



*Keep the dream alive.*

MATHEMATICS DEPARTMENT

Grade 11: Mathematics Paper II

November Exam 2016

Time: 120 min

150 marks

NAME: MEMO

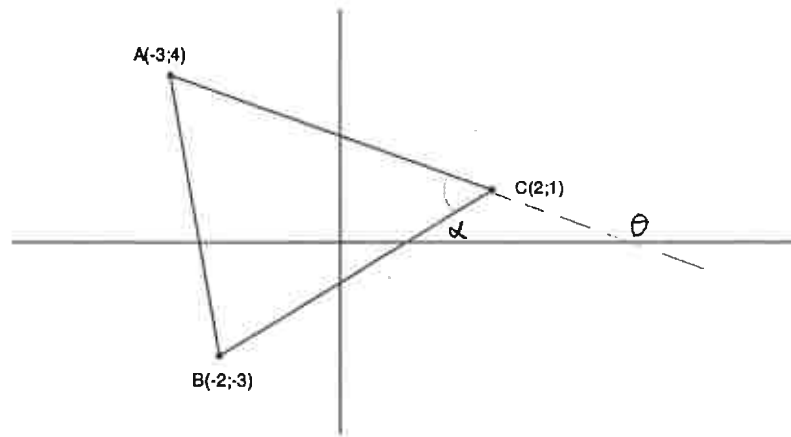
PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 24 pages and 12 questions. Please check that your paper is complete. A separate formula sheet will be handed out.
2. Read the questions carefully.
3. Answer all the questions.
4. Number your answers exactly as the questions are numbered.
5. You may use an approved non-programmable and non-graphical calculator, unless a specific question prohibits the use of a calculator.
6. Round off your answers to two decimal digits where necessary.
7. All the necessary working details must be clearly shown.
8. It is in your own best interest to present your work neatly.

## Section A

## Question 1

In the diagram below  $\Delta ABC$  is given with  $A(-3;4)$ ,  $B(-2;-3)$  and  $C(2;1)$ .



1.1 Determine:

1.1.1 The lengths of  $AC$  and  $AB$

$$AC = \sqrt{(-3-2)^2 + (4-1)^2} = \sqrt{34} \quad \checkmark \quad \text{formula} \quad (3) \quad 3$$

$$AB = \sqrt{(-3+2)^2 + (4+3)^2} = 5\sqrt{2} \quad \checkmark$$

1.1.2 The median of  $\Delta ABC$  from point  $C$ .

$$M_{AB} = \left( \frac{-3-2}{2}, \frac{4-3}{2} \right) = \left( -\frac{5}{2}, \frac{1}{2} \right) \quad \checkmark \quad \checkmark \quad (6)$$

$$m = \frac{1 - \frac{1}{2}}{2 + \frac{5}{2}} = \frac{1}{9} \quad \checkmark \quad \checkmark$$

$$y - \frac{1}{2} = \frac{1}{9} \left( x + \frac{5}{2} \right) \quad \checkmark \quad 6$$

$$y = \frac{1}{9}x + \frac{7}{9} \quad \checkmark$$

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1.2 Calculate:

1.2.1 The angle of inclination of  $AC$ . (3)

$$m_{AC} = \frac{4-1}{-3-2} = -\frac{3}{5} \checkmark$$

$$\tan \theta = -\frac{3}{5} \checkmark$$

$$\theta = 149,04^\circ \checkmark$$

3

1.2.2 The angle of inclination of  $BC$ . (3)

$$m_{BC} = \frac{1+3}{2+2} = 1 \checkmark$$

$$\tan \alpha = 1 \checkmark$$

$$\alpha = 45^\circ \checkmark$$

3

1.2.3  $\hat{ACB}$  (3)

$$\hat{ACB} = \alpha + 180^\circ - \theta \checkmark$$

$$= 45^\circ + 180^\circ - 149,04^\circ$$

$$= 75,96^\circ \checkmark$$

3

1.3 Classify  $\triangle ABC$  fully. (4)

$$AC = \sqrt{34} = 5,83 \checkmark$$

$$AB = 5\sqrt{2} = 7,07 \checkmark$$

$$BC = 4\sqrt{2} = 5,66 \checkmark$$

Acute L'd scalene  $\triangle$  ✓ ✓

4

1.4 Determine the coordinates of  $D$  if  $ACDB$  is a parallelogram. (2)

$$D(3, -6) \checkmark \checkmark$$

2

[24]

