NATIONAL SENIOR CERTIFICATE EXAMINATION
EXEMPLAR PAPER 2014

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours 150 marks

QUESTION 1

(a)

(1) $x^2 - 3x = 88$
$(x-11)(x+8) = 0$
$x = 11; x = 8$ (3)

(2) $3 \times 2^{2x-1} = 96$
$2^{2x-1} = 32$
$2^{2x-1} = 2^5$
$2x = 6$
$x = 3$ (3)

(3) $\log_9 x = \frac{1}{2}$
$9^{\frac{1}{2}} = x$
$x = 3$ (2)

(4) $(1-x)(x+2) > 0$
$-2 < x < 1$ (2)

(b)

$4^x \times 6^y = 48^{12}$
$2^{2x} \times 3^y . 2^y = (2^4 \times 3)^{12}$
$2^{2x+y-48} = 3^{12-y}$ (4)
$2x + y - 48 = 0 = 12 - y$
$2x + 2y = 60$
$x + y = 30$

[14]

QUESTION 2

Shape $1 = 6$; Shape $2 = 10$; Shape $3 = 14$; ...
$T_n = 4n + 2$
$4n + 2 = 526$
$n = 131$
131^{st} shape would have 526 matches.

[4]

QUESTION 3

(a) $298 = a(1) + b + 339$
$259 = 4a + 2b + 339$
$b = -42$
$a = 1$ (4)

(b) $T_n = n^2 - 42n + 239$ (1)

[5]

QUESTION 4

(a) $T_3 = \frac{9}{2}$ (1)

(b) $S_n = \frac{2\left(\left(\frac{3}{2}\right)^n - 1\right)}{\frac{3}{2} - 1}$

$4\left(\left(\frac{3}{2}\right)^n - 1\right) = 16\frac{1}{4}$

$\left(\frac{3}{2}\right)^n = \frac{65}{16} + 1$

$\left(\frac{3}{2}\right)^n = \frac{81}{16}$

$n = 4$ (5)

[6]

QUESTION 5

(a) (1) $110\,000 = 160\,000(1 - 4i)$

$0,6875 = 1 - 4i$

$-0,3125 = -4i$

$0,078125 = i$

$7,8\% = r$ (2)

(2) $110\,000 = 160\,000(1 - i)^4$

$0,6875 = (1 - i)^4$

$0,91058... = 1 + i$ (3)

$0,08941 = i$

$8,9\% = r$

(b) $50\,000 = \frac{\left[\left(1 + \frac{0,0825}{12}\right)^{180} - 1\right]}{\frac{0,0825}{12}}$

$x = R\,141,32$ (4)

(c) (1) $45\,000 = \frac{839,35\quad\left[1 - \left(1 + \frac{0,1025}{12}\right)^{-n}\right]}{\frac{0,1025}{12}}$

$0,4579 = [1 - \left(1 + \frac{0,1025}{12}\right)^{-n}$

$\log_{(1,008541667)} 0,542056.... = -n$

$-71,999.... = -n$

$n = 72$ months / 6years (4)

(2) $OB = 45\,000\left(1+\frac{0{,}1025}{12}\right)^{36} - \frac{839{,}35\left[\left(1+\frac{0{,}1025}{12}\right)^{36}-1\right]}{\frac{0{,}1025}{12}}$

$OB = 61121{,}07 - 35\,203{,}17$

R25 917,90

Alteratively: $\frac{839{,}35\left[1-\left(1+\frac{0{,}1025}{12}\right)^{-36{,}000278}\right]}{\frac{0{,}1025}{12}}$

$= R\ 25918{,}78$ (4)

[17]

QUESTION 6

(a) $x = -1;\ y = -3$ (2)

(b) For A: let $y = 0$ $\frac{-4}{x+1} - 3 = 0$ For B: let $y = 0$; $y = -7$

$x = -2\frac{1}{3}$

$AB^2 = \left(-2\frac{1}{3}\right)^2 + (-7)^2$

$= \frac{7\sqrt{10}}{3}$ (6)

(c) $y = -x - 4$ (2)

[10]

QUESTION 7

(a) $f(x+h) = 2(x+h)^3$

$= 2(x^3 + 3x^2h + 3xh^2 + h^3)$

$= 2x^3 + 6x^2h + 6xh^2 + 2h^3$

$\lim_{h\to 0} \frac{2x^3 + 6x^2h + 6xh^2 + 2h^3 - 2x^3}{h}$

$\lim_{h\to 0} \frac{h(6x^2 + 6xh + 2h^2)}{h}$

$\lim_{h\to 0} (6x^2 + 6xh + 2h^2)$

$= 6x^2$ (6)

