NATIONAL SENIOR CERTIFICATE EXAMINATION
EXEMPLAR 2014

MATHEMATICS: PAPER I

Time: 3 hours | 150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 11 pages. Please check that your paper is complete.
2. Read the questions carefully.
3. Answer all the questions.
4. Number your answers exactly as the questions are numbered.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Round off your answers to one decimal digit where necessary.
7. All the necessary working details must be clearly shown.
8. It is in your own interest to write legibly and to present your work neatly.

SECTION A

QUESTION 1

(a) Solve for x:

(1) $x(x-3)=88$ (3)

(2) $3\times 2^{2x-1}=96$ (3)

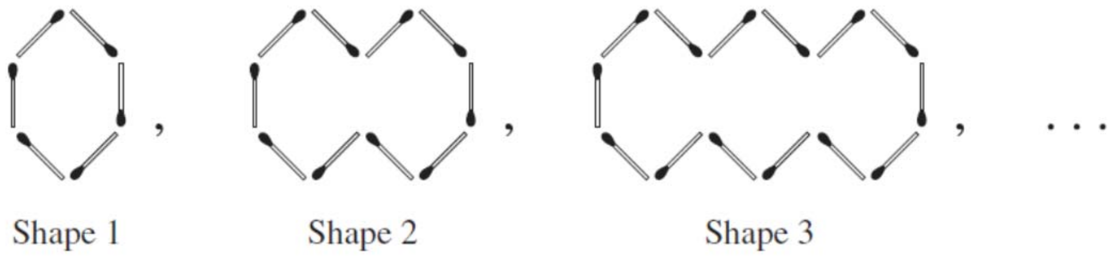
(3) $\log_9 x=\frac{1}{2}$ (2)

(4) $(1-x)(x+2)>0$ (2)

(b) Given $4^x\times 6^y=48^{12}$.
Find the value of $x+y$. Show all your working. (4)

[14]

QUESTION 2



	Shape 1	Shape 2	Shape 3
Number of matchsticks	6	10	14

If the pattern is continued, which shape would use exactly 526 matchsticks?

[4]

QUESTION 3

Consider the sequence 298 ; 259 ; 222 ; 187 ; 154 ; 123 ; …………

The nth term for the sequence is given by $T_n = an^2 + bn + 339$.

(a) Find the values of a and b. (4)

(b) Write down the nth term of the sequence 198; 159; 122; 87; 54; 23; ….. (1)

[5]

QUESTION 4

The first term (T_1) and the common ratio (r) of a geometric series are 2 and $\frac{3}{2}$ respectively.

(a) Determine the value of the third term (T_3) of the series. (1)

(b) Use an appropriate formula to calculate n, for which $S_n = 16\frac{1}{4}$. (5)

[6]

QUESTION 5

(a) Peter buys a new car for R160 000. Once he takes delivery of the car, it starts to depreciate in value.

If the car depreciates on a straight line balance method or on the reducing balance method it reaches the same value of R110 000 after 4 full years.

(1) What is the percentage rate of depreciation, r %, according to the straight line method of depreciation? Give your answer correct to 1 decimal digit. (2)

(2) What is the percentage rate of depreciation, r %, according to the reducing balance method? Give your answer correct to 1 decimal digit. (3)

(b) Jabu decides to start saving for an overseas trip to be taken to celebrate his 60th birthday. Exactly one month after his 45th birthday he starts to save a constant monthly amount, up to and including a payment made on the day of his 60th birthday.

Determine the amount he needs to save each month in order to be able to withdraw R50 000 on his 60th birthday if the interest rate is 8,25% per annum compounded monthly. (4)

(c) In order to buy furniture for her new apartment, Noma takes out a loan of R45 000 from the bank.

The bank charges an annual interest rate of 10,25% p.a. compounded monthly.

The instalment of R839,35 starts one month after she has received the money from the bank.