## Question 2

2.1 Simplify the following:

$$\frac{\sin(\beta-90^\circ)\tan(-\beta-180^\circ)\cos(180^\circ+\beta)}{\cos(-\beta)\sin(720^\circ+\beta)} \quad (6)$$

$$= \frac{-\cancel{\cos\beta} \cdot -\cancel{\tan\beta} \cdot -\cos\beta}{\cancel{\cos\beta} \cdot \sin\beta}$$

$$= -\frac{\sin\beta \cdot \cos\beta}{\cancel{\cos\beta} \cdot \sin\beta}$$

$$= -1 \quad \checkmark$$

2.2 Prove the following identity:

$$\tan^2 A \cdot \cos^2 A + \frac{\sin^2 A}{\tan^2 A} = 1 \quad (4)$$

$$\text{LHS: } \frac{\sin^2 A}{\cancel{\cos^2 A}} \cdot \sqrt{\cancel{\cos^2 A}} + \frac{\sin^2 A}{\cancel{\sin^2 A}} \cdot \sqrt{\cancel{\cos^2 A}}$$

$$= \sin^2 A + \sqrt{\cos^2 A}$$

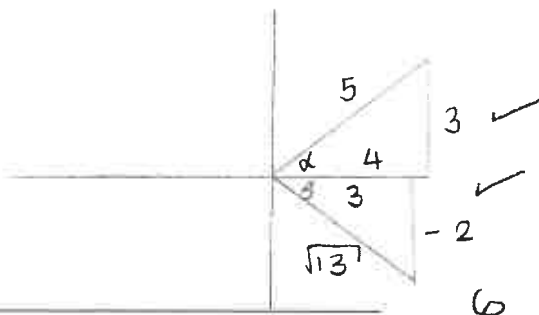
$$= 1 \quad \checkmark$$

$$= \text{RHS}$$

2.3 If  $\sin \alpha = \frac{3}{5}$  and  $\tan \beta = \frac{-2}{3}$ ;  $\alpha < 90^\circ$ ;  $180^\circ < \beta < 360^\circ$ ,

determine the value of:  $25 \cos^2 \alpha - 13 \sin^2 \beta$ , without the use of a calculator (6)

$$\begin{aligned}
 & 25 \left( \frac{4}{5} \right)^2 - 13 \left( \frac{-2}{\sqrt{13}} \right)^2 \\
 &= \frac{25 \cdot 16}{25} - \frac{13 \cdot 4}{13} \\
 &= 16 - 4 \\
 &= 12
 \end{aligned}$$



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## Question 3

3.1 Solve for  $A$ ;  $A \in [-180^\circ; 180^\circ]$  if

$$4 \cos^2 A + 2 \sin A \cos A - 1 = 0$$

$$4 \cos^2 A + 2 \sin A \cos A - \sin^2 A - \cos^2 A = 0 \quad (7)$$

$$3 \cos^2 A + 2 \cos A \sin A - \sin^2 A = 0$$

$$(3 \cos A - \sin A)(\cos A + \sin A) = 0$$

$$3 \cos A = \sin A \quad \text{OR} \quad \cos A = -\sin A$$

$$3 = \tan A \quad \text{OR} \quad -1 = \tan A$$

$$A = 71,57^\circ + k \cdot 180^\circ \quad \text{OR} \quad A = -45^\circ + k \cdot 180^\circ$$

$$\therefore A = -45^\circ; 135^\circ; 71,57^\circ; -108,43^\circ$$

KEZ

3.2 If  $\sin \theta = k$ , calculate, in terms of  $k$ , the value of:

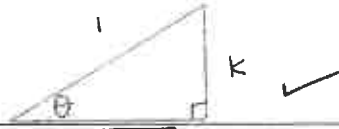
3.2.1  $\cos(90^\circ + \theta)$  (2)

$$= -\sin \theta \quad \checkmark$$

$$= -k \quad \checkmark$$

2

3.2.2  $\cos \theta$  (3)