(b) $y = 8x^3 - 4x^{\frac{1}{2}} + 4x + 3x^{-1}$

$\frac{dy}{dx} = 24x^2 - \frac{2}{x^{\frac{1}{2}}} + 4 - \frac{3}{x^2}$ (6)

(c) (1) $h(t) = 8t + 24 - t^3 + t^2$

$h'(t) = 8 - 3t^2 + 2t$

want $-(3t^2 - 2t - 8) > 0$

$(3t + 4)(t - 2) < 0$

$\frac{-4}{3} < t < 2$

$\therefore 0 \le t < 2$ (4)

(2) $h(0) = 24$

$h(3) = 30$

$\frac{30 - 24}{3 - 0} = 2$ cm / minute (3)

[19]

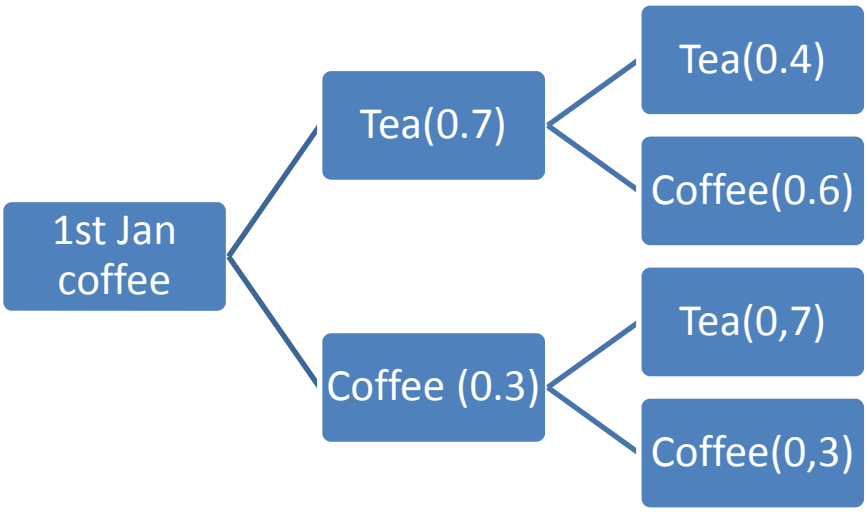
75 marks

QUESTION 8

(a) (1) 6 ! or 720 (1)

(2) Are next to each other $\frac{5! \times 2!}{6!} = \frac{1}{3}$

So not next to each other $1 - \frac{1}{3} = \frac{2}{3}$ (6)



(b)

$P(C,T,T) + P(C,C,T)$

$= 0,7 \times 0,4 + 0,3 \times 0,7$

$= 0,28 + 0,21$

$= 0,49$

(5)

8(c)

(1) $P(A \cup B) = \frac{1}{4} + \frac{1}{3} = \frac{7}{12}$

$P(A \cup B)' = \frac{5}{12}$ (3)

(2) $P(A \cap B) = \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$

$P(A \cup B) = \frac{1}{4} + \frac{1}{3} - \frac{1}{12} = \frac{1}{2}$ (3)

[18]

QUESTION 9

(a)

$x^2 + 2xp + 7p + 8$

$x^2 + 2xp + p^2 - p^2 + 7p + 8$

$(x + p)^2 - (p^2 - 7p - 8)$

$p^2 - 7p - 8 = 0$

$(p - 8)(p + 1) = 0$

$p = 8 \text{ or } p = -1$

OR

$b^2 - 4ac = 0$

$(2p)^2 - 4(1)(7p + 8) = 0$

$4p^2 - 28p - 52 = 0$

$4(p - 8)(p + 1) = 0$

(4)

(b)(1) $3b = 9$

$b = 3$

$ax^2 + 3x^2 = 8x^2$

$a = 5$ (3)

(2) $(x + 3)(x^2 + 5x + 3) = 0$

$x = -3; x = -4{,}3; x = -0{,}7$

sum of the roots is - 8 (4)

[11]

QUESTION 10

(a) (1) $T_{30} = a + 29d = 482{,}50$ (1)

