basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 11

MATHEMATICS P2

NOVEMBER 2015

MARKS: 150

TIME: 3 hours

This question paper consists of 15 pages and a 24-page answer book.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs et cetera that you used to determine the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. If necessary, round off answers to TWO decimal places, unless stated otherwise.
6. Diagrams are NOT necessarily drawn to scale.
7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
8. Write neatly and legibly.

QUESTION 1

The table below shows the weight (to the nearest kilogram) of each of the 27 participants in a weight-loss programme.

56	68	69	71	71	72	82	84	85
88	89	90	92	93	94	96	97	99
102	103	127	128	134	135	137	144	156

1.1 Calculate the range of the data. (2)

1.2 Write down the mode of the data. (1)

1.3 Determine the median of the data. (1)

1.4 Determine the interquartile range of the data. (3)

1.5 Use the number line provided in the ANSWER BOOK to draw a box and whisker diagram for the data above. (2)

1.6 Determine the standard deviation of the data. (2)

1.7 The person weighing 127 kg states that she weighs more than one standard deviation above the mean. Do you agree with this person? Motivate your answer with calculations. (3)

[14]

QUESTION 2

The table below shows the weight (in grams) that each of the 27 participants in the weight-loss programme lost in total over the first 4 weeks.

WEIGHT LOSS OVER 4 WEEKS (IN GRAMS)	**FREQUENCY**
$1\ 000 < x \leq 1\ 500$	2
$1\ 500 < x \leq 2\ 000$	3
$2\ 000 < x \leq 2\ 500$	3
$2\ 500 < x \leq 3\ 000$	4
$3\ 000 < x \leq 3\ 500$	5
$3\ 500 < x \leq 4\ 000$	7
$4\ 000 < x \leq 4\ 500$	2
$4\ 500 < x \leq 5\ 000$	1

2.1 Estimate the average weight loss, in grams, of the participants over the first 4 weeks. (2)

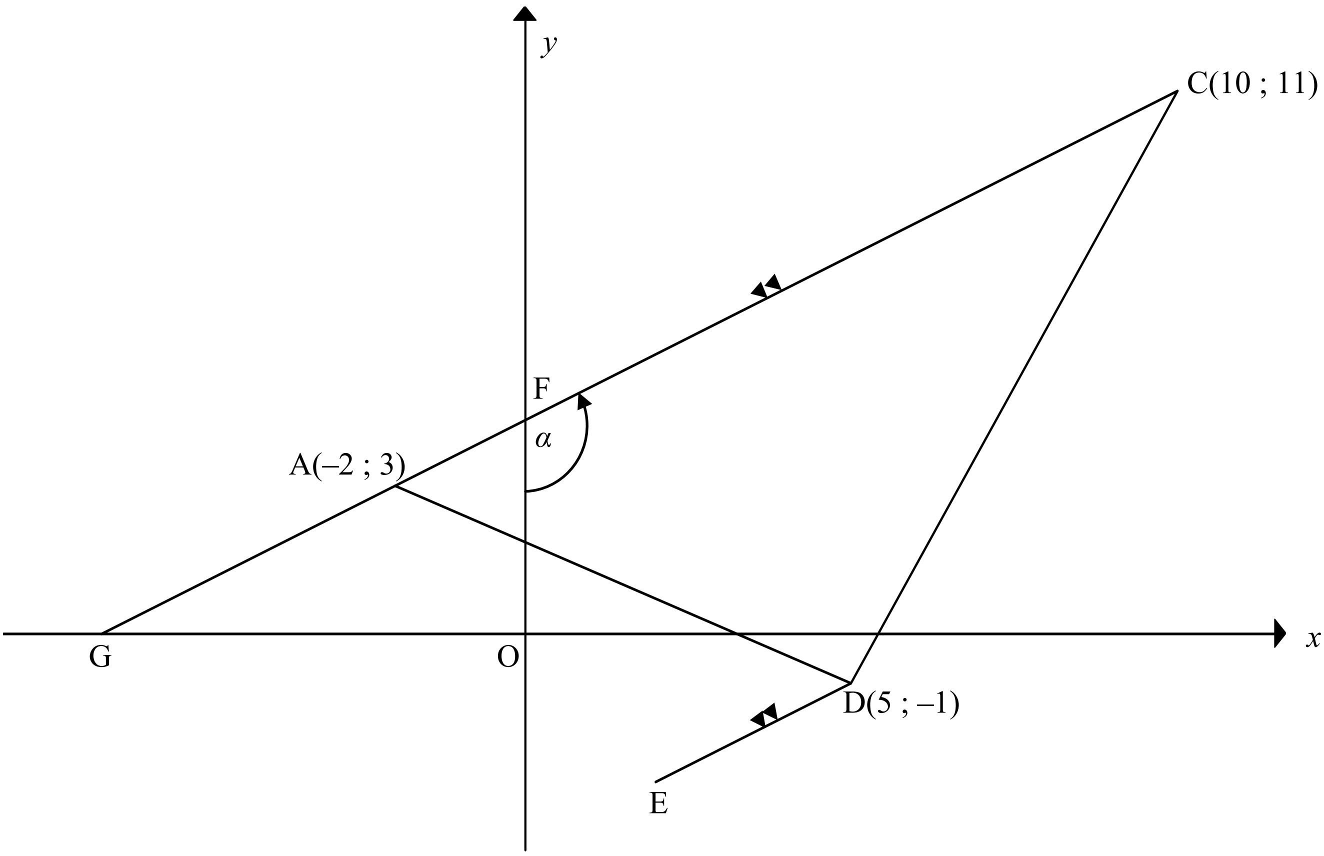
2.2 Draw an ogive (cumulative frequency graph) of the data on the grid provided. (4)