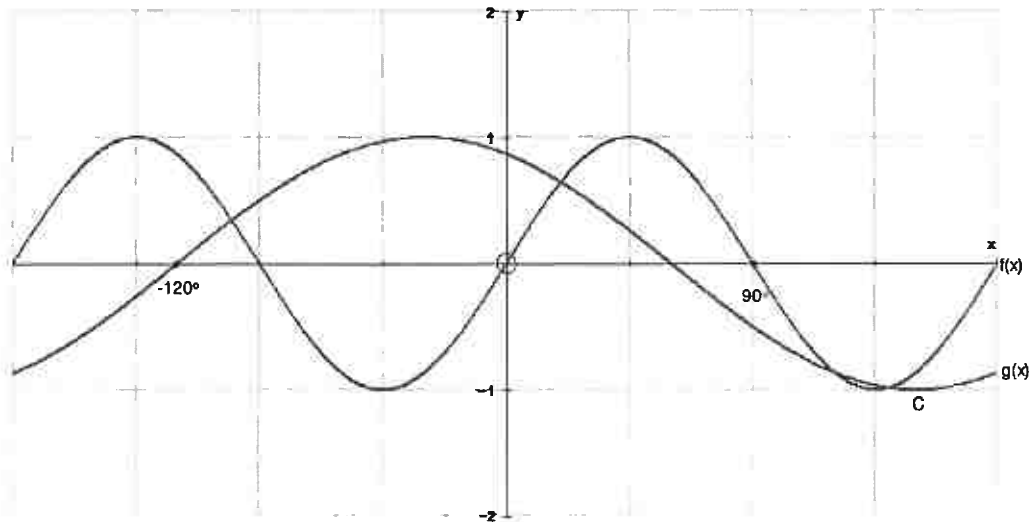


$$\frac{\sqrt{1-k^2}}{1} \quad \checkmark \checkmark$$

1

3

3.3 The sketch of  $f(x) = \sin ax$  and  $g(x) = \cos(x - b)$  is given below.



3.3.1 Find the values of  $a$  and  $b$ . (2)

$$a = 2 \quad \checkmark \quad b = -30^\circ \quad \checkmark$$

2

3.3.2 Determine the coordinates of  $C$ , a turning point on  $g(x)$ . (2)

$$C(150; -1)$$

2

3.3.3 For which values of  $x; x < 0$  is  $f(x) \cdot g(x) \geq 0$ ? (3)

$$x \in [-180^\circ] \cup [-120^\circ; -90^\circ]$$

3

3.3.4 Determine the equation of  $f(x)$  if the  $y$ -axis is moved  $30^\circ$  to the left. (1)

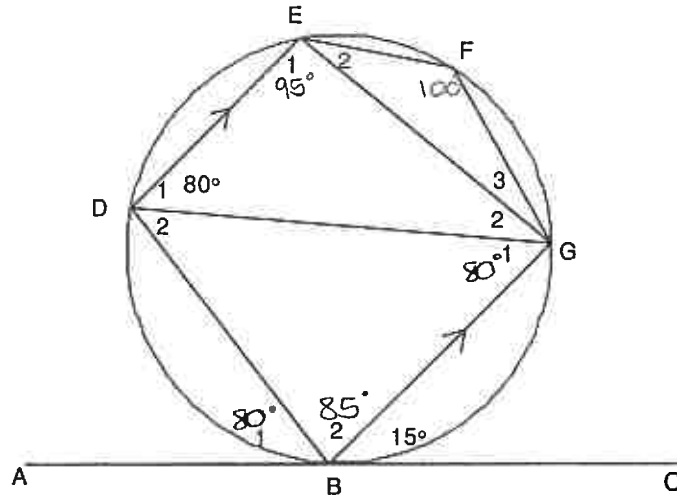
$$f(x) = \sin 2(x - 30^\circ)$$

1

[20]

**Question 4**

$ABC$  is a tangent at  $B$ .  $DE \parallel BG$ .  $\widehat{GBC} = 15^\circ$  and  $\widehat{EDG} = 80^\circ$ .