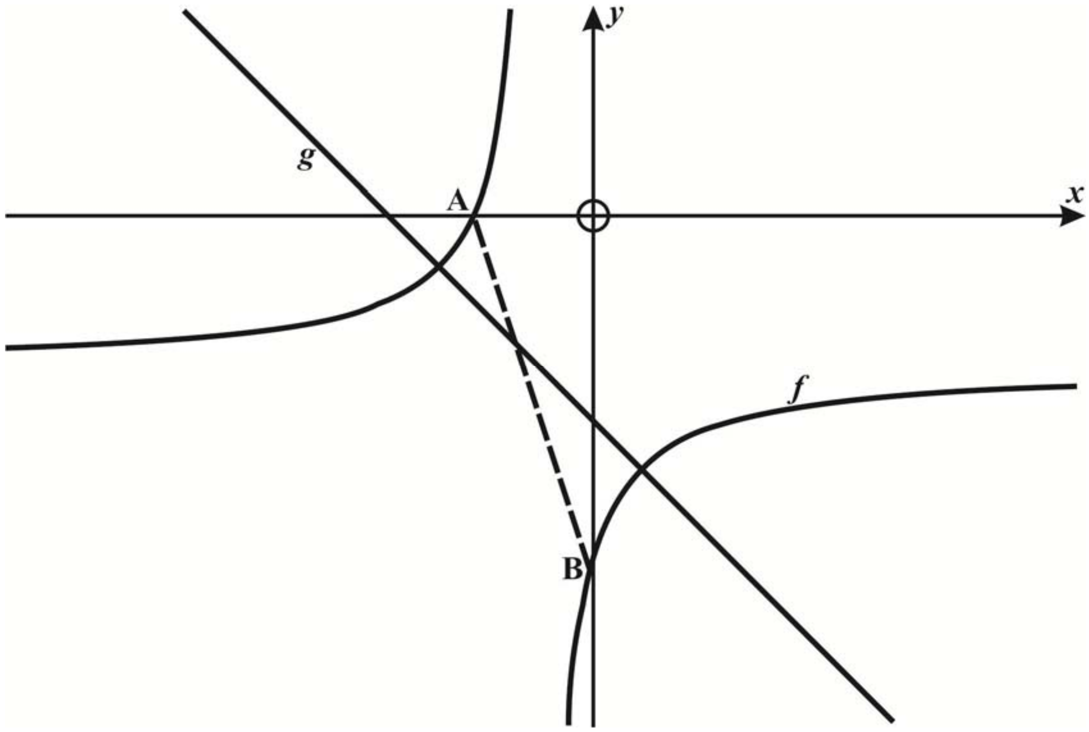
(1) Calculate the duration period of the loan, to the closest number of months. (4)

(2) Calculate the outstanding balance of her loan after three years (immediately after the 36th instalment). (4)

[17]

QUESTION 6

Sketched are the graphs of $y = f(x)$ and $y = g(x)$ where $f(x) = \dfrac{-4}{x+1} - 3$.



Determine:

(a) The equations of the asymptotes of *f*. (2)

(b) The length of AB, if A and B are the *x* and *y* intercepts of *f*. Leave answer in surd form. (6)

(c) The equation of *g*, if *g* is an axis of symmetry of *f*. (2)

[10]

QUESTION 7

(a) Determine $f\,'(x)$ from first principles if $f(x) = 2x^3$. (6)

(b) Find $\dfrac{dy}{dx}$ if $y = 8x^3 - 4\sqrt{x} + \dfrac{4x^2 + 3}{x}$; $x > 0$. (6)

Leave answers with positive exponents.

(c) The volume of water in a tank changes at a variable rate. The height of the water, h cm, at any time, t minutes can be described by $h(t) = 4(2t + 6) - t(t^2 - t)$

Find:

(1) the time interval for t, during which the height of the water is increasing. (4)

(2) the average rate at which the height has changed over the first 3 minutes. (3)

[19]

75 marks

SECTION B

QUESTION 8

(a) The letters L, M, N, O, P, Q are randomly arranged in a straight line.

(1) How many different arrangements can be created? Answer may be left in factorial form. (1)

(2) What is the probability that the letters 'L' and 'M' are not next to each other? (6)

(b) Sam either has tea or coffee at morning break. If she has tea one morning, the probability that she has tea the next morning, is 0,4. If she has coffee one morning the probability she has coffee the next morning, is 0,3.