(2) $S_{30} = \frac{30}{2}(a + 482{,}50] = 11\,212{,}50$

$a = 265$

$265 + 29d = 482{,}50$

$d = R7{,}50$

first days pay R265

daily increase R7,50 (5)

(b)(1) If $K = 1:1+2+3+4+\ldots 100$ (100 terms)
If $K = 2:2+4+6+8+\ldots +100$ (50 terms)
If $K = 4:4+8+12+\ldots +100$ (25 terms)
Number of terms $= \frac{100}{K}$ (2)

(2) $\frac{\frac{100}{K}}{2}[K+100)$

$\frac{100}{2}+\frac{5000}{K}$

$50+\frac{5000}{K}$ (3)

[11]

QUESTION 11

(a) $-1 = \log_a 3$

$a^{-1} = 3$

$a = \frac{1}{3}$ (2)

(b) $inverse: y = \left(\frac{1}{3}\right)^x = 3^{-x}$ (2)

(c) $(1;0)$ and $(3;-1)$ lie on $g(x)$

$(0:1)$ and $(-1;3)$ lie on $g^{-1}(x)$

$-1 \le x \le 0$ (2)

(d) $x \ge 3$ or $x \le 3$ (1)

(e) $0 < x < 1$ (2)

[9]

QUESTION 12

(a) $PB^2 = x^2 + (y-3)^2$

$PB = \sqrt{x^2 + (y-3)^2}$ (3)

(b) $y = x^2 + 1$

$PB = \sqrt{x^2 + (x^2-2)^2}$

$\text{PB}^2 = \sqrt{x^4 - 3x^2 + 4}$

$(PB^2)' = 4x^3 - 6x = 0$

$2x(2x^2 - 3) = 0$

$x = 0$ or $x = \pm\sqrt{\frac{3}{2}}$

$\therefore x = \sqrt{\frac{3}{2}};\ x > 0$ (6)

[9]

QUESTION 13

(a) $f(x) = ax^3 - 3x + b$

$f'(x) = 3ax^2 - 3$

at $A(2;3)$

$3 = 8a - 6 + b$

$8a + b = 9....(1)$

and $12a - 3 = \frac{3-0}{2-1} = 3$

$a = \frac{1}{2}$ so $b = 5$ (4)

(b) gradient of perp line $= -\frac{1}{3}$

Equation is $y - 3 = -\frac{1}{3}(x - 2)$

at $C; y = 0$

$-3 = -\frac{1}{3}(x - 2)$

$x = 11$

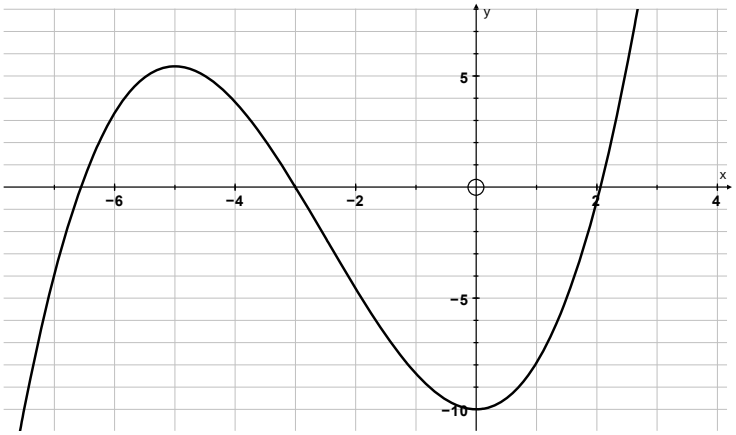
C is $(11; 0)$ (3)

(c) Area $= 0,5 \times 10 \times 3 = 15$ square units (2)

QUESTION 14

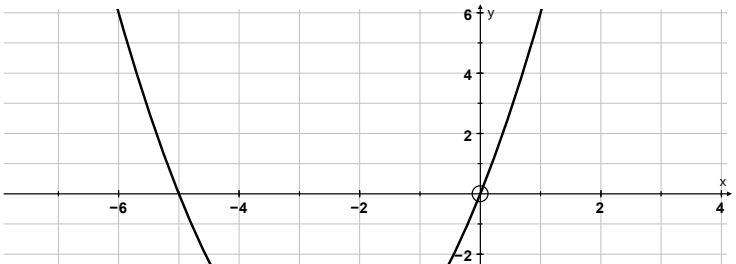
(a) $a = 20$ (1)

(b) (1)



(3)

(2)



(4)
[10]

75 marks