Give reasons and calculate the size of:

4.1  $\widehat{F} = 100^\circ$  ✓ (2)  
 opp  $\angle$ s cyclic quad. ✓

2

4.2  $\widehat{C}_1 = 80^\circ$  ✓ (4)  
 Alt  $\angle$ s  $DE \parallel BG$   
 $\widehat{B}_1 = 80^\circ$  ✓ tan-chord

4

4.3  $\widehat{E}_1 = 95^\circ$  ✓ (4)  
 $\widehat{B}_2 = 85^\circ$  ✓  $\angle$ s on str line  
 $\widehat{E}_1 = 95^\circ$  ✓ opp  $\angle$ s cyclic quad.

4



4.4  $\hat{G}_2 = 5^\circ$  ✓  $LS$  in  $\Delta$  ✓ (2)

2

[12]

**Question 5**

The approximate electricity charges in Rands of 27 flats in a particular block of flats in a given month were recorded as follows:

780	750	880	800	900	810	600	830	690
970	680	810	770	750	690	760	910	700
950	800	700	810	640	700	580	680	870

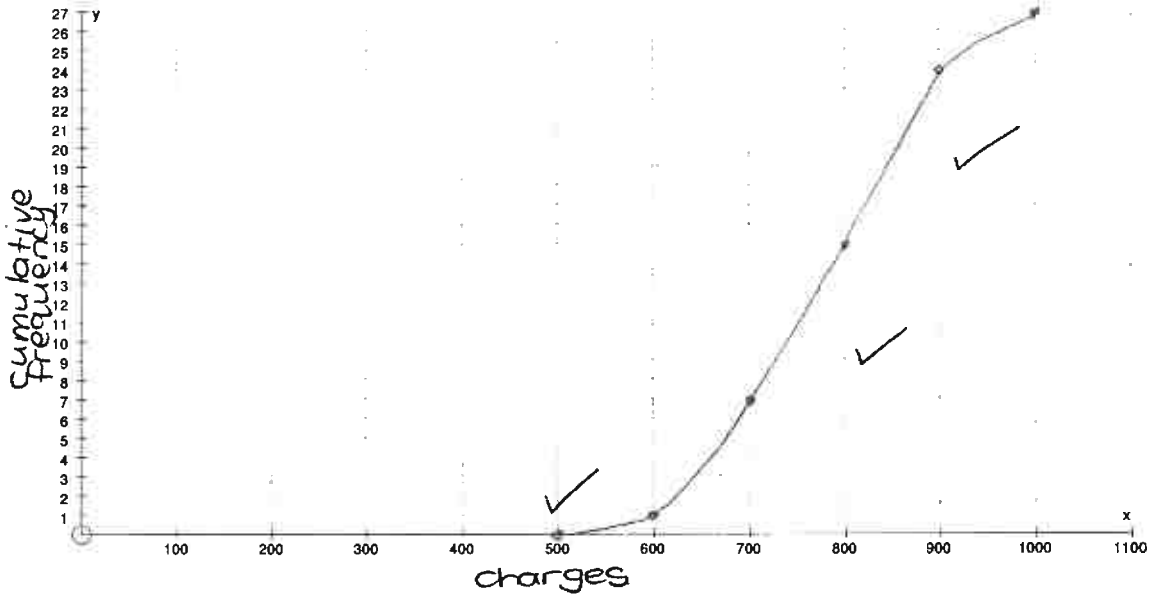
5.1 Complete the table:

Cost in Rands	Frequency	Cumulative Frequency
$500 \leq x < 600$	1	1
$600 \leq x < 700$	6	7
$700 \leq x < 800$	8	15
$800 \leq x < 900$	9	24
$900 \leq x < 1\ 000$	3	27

(2)

2

5.2 Draw an ogive on the given set of axes to represent the above information. (3)



3

5.3 Use the raw data given in the original table to calculate:

5.3.1 The mean cost of electricity in this particular month for the residents of the block of flats. (1)

$$\bar{x} = 770,74 \quad \checkmark$$

1

5.3.2 The standard deviation. (1)

$$\sigma = 99,59 \quad \checkmark$$

1

5.4 How many household's electricity charges for this particular month lie within one standard deviation of the mean? (2)

$$671,15 - 870,33 \quad \checkmark$$

19 households  $\checkmark$

2

[9]