Suppose that Sam has coffee on Monday 1st January. Draw a tree diagram to represent the situation and hence determine the probability that she has tea on Wednesday 3rd January.

(5)

(c) Let A and B be two events in a sample space such that $P(A) = \frac{1}{4}$ and $P(B) = \frac{1}{3}$.

(1) If A and B are mutually exclusive find $P(A \cup B)'$ (3)

(2) If A and B are not mutually exclusive, but are independent, find $P(A \cup B)$ (3)

[18]

QUESTION 9

(a) Find two integer values for p so that $x^2 + p(2x+7) + 8$ is a perfect square. (4)

(b) $f(x) = x^3 + 8x^2 + 18x + 9 = (x+3)(x^2 + ax + b)$ where a and b are real values.

(1) Find the values of a and b. (3)

(2) Find the value of the sum of the roots of $f(x) = 0$. (4)

[11]

QUESTION 10

(a) A farmer develops a scheme to keep his fruit pickers working for an entire 30 day season without taking off unnecessary rest days in between.

For DAY 1 he pays: R a

For DAY 2 he pays: R $(a+d)$

For DAY 3 he pays R $(a+2d)$, etc.

He increases the daily payment by an amount of R d for each consecutive extra day worked.

A picker who works for all consecutive 30 days will earn R482 ,50 on the final 30th day.

A picker who works for all consecutive 30 days will earn a total sum of R11 212,50.

(1) Write down an equation for the workers earnings on DAY 30. (1)

(2) Find the value of the first day's pay and the daily increase. (5)

(b) The series K + 2K + 3K + 4K + … + 100 is given. K is a positive integer and K is a factor of 100.

Find, in terms of K,

(1) an expression for the number of terms in the series. (2)

(2) an expression for the sum of the series. (3)

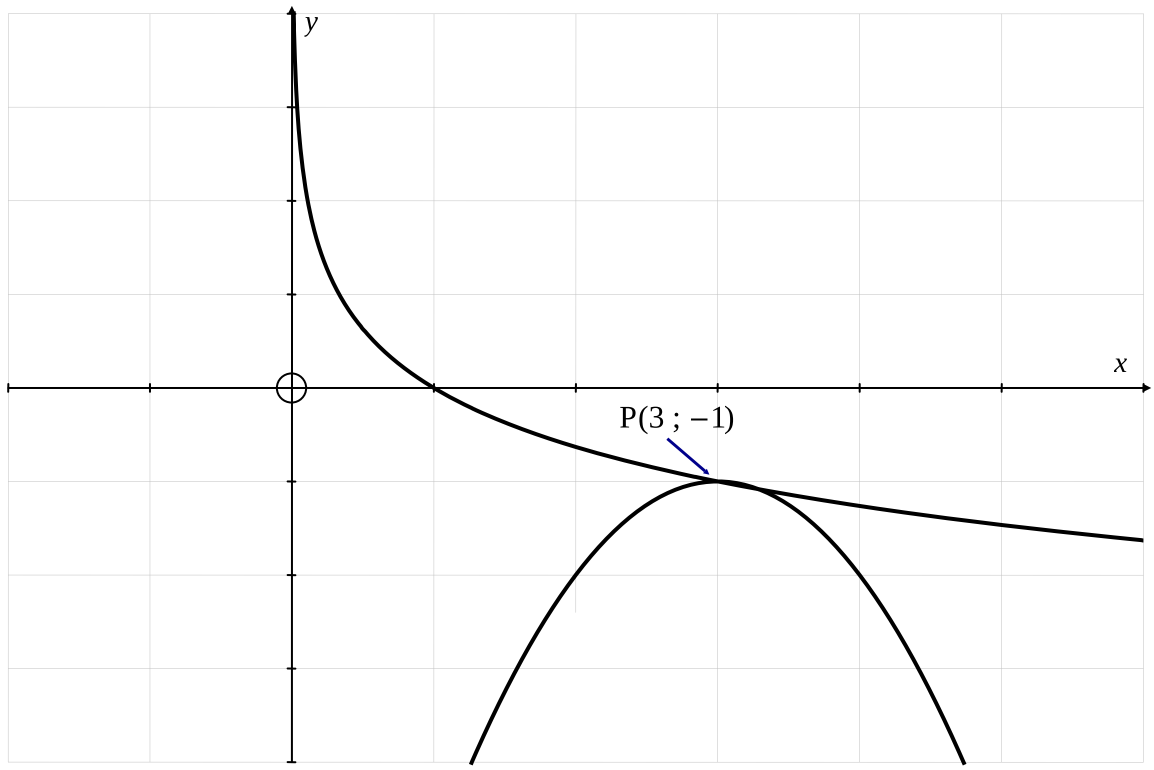
[11]