2.3 The weight-loss programme guarantees a loss of 800 g per week if a person follows the programme without cheating. Hence, determine how many of the participants had an average weight loss of 800 g or more per week over the first 4 weeks. (2)

[8]

QUESTION 3

In the diagram, A(–2 ; 3), C(10 ; 11) and D(5 ; –1) are the vertices of ΔACD. CA intersects the y-axis in F and CA produced cuts the x-axis in G. The straight line DE is drawn parallel to CA. $C\hat{F}O = \alpha$.



3.1 Calculate the gradient of the line AC. (2)

3.2 Determine the equation of line DE in the form $y = mx + c$. (3)

3.3 Calculate the size of α. (3)

3.4 B is a point in the first quadrant such that ABDE, in that order, forms a rectangle. Calculate, giving reasons, the:

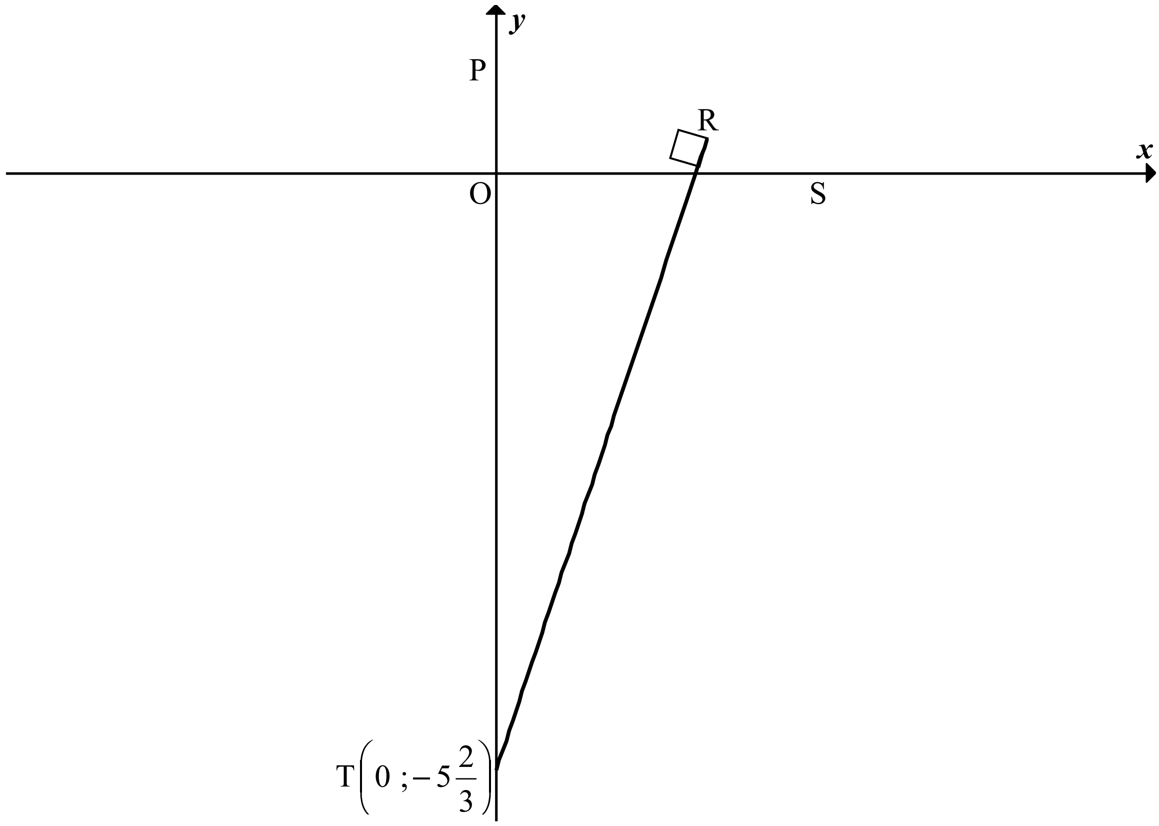
3.4.1 Coordinates of M, the midpoint of BE (3)

