



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 11

MATHEMATICS P2

EXEMPLAR 2013

MEMORANDUM

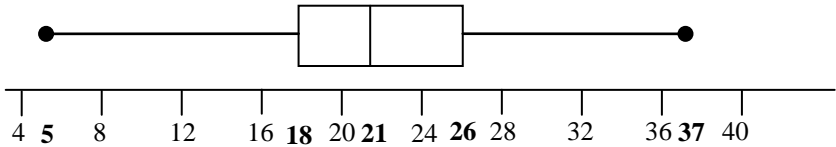
MARKS: 150

This memorandum consists of 13 pages.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum.
- Assuming answers/values in order to solve a problem is NOT acceptable.

QUESTION 1

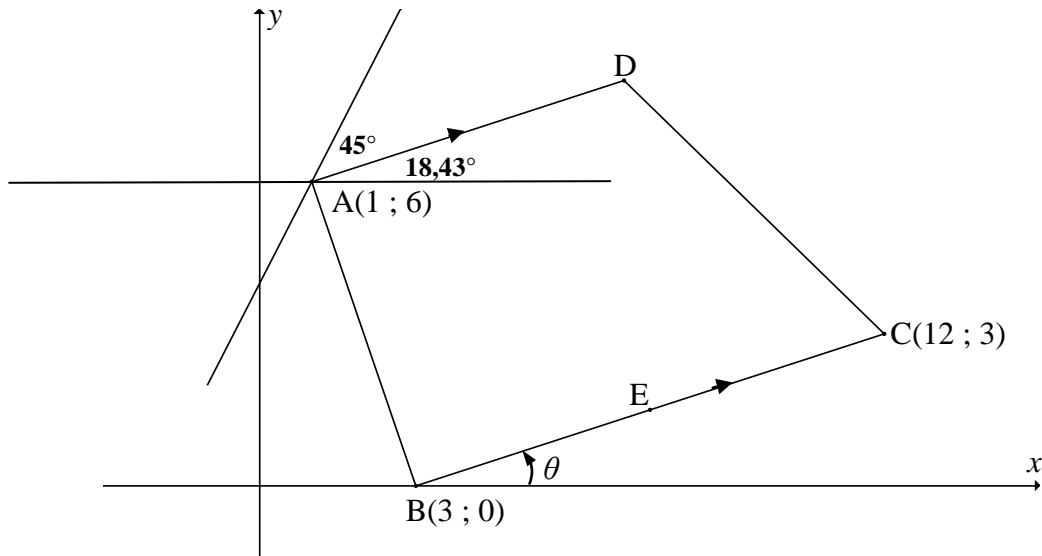
1.1	$\text{Mean} = \frac{\sum_{i=1}^n x_i}{n} = \frac{408}{19} = 21,47$	$\checkmark \frac{408}{19}$ \checkmark answer (2)																				
1.2	Standard deviation = 7,81	$\checkmark \checkmark$ answer (2)																				
1.3	The one standard deviation limits are $(\bar{x} - 1\sigma; \bar{x} + 1\sigma)$ $= (21,47 - 7,81; 21,47 + 7,81) = (13,66 ; 29,28)$ \therefore 13 people lie within 1 standard deviation of the mean.	\checkmark interval \checkmark 13 people (2)																				
1.4	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="text-align: center;">5</td> <td style="text-align: center;">12</td> <td style="text-align: center;">13</td> <td style="text-align: center;">15</td> <td style="text-align: center;">18</td> <td style="text-align: center;">18</td> <td style="text-align: center;">18</td> <td style="text-align: center;">19</td> <td style="text-align: center;">20</td> <td style="text-align: center;">21</td> </tr> <tr> <td style="text-align: center;">21</td> <td style="text-align: center;">22</td> <td style="text-align: center;">23</td> <td style="text-align: center;">23</td> <td style="text-align: center;">26</td> <td style="text-align: center;">29</td> <td style="text-align: center;">33</td> <td style="text-align: center;">35</td> <td style="text-align: center;">37</td> <td></td> </tr> </table> <p>IQR = 26 – 18 = 8</p>	5	12	13	15	18	18	18	19	20	21	21	22	23	23	26	29	33	35	37		\checkmark $Q_1 = 18$ \checkmark $Q_3 = 26$ \checkmark IQR = 8 (3)
5	12	13	15	18	18	18	19	20	21													
21	22	23	23	26	29	33	35	37														
1.5		$\checkmark \checkmark$ box \checkmark whiskers (3)																				
1.6	There is a marked difference between the lowest value (5) and the next lowest value (12) whilst the differences between all other data points are within at most 3 values. \therefore 5 is an outlier	\checkmark reason \checkmark 5 is an outlier (2) [14]																				