QUESTION 11

Refer to the figure below.

The graphs of $y = g(x) = \log_a x$ and $y = h(x) = -(x-3)^2 - 1$ are given.

The point $P(3;-1)$ lies on the graph of both *g* and *h*.



Determine:

(a) the value of *a*. (2)

(b) the equation which defines $g^{-1}(x)$ in the form $y = ...$ (2)

(c) the $x-$values for which $1 \leq g^{-1}(x) \leq 3$. (2)

(d) a possible restriction that could be placed on $h(x)$ to ensure that $h^{-1}(x)$ is a function. (1)

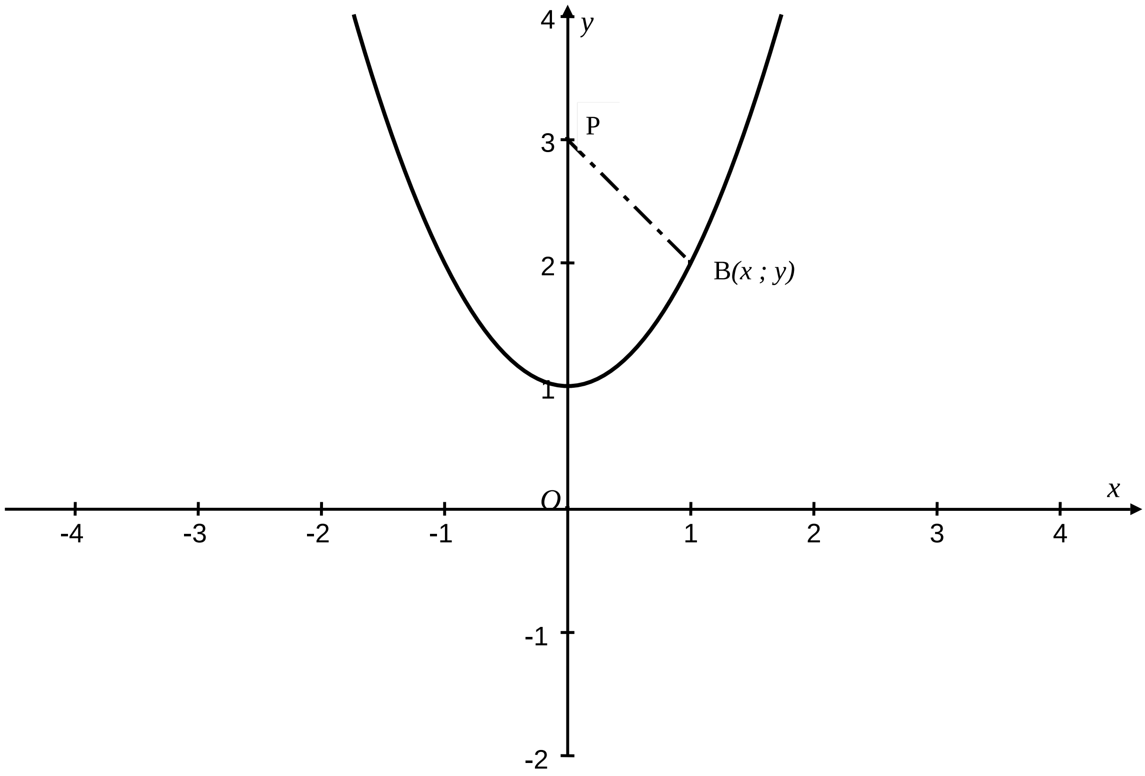
(e) the values of *x* for which $g(x).h(x) < 0$. (2)

[9]

QUESTION 12

B is a movable point on the parabola $y = x^2 + 1$, where $x > 0$.