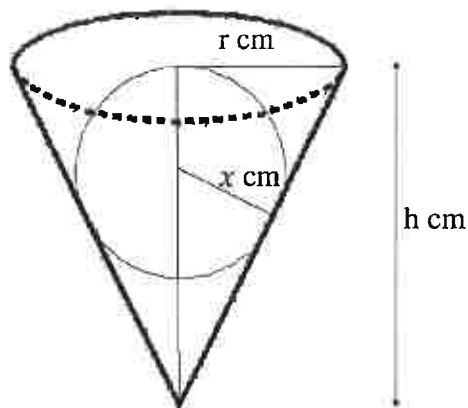
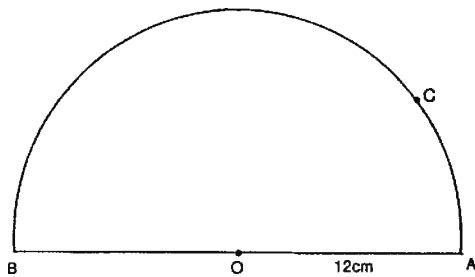
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**Question 6**

In the diagram below,  $ACBO$  is a piece of paper plate in the shape of a semi-circle of radius  $12\text{ cm}$ .

The paper plate is folded so that  $OA$  and  $OB$  coincide to form a cone with radius of base  $r\text{ cm}$  and height  $h\text{ cm}$ .

A metal ball of radius  $x\text{ cm}$  is put into the cone such that the ball touches the base of the cone.



6.1 Determine  $r$  and  $h$ , leaving answers in surd form if necessary. (6)

$$C = \frac{2\pi \cdot 12}{2} = \pi \cdot 12 \checkmark$$

$$2\pi r = \pi \cdot 12 \checkmark$$

$$r = 6\text{ cm} \checkmark$$

$$h^2 = 12^2 - 6^2 \text{ Pyth}$$

$$h = 6\sqrt{3} \text{ cm} \checkmark$$

6

Volume of a cone =  $\frac{1}{3}\pi r^2 h$

Volume of a sphere =  $\frac{4}{3}\pi r^3$

6.2 The cone containing the sphere is then filled with water. Assuming the sphere remains in its position, determine the volume of the water in the cone.

You may assume that  $x = 2\text{cm}$ .

(3)

$$V = \frac{1}{3}\pi(6)^2 \cdot 6\sqrt{3} - \frac{4}{3}\pi(2)^3$$

$$= 358.27 \text{ cm}^3$$

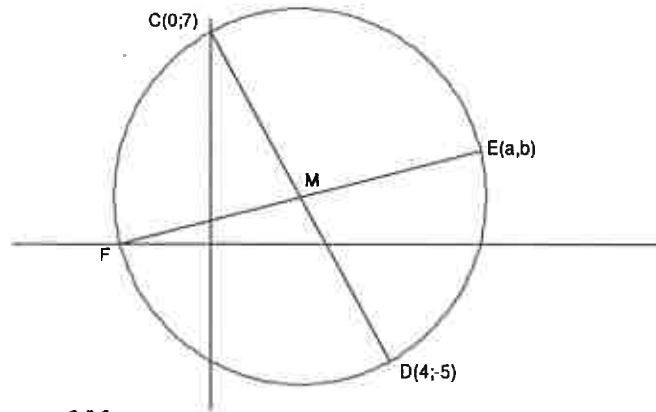
3

[9]

## Section B

## Question 7

A circle with centre  $M$  passes through the points  $C(0; 7)$ ,  $D(4; -5)$  and  $E(a; b)$ .  $F$  is on the  $x$ -axis.



7.1 Determine the coordinates of  $M$ . (2)

$$M = \left( \frac{0+4}{2}, \frac{7-5}{2} \right)$$

$$= (2, 1)$$

2

7.2 Calculate the distance of  $CD$ . (2)

$$CD = \sqrt{(4-0)^2 + (-5-7)^2}$$

$$= 4\sqrt{10}$$

2

7.3 Determine the value of  $b$ , the  $y$  value of coordinate  $E$ . (2)

$$\frac{b+0}{2} = 1$$

$$\therefore b = 2$$

2

7.4 Determine the value of  $a$ , the  $x$  value of coordinate  $E$ . (HINT:  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ )

$$E(a; 2) \quad (5)$$

$$EM = CM$$

$$\sqrt{(a-2)^2 + (2-1)^2} = 2\sqrt{10} \quad \checkmark$$

$$a^2 - 4a + 4 + 1 = 40$$

$$a^2 - 4a - 35 \quad \checkmark = 0$$

$$a = \underline{8, 24} \quad \checkmark \text{ OR } a = -4, 24$$

5

[11]