3.4.2 Length of diagonal BE (3)

[14]

QUESTION 4

In the diagram, the straight line SP is drawn having S and P as its x- and y-intercepts respectively. The equation of SP is $x + ay - a = 0$, $a > 0$. It is also given that $OS = 3OP$.
The straight line RT is drawn with R on SP and $RT \perp PS$. RT cuts the y-axis in $T\left(0\,;-5\frac{2}{3}\right)$.



4.1 Calculate the coordinates of P. (2)

4.2 Calculate the value of a. (2)

4.3 Determine the equation of RT in the form $y = mx + c$ if it is given that $a = 3$. (3)

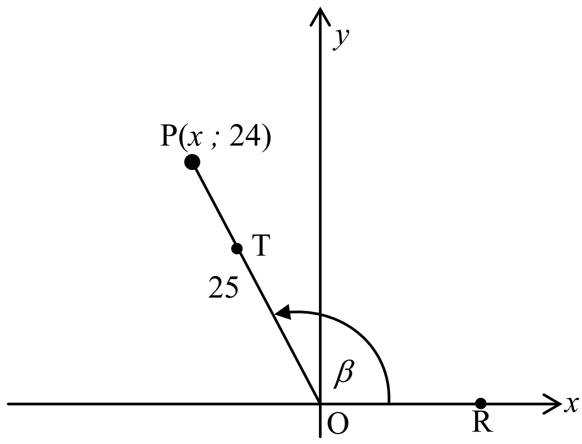
4.4 Calculate the coordinates of R, the point where PS and TR meet. (4)

4.5 Calculate the area of ΔPRT if it is given that $R\left(2\,;\,\frac{1}{3}\right)$. (3)

4.6 Calculate, giving reasons, the radius of a circle passing through the points P, R and T. (2)
[16]

QUESTION 5

5.1 In the diagram below, $P(x\,;24)$ is a point such that $OP = 25$ and $R\hat{O}P = \beta$, where β is an obtuse angle.



5.1.1 Calculate the value of x. (2)

5.1.2 Determine the value of each of the following WITHOUT using a calculator:

(a) $\sin\beta$ (1)

(b) $\cos(180° - \beta)$ (2)

(c) $\tan(-\beta)$ (2)

5.1.3 T is a point on OP such that $OT = 15$. Determine the coordinates of T WITHOUT using a calculator. (4)

5.2 Determine the value of the following expression:

$$\frac{2\sin x.\cos x\ (1+\tan^2 x)}{\tan x}$$ (4)