PB is a line segment joining P(0 ; 3) to B, on the parabola.



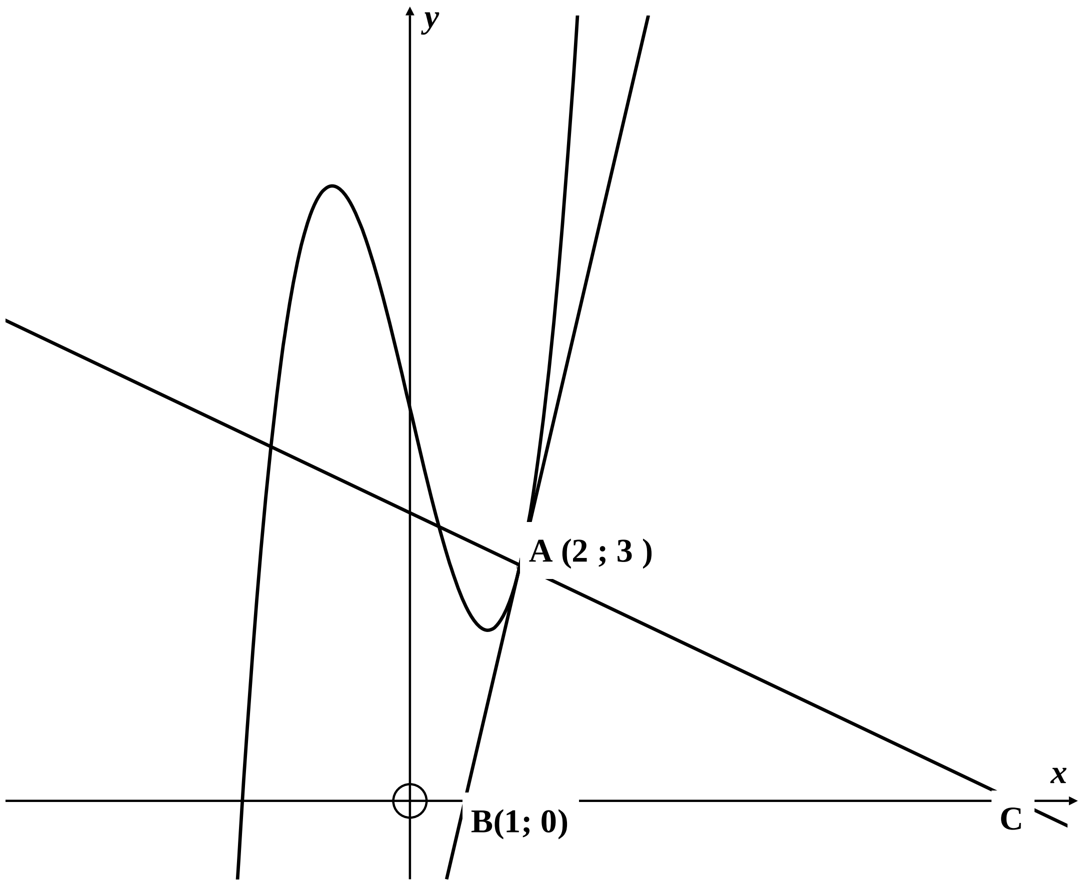
(a) Find an expression for the length of PB. (3)

(b) Hence find the value of *x*, which results in a minimum length of PB^2. (6)

[9]

QUESTION 13

In the diagram, the tangent to the curve $y = f(x) = ax^3 - 3x + b$ at $A(2\ ;\ 3)$ meets the x-axis at $B(1\ ;\ 0)$. The line perpendicular to the tangent at A meets the x- axis at C.