QUESTION 2

<p>2.1</p>	<table border="1"> <thead> <tr> <th>Class</th> <th>Frequency</th> <th>Cumulative frequency</th> </tr> </thead> <tbody> <tr> <td>$0 \leq m < 2$</td> <td>7</td> <td>7</td> </tr> <tr> <td>$2 \leq m < 4$</td> <td>15</td> <td>22</td> </tr> <tr> <td>$4 \leq m < 6$</td> <td>26</td> <td>48</td> </tr> <tr> <td>$6 \leq m < 8$</td> <td>29</td> <td>77</td> </tr> <tr> <td>$8 \leq m < 10$</td> <td>36</td> <td>113</td> </tr> <tr> <td>$10 \leq m < 12$</td> <td>31</td> <td>144</td> </tr> <tr> <td>$12 \leq m < 14$</td> <td>14</td> <td>158</td> </tr> <tr> <td>$14 \leq m < 16$</td> <td>2</td> <td>160</td> </tr> </tbody> </table>	Class	Frequency	Cumulative frequency	$0 \leq m < 2$	7	7	$2 \leq m < 4$	15	22	$4 \leq m < 6$	26	48	$6 \leq m < 8$	29	77	$8 \leq m < 10$	36	113	$10 \leq m < 12$	31	144	$12 \leq m < 14$	14	158	$14 \leq m < 16$	2	160	<p>✓ first three cumulative frequencies correct ✓ remainder correct (total = 160) (2)</p>
Class	Frequency	Cumulative frequency																											
$0 \leq m < 2$	7	7																											
$2 \leq m < 4$	15	22																											
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$12 \leq m < 14$	14	158																											
$14 \leq m < 16$	2	160																											
<p>2.2</p>		<p>✓ grounding at 0 ✓ plotting cumulative frequencies at upper limits ✓ smooth shape of curve (3)</p>																											

2.3	The median for the data is approximately 8 messages.	✓Median (1)
2.4	Approximately 130 learners sent 11 or fewer messages. Therefore 30 learners sent more than 11 messages. $\frac{30}{160} \times 100\% = 18,75\%$	✓30 learners ✓answer (2)
2.5	Skewed to the left or negatively skewed	✓answer (1) [9]

QUESTION 3



3.1	$E\left(\frac{3+12}{2}; \frac{0+3}{2}\right)$ $= \left(7\frac{1}{2}; 1\frac{1}{2}\right)$	✓ substitution into midpoint formula ✓ answer (2)
3.2	$m_{BC} = \frac{3-0}{12-3}$ $= \frac{1}{3}$	✓ substitution into gradient formula ✓ answer (2)
3.3	$\tan \theta = m_{BC} = \frac{1}{3}$ $\theta = \tan^{-1}\left(\frac{1}{3}\right) = 18,43^\circ$	✓ $\tan \theta = m_{BC}$ ✓ answer (2)
3.4	$m_{AD} = m_{BC} = \frac{1}{3} \quad AD \parallel BC, \text{ equal gradients}$ $m_{AB} = \frac{6-0}{1-3} = -3$ $\therefore m_{AD} \times m_{AB} = \frac{1}{3} \times -3 = -1$ $\therefore AD \perp AB$	✓ $m_{AD} = \frac{1}{3}$ ✓ $m_{AB} = -3$ ✓ $m_{AD} \times m_{AB} = -1$ (3)
3.5	inclination of new line = $45^\circ + 18,43^\circ = 63,43^\circ$ $\therefore \tan 63,43^\circ = 2 = m_{line}$ $\therefore y - 6 = 2(x - 1)$ $y = 2x + 4$	✓ $18,43^\circ$ ✓ $63,43^\circ$ ✓ $m = 2$ ✓ subst of (1 ; 6) ✓ equation (5) [14]