## Question 8

If  $x + \frac{1}{x} = 2 \cos A$  ;  $0 \leq A \leq 180^\circ$  and  $x^2 + \frac{1}{x^2} = 1$ , Calculate  $A$ .

$$\left(x + \frac{1}{x}\right)^2 = (2 \cos A)^2$$

$$x^2 + 2 + \frac{1}{x^2} = 4 \cos^2 A$$

$$x^2 + \frac{1}{x^2} = 4 \cos^2 A - 2 \checkmark$$

$$\therefore 4 \cos^2 A - 2 = 1$$

$$\cos^2 A = \frac{3}{4}$$

$$\cos A = \pm \frac{\sqrt{3}}{2} \checkmark$$

$$A = \pm 30^\circ + k \cdot 360^\circ \text{ OR } A = \pm 150^\circ + k \cdot 360^\circ \text{ } k \in \mathbb{Z}$$

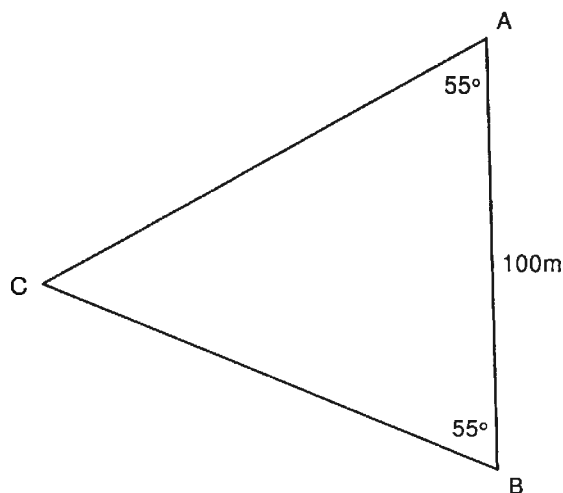
$$A = 30^\circ \checkmark ; 150^\circ \checkmark$$

5

[5]

## Question 9

9.1 A is a point 100m north of B. C is a beacon on an island so that  $\hat{CAB} = \hat{CBA} = 55^\circ$ .



Calculate:

9.1.1 The distance of  $BC$ . (2)

$$\frac{BC}{\sin 55^\circ} = \frac{100}{\sin 70^\circ} \checkmark$$

$$BC = 87,17 \checkmark$$

2

9.1.2 The shortest distance from the beacon to the bank  $AB$ . (3)

$$A = \frac{1}{2} \cdot 87,17 \cdot 100 \cdot \sin 55^\circ$$

$$= 3570,27 \checkmark$$

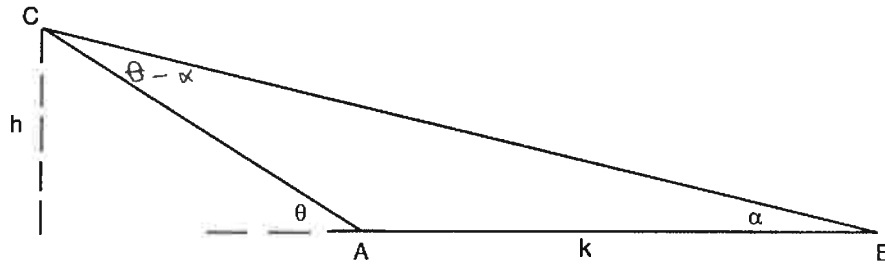
$$A = \frac{1}{2} \cdot 100 \cdot h \checkmark = 3570,27$$

$$h = 71,41 \text{ m} \checkmark$$

3



- 9.2 Joshua stands at a point  $A$  and observes that the angle of elevation of the top of a church spire ( $C$ ) is  $\theta$ . He then walks  $k$  metres east and now finds the angle of elevation from point  $B$  to be  $\alpha$ .