Find:

(a) the value of *a* and *b*. (4)

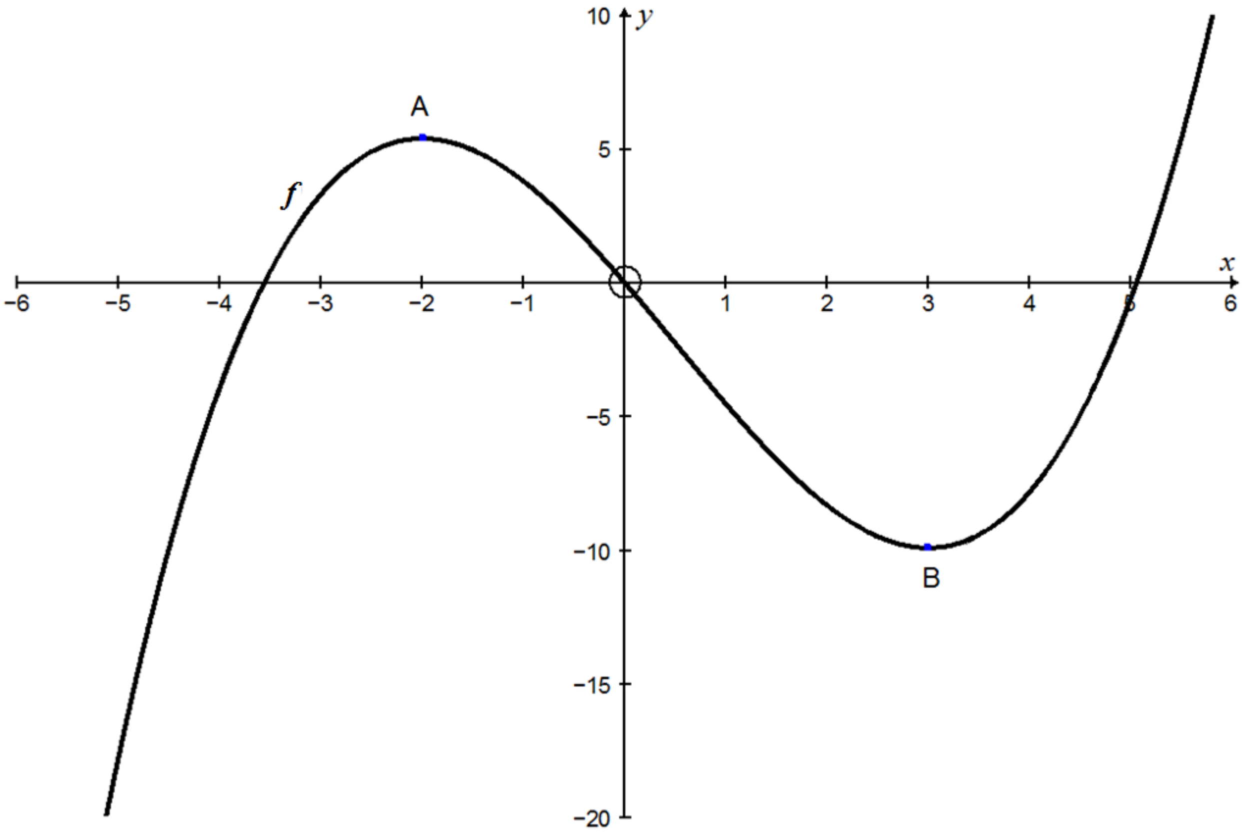
(b) the coordinates of C. (3)

(c) the area of triangle ABC. (2)
[9]

QUESTION 14

The figure shows a sketch of a function with equation $y = f(x)$.

$f(x)$ has a local maximum value at $A\left(-2\ ;\ 5\frac{1}{2}\right)$ and a local minimum value at $B(3\ ;\ -10)$.



(a) The graph of $y = 2f(x) + a$ has a minimum at (3 : 0) where a is a constant. Write down the value of a. (1)

(b) On a separate axes for each, on your diagram sheet, sketch a function with the equation

(1) $y = f(x+3)$ (3)

Indicate clearly the co-ordinates of the new maximum and minimum values.

(2) $y = f\,'(x+3)$ (4)

Indicate only the x-intercepts of the graph.

[8]

75 marks

Total: 150 marks