5.3 Consider: $\dfrac{1-\cos^2 A}{4\cos(90°+A)}$

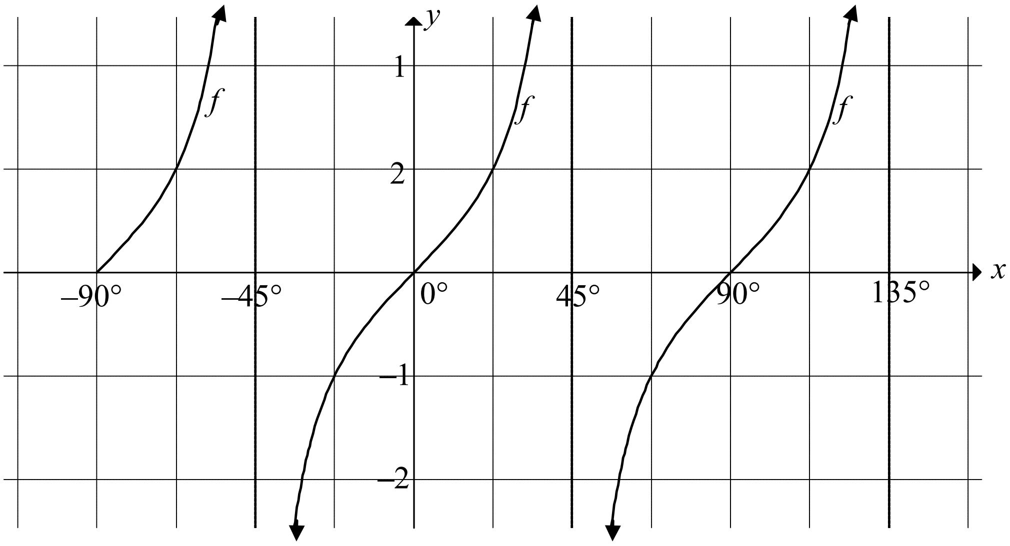
5.3.1 Simplify the expression to a single trigonometric term. (3)

5.3.2 Hence, determine the general solution of $\dfrac{1-\cos^2 2x}{4\cos(90°+2x)} = 0{,}21$. (6)

[24]

QUESTION 6

6.1 In the diagram, the graph of $f(x) = \tan bx$ is drawn for the interval $-90° \leq x \leq 135°$.

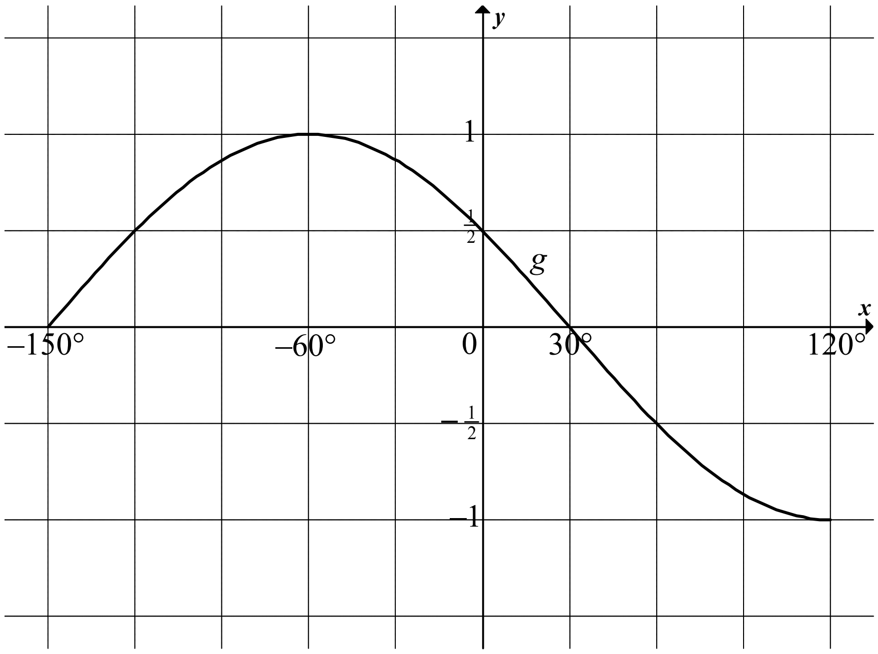


6.1.1 Determine the value of b. (1)

6.1.2 Determine the values of x in the interval $0° \leq x \leq 135°$ for which $f(x) \leq -1$. (2)

6.1.3 Graph h is defined as $h(x) = \tan b(x + 55°)$. Write down the equations of the asymptotes of h in the interval $-90° \leq x \leq 135°$. (2)

6.2 In the diagram, the graph of $g(x) = \cos(x + 60°)$ is drawn for the interval $-150° \leq x \leq 120°$.



6.2.1 On the same system of axes, draw the graph of $k(x) = -\sin x$ for the interval $-150° \leq x \leq 120°$. Show ALL the intercepts with the axes as well as the coordinates of the turning points and end points of the graph. (4)