Show that the height of the church is given as:

$$h = \frac{k \cdot \sin \theta \cdot \sin \alpha}{\sin(\theta - \alpha)}$$

$$\sin \theta = \frac{h}{AC} \quad \checkmark \quad (6)$$

$$AC = \frac{h}{\sin \theta} \quad \checkmark$$

$$\frac{k}{\sin(\theta - \alpha)} = \frac{AC}{\sin \alpha} \quad \checkmark$$

$$AC = \frac{k \sin \alpha}{\sin(\theta - \alpha)} \quad \checkmark$$

$$\therefore \frac{h}{\sin \theta} = \frac{k \sin \alpha}{\sin(\theta - \alpha)} \quad \checkmark$$

$$h = \frac{k \cdot \sin \theta \cdot \sin \alpha}{\sin(\theta - \alpha)} \quad \checkmark$$

6

[11]

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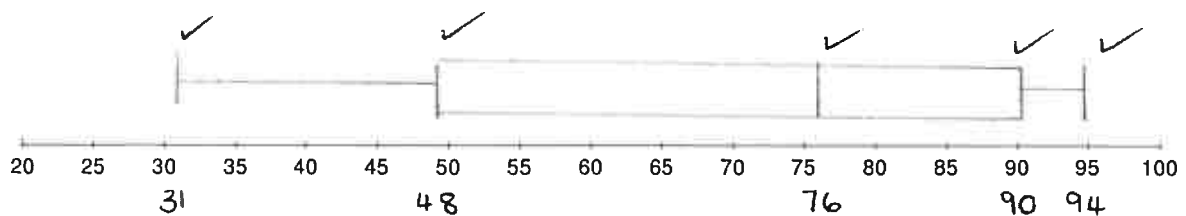
**Question 10**

The Mathematics marks attained by a group of learners is given below:

Mathematics marks

94	92	90	78	78	76
49	48	48	38	31	

- 10.1 Determine the five number summary for Set B and represent it using a box and whisker diagram. (5)



- 10.2 Is the data of Mathematics results skewed to the left, to the right, or neither? Give a reason. (2)

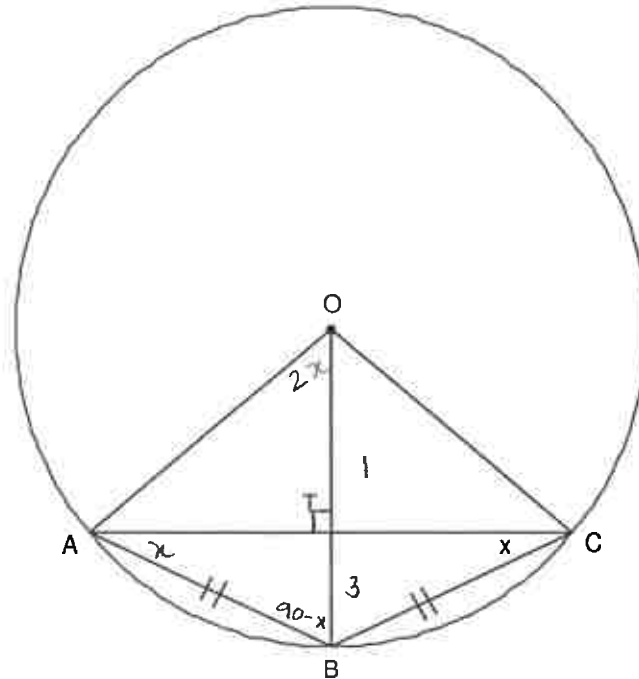
mean < median ✓

skewed to left ✓

[7]

**Question 11**

In the figure  $O$  is the centre of the circle.  $AB = BC$ ,  $\hat{A}BO = \hat{C}BO$  and  $\hat{B}CA = x$ .  $OB \perp AC$



11.1 Determine  $\hat{A}OB$  in terms of  $x$ . (2)

$\hat{A}OB = 2x$  ✓ ∠ at centre ✓

2

11.2 Hence, determine  $\hat{A}BO$  in terms of  $x$ . (3)

$\frac{180 - 2x}{x} = 90 - x = \hat{A}BO$  Base Ls ISOS Δ ✓ (3)

