$ (S/R) ✓ reason (3)
9.4	$DC^2 = MC^2 - MD^2$ (Theorem of Pythagoras) $= (3BC)^2 - (2BC)^2$ (MB = MD = radii) $= 9BC^2 - 4BC^2$ $= 5BC^2$	✓ Th of Pythagoras ✓ substitution ✓ $9BC^2 - 4BC^2$ (3)
9.5	In ΔDBC and ΔDFM: $\hat{D}_4 = \hat{D}_2 = x$ (proven in 9.1) $\hat{B}_1 = \hat{F}_2$ (ext $\angle$ of cyclic quad) $\hat{C} = \hat{M}_2$ $\therefore \Delta DBC \mid\mid\mid \Delta DFM$ ($\angle$; $\angle$; $\angle$)	✓ $\hat{D}_4 = \hat{D}_2$ ✓ $\hat{B}_1 = \hat{F}_2$ ✓ reason ✓ $\hat{C} = \hat{M}_2$ or ($\angle$; $\angle$; $\angle$) (4)
9.6	$\dfrac{DM}{FM} = \dfrac{DC}{BC}$ ($\Delta DBC \mid\mid\mid \Delta DFM$) $= \dfrac{\sqrt{5}BC}{BC}$ $= \sqrt{5}$	✓ S ✓ answer (2) **[19]**

QUESTION 10

10.1	 Construction: Join DC and BE and heights k and h $\frac{\text{area } \Delta\text{ADE}}{\text{area } \Delta\text{DEB}} = \frac{\frac{1}{2}.\text{AD}.k}{\frac{1}{2}.\text{DB}.k} = \frac{\text{AD}}{\text{DB}}$ (equal heights) $\frac{\text{area } \Delta\text{ADE}}{\text{area } \Delta\text{DEC}} = \frac{\frac{1}{2}.\text{AE}.h}{\frac{1}{2}.\text{E}C.h} = \frac{\text{AE}}{\text{EC}}$ (equal heights) But Area ΔDEB = Area ΔDEC (same base, same height) $\therefore \frac{\text{area } \Delta\text{ADE}}{\text{area } \Delta\text{DEB}} = \frac{\text{area } \Delta\text{ADE}}{\text{area } \Delta\text{DEC}}$ $\therefore \frac{\text{AD}}{\text{DB}} = \frac{\text{AE}}{\text{EC}}$	✓ construction ✓ $\frac{\text{area } \Delta\text{ADE}}{\text{area } \Delta\text{DEB}} = \frac{\text{AD}}{\text{DB}}$ ✓ reason ✓ $\frac{\text{area } \Delta\text{ADE}}{\text{area } \Delta\text{DEC}} = \frac{\text{AE}}{\text{EC}}$ ✓ Area ΔDEB = Area ΔDEC (S/R) ✓ $\frac{\text{area } \Delta\text{ADE}}{\text{area } \Delta\text{DEB}} = \frac{\text{area } \Delta\text{ADE}}{\text{area } \Delta\text{DEC}}$ (6)

10.2.1	$\frac{AB}{BE}=\frac{AC}{CD}$ (Prop Th; BC | | ED) $\frac{1}{3}=\frac{3}{CD}$ $\therefore$ CD = 9 units	✓ $\frac{AB}{BE}=\frac{AC}{CD}$ (S/R) ✓ substitution ✓ answer (3)
10.2.2	$\frac{DG}{GA}=\frac{FD}{FE}$ (Prop Th; FG | | EA) $\frac{9-x}{3+x}=\frac{3}{6}$ $54-6x=9+3x$ $-9x=-45$ $x=5$	✓ $\frac{DG}{GA}=\frac{FD}{FE}$ (S/R) ✓ substitution ✓ simplification ✓ answer (4)
10.2.3	In ΔABC and ΔAED: $\hat{A}$ is common $A\hat{B}C=\hat{E}$ (corres $\angle$s; BC | | ED) $A\hat{C}B=\hat{D}$ (corres $\angle$s; BC | | ED) $\Delta ABC \mid\mid\mid \Delta AED$ ($\angle$, $\angle$, $\angle$) $\therefore \frac{BC}{ED}=\frac{AC}{AD}$ $\frac{BC}{9}=\frac{3}{12}$ BC = $2\frac{1}{4}$ units	✓ $\hat{A}$ is common ✓ $A\hat{B}C=\hat{E}$ (S/R) ✓ $A\hat{C}B=\hat{D}$ (S/R) or ($\angle$; $\angle$; $\angle$) ✓ $\frac{BC}{ED}=\frac{AC}{AD}$ ✓ answer (5)
10.2.4	$\frac{\text{area }\Delta ABC}{\text{area }\Delta GFD}=\frac{\frac{1}{2}AC.BC.\sin A\hat{C}B}{\frac{1}{2}GD.FD.\sin\hat{D}}$ $=\frac{\frac{1}{2}(3)(2\frac{1}{4})\sin\hat{D}}{\frac{1}{2}(4)(3)\sin\hat{D}}$ (corres $\angle$s; BC | | ED) $=\frac{9}{16}$	✓ use of area rule ✓ correct sides and angles ✓ substitution of values ✓ $\sin A\hat{C}B=\sin\hat{D}$ (S/R) ✓ answer (5) **[23]**

TOTAL: 150