6.2.2 Determine the minimum value of $h(x) = \cos(x + 60°) - 3$. (2)

6.2.3 Solve the equation $\cos(x + 60°) + \sin x = 0$ for the interval $-150° \leq x \leq 120°$. (6)

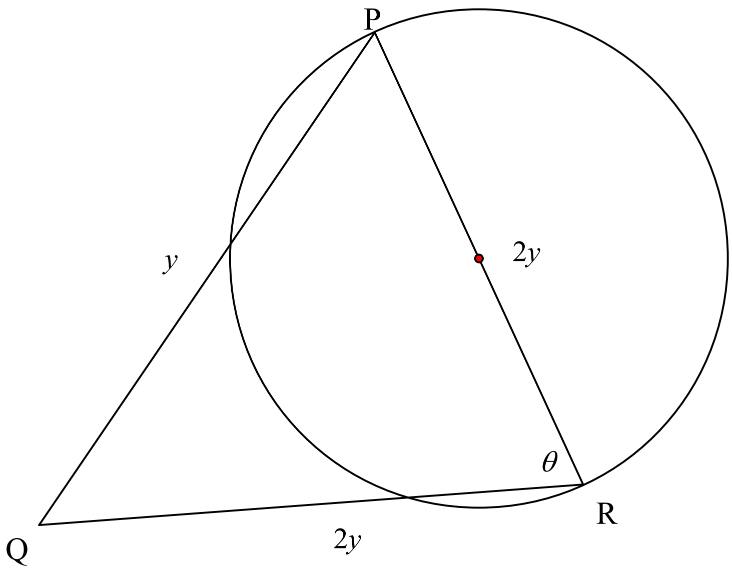
6.2.4 Determine the values of x for the interval $-150° \leq x \leq 120°$, for which $\cos(x + 60°) + \sin x > 0$. (2)

6.2.5 The function g can also be defined as $y = -\sin(x - \theta)$, where θ is an acute angle. Determine the value of θ. (2)

[21]

QUESTION 7

In the diagram, PR is the diameter of the circle. Triangle PQR is drawn with vertex Q outside the circle. $\hat{R} = \theta$, PR = QR = $2y$ and PQ = y.



7.1 Determine the value of $\cos\theta$. (4)

7.2 If QR cuts the circumference of the circle at T, determine PT in terms of y and θ. (3)

[7]

QUESTION 8

A cylindrical aerosol can has a lid in the shape of a hemisphere that fits exactly on the top of the can. The height of the can is 16 cm and the radius of the base of the can is 2,9 cm.