3

11.3 If  $OT:TB = 1:3$  and  $OA = r$ , show that  $AC = \frac{r\sqrt{15}}{2}$  units. (6)

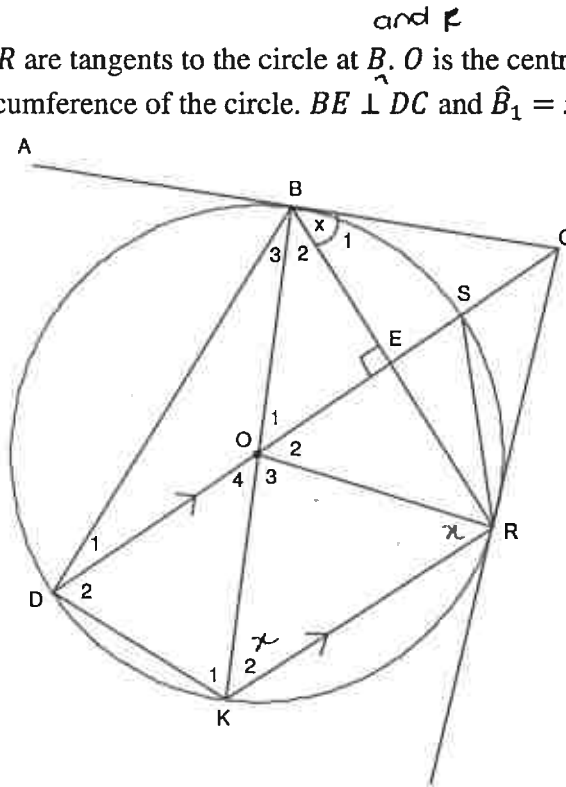
$$\begin{aligned}
 AT^2 &= r^2 - \left(\frac{r}{4}\right)^2 \quad \checkmark \text{ pyth} \\
 AT^2 &= r^2 - \frac{r^2}{16} \\
 AT &= \sqrt{r^2 - \frac{r^2}{16}} \\
 &= r \sqrt{1 - \frac{1}{16}} \\
 &= r \sqrt{\frac{15}{16}} \quad \checkmark \\
 AC &= 2 \times AT \quad \text{line perp to chord} \quad \checkmark \\
 &= 2 \times r \sqrt{\frac{15}{16}} \\
 &= \frac{r \sqrt{15}}{2} \quad \checkmark \text{ units}
 \end{aligned}$$

6

[11]

**Question 12**

In the figure  $AC$  and  $CR$  are tangents to the circle at  $B$ .  $O$  is the centre of the circle.  $B, R, K, S$  and  $D$  are points on the circumference of the circle.  $BE \perp DC$  and  $\hat{B}_1 = x$ .  $BK$  is a diameter and  $DC \parallel KR$ .



12.1 Find 3 other angles equal to  $x$ . (6)

$\hat{K}_2 = x$  ✓ tan-chord  
 $\hat{KRO} = x$  ✓ radii isos  $\Delta$   
 $\hat{O}_4 = x$  ✓ Alt  $\angle$ s  $\checkmark DC \parallel KR$   
 OR  $\hat{O}_1 = x$  vert opp  $\angle$ s

6

12.2 Determine the value of  $\hat{B}_3$  in terms of  $x$ . (2)

$\hat{B}_3 = \frac{1}{2}x$  ✓ Ext  $\angle \Delta$  base  $\angle$ s isos  $\Delta$

2

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Prove that:

12.3  $BCRO$  is a cyclic quadrilateral (4)

$$\hat{O}_2 = x \quad \checkmark \quad \text{Alt } \angle\text{s} \quad \checkmark \quad DC \parallel KR$$

$$\hat{B}_1 = x = \hat{O}_2 \quad \checkmark$$

$\therefore BCRO$  is a cyclic quad  $\angle$ s subtended by line  $CR$  are = 4

12.4  $ABC$  is a tangent to the circle through  $B, O$  and  $E$  (3)

$$\hat{O}_1 = x \quad \checkmark \quad \text{vert opp } \angle\text{s}$$

$$\hat{B}_1 = x = \hat{O}_1 \quad \checkmark$$

$\therefore ABC$  is a tangent to circle  $BOE$  3

$\angle$  between  $BC$  and  $B$  =  $\angle$  subtended by  $BE$ .

[15]

**TOTAL FOR THIS PAPER: [150]**