Volume of sphere = $\frac{4}{3}\pi r^3$
Surface area of sphere = $4\pi r^2$

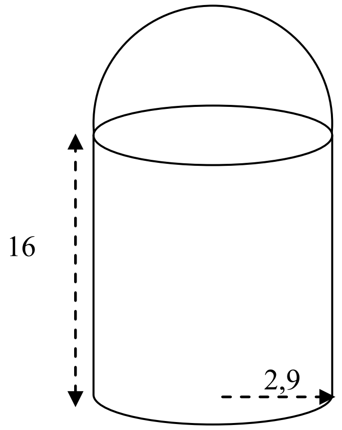


FIGURE 1

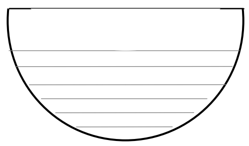


FIGURE 2

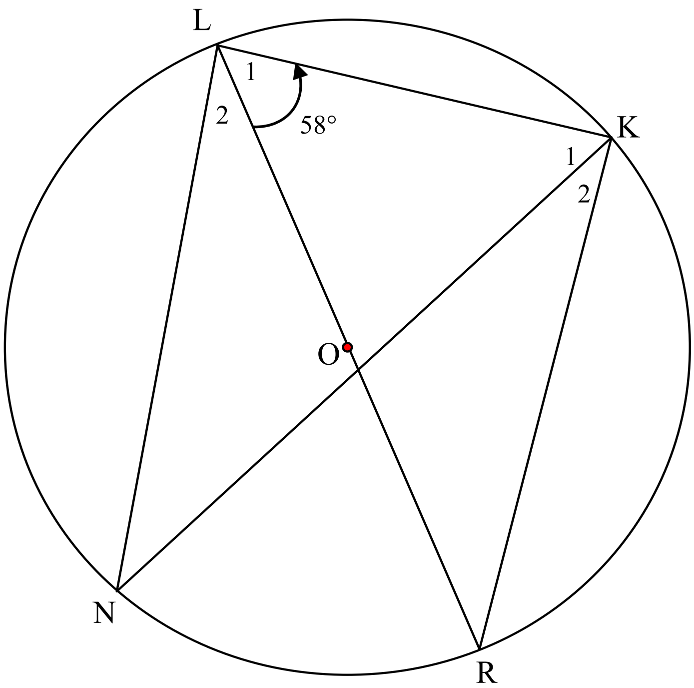
8.1 Calculate the surface area of the can with the lid in place, as shown in FIGURE 1. (5)

8.2 If the lid is 80% filled with a liquid, as shown in FIGURE 2, calculate the volume of the liquid in the lid. (3)
[8]

Give reasons for your statements and calculations in QUESTIONS 9, 10 and 11.

QUESTION 9

In the diagram, O is the centre of the circle. Diameter LR subtends $L\hat{K}R$ at the circumference of the circle. N is another point on the circumference and chords LN and KN are drawn. $\hat{L}_1 = 58°$.



Calculate, giving reasons, the size of:

9.1 $L\hat{K}R$ (2)

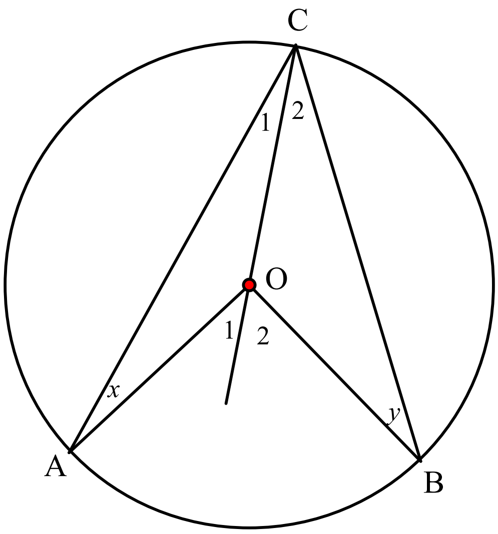
9.2 $\hat{R}$ (2)

9.3 $\hat{N}$ (2)

[6]

QUESTION 10

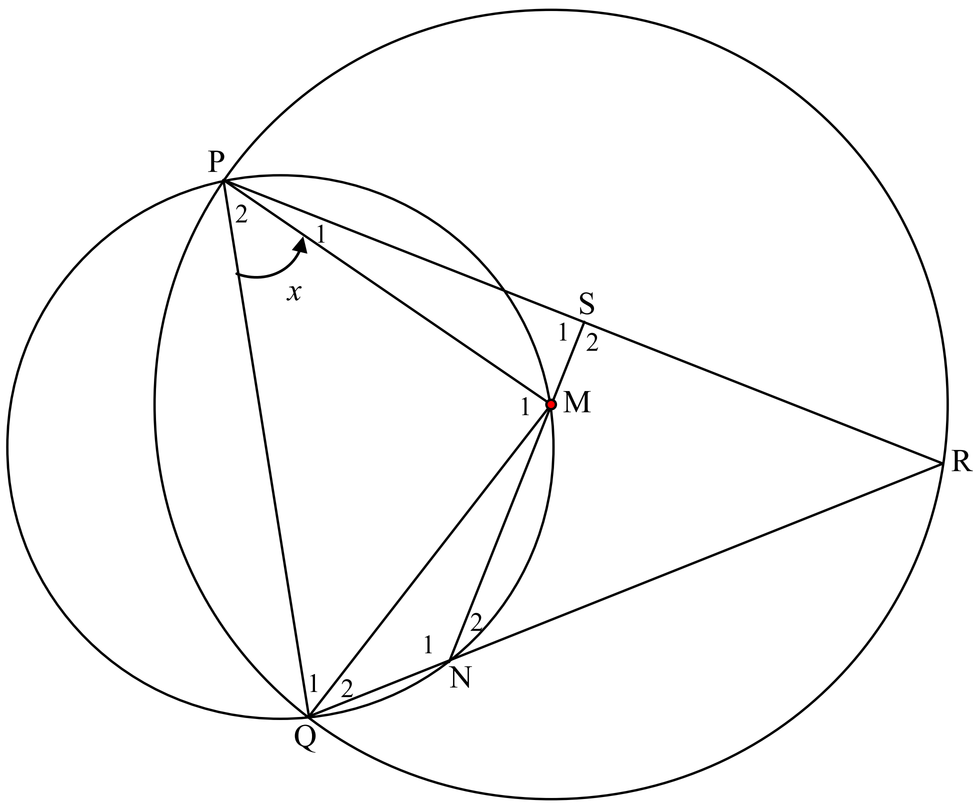
10.1 In the diagram, O is the centre of the circle. A, B and C are points on the circumference of the circle. Chords AC and BC and radii AO, BO and CO are drawn. $\hat{A} = x$ and $\hat{B} = y$.



10.1.1 Determine the size of $\hat{O}_1$ in terms of x. (3)

10.1.2 Hence, prove the theorem that states that the angle subtended by an arc at the centre is equal to twice the angle subtended by the same arc at the circumference, that is $A\hat{O}B = 2A\hat{C}B$. (3)

10.2 In the diagram, PQ is a common chord of the two circles. The centre, M, of the larger circle lies on the circumference of the smaller circle. PMNQ is a cyclic quadrilateral in the smaller circle. QN is produced to R, a point on the larger circle. NM produced meets the chord PR at S. $\hat{P}_2 = x$.



10.2.1 Give a reason why $\hat{N}_2 = x$. (1)

10.2.2 Write down another angle equal in size to x. Give a reason. (2)

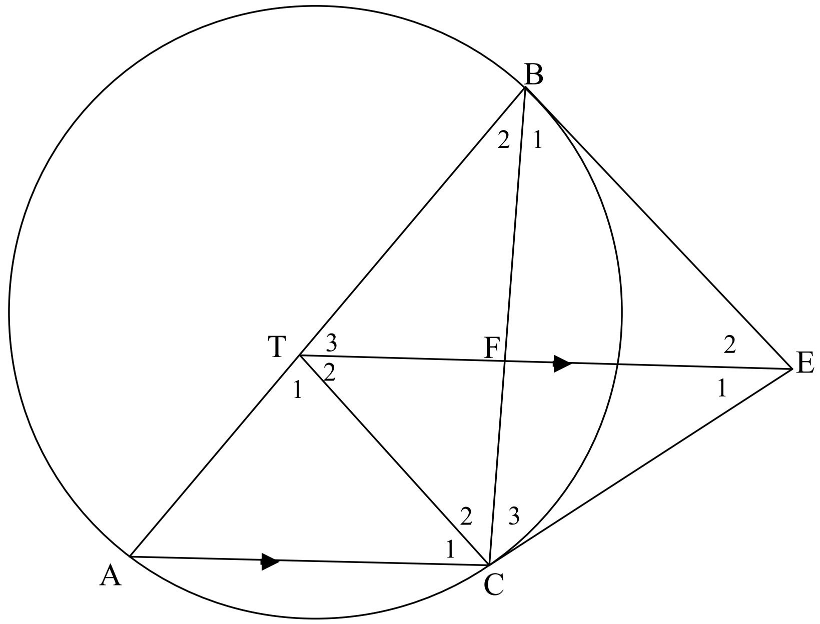
10.2.3 Determine the size of $\hat{R}$ in terms of x. (3)

10.2.4 Prove that PS = SR. (3)

[15]

QUESTION 11

In the diagram, the vertices A, B and C of ΔABC are concyclic. EB and EC are tangents to the circle at B and C respectively. T is a point on AB such that TE | | AC. BC cuts TE in F.



11.1 Prove that $\hat{B}_1 = \hat{T}_3$. (4)

11.2 Prove that TBEC is a cyclic quadrilateral. (4)

11.3 Prove that ET bisects $B\hat{T}C$. (2)

11.4 If it is given that TB is a tangent to the circle through B, F and E, prove that TB = TC. (4)

11.5 Hence, prove that T is the centre of the circle through A, B and C. (3)
[17]

TOTAL: 150