QUESTION 4

4.1	$m_{QP} = m_{OS} = 6$ $QP \parallel OS$, equal gradients $y - 17 = 6(x + 3)$ $y = 6x + 35$	$\checkmark m_{QP} = 6$ \checkmark subst $(-3 ; 17)$ into formula \checkmark equation (3)
4.2	$6x + 35 = -x$ $7x = -35$ $x = -5$ $y = -(-5) = 5$ $\therefore Q(-5 ; 5)$	OR $y = 6(-5) + 35 = 5$ \checkmark setting up equation $\checkmark x = -5$ $\checkmark y = 5$ \checkmark coordinates of Q (4)
4.3	$OQ^2 = (-5 - 0)^2 + (5 - 0)^2$ $= 50$ $OQ = \sqrt{50} = 5\sqrt{2}$ units	\checkmark substitution into distance formula $\checkmark 5\sqrt{2}$ (2)
4.4	$m_{OS} = 6$ \therefore inclination of OS is $\tan^{-1}(6) = 80,54^\circ$ $m_{OQ} = -1$ \therefore inclination of QO is $180^\circ - \tan^{-1}(1) = 135^\circ$ $\alpha = 135^\circ - 80,54\dots^\circ$ $= 54,46^\circ$	$\checkmark 80,54^\circ$ $\checkmark 135^\circ$ $\checkmark 54,46^\circ$ (3)
4.5	$QS^2 = OS^2 + OQ^2 - 2OS \cdot OQ \cdot \cos \alpha$ $= 148 + 50 - 2(\sqrt{148})(\sqrt{50} \cdot \cos 54,46^\circ)$ $QS = 9,90$ units	\checkmark correct use of cosine rule \checkmark substitution into formula $\checkmark 9,90$ (3) [15]

QUESTION 5

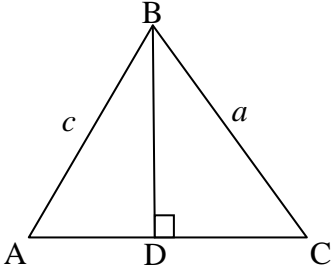
5.1.1	$\cos \alpha = -\frac{5}{13}$	$\checkmark -\frac{5}{13}$ (1)
5.1.2	$(-5)^2 + b^2 = 13^2$ $b^2 = 169 - 25 = 144$ $b = 12$ $\tan (180^\circ - \alpha)$ $= -\tan \alpha$ $= -\left(-\frac{12}{5}\right)$ $= \frac{12}{5}$	$\checkmark b = 12$ $\checkmark -\tan \alpha$ $\checkmark \frac{12}{5}$ (3)
5.2.1	$\frac{\sin(\theta - 360^\circ) \sin(90^\circ - \theta) \tan(-\theta)}{\cos(90^\circ + \theta)}$ $= \frac{\sin \theta \cos \theta (-\tan \theta)}{-\sin \theta}$ $= -\cos \theta \left(-\frac{\sin \theta}{\cos \theta} \right)$ $= \sin \theta$	$\checkmark \checkmark \checkmark$ reductions $\checkmark \tan \theta = \frac{\sin \theta}{\cos \theta}$ $\checkmark \sin \theta$ (5)
5.2.2	From 5.2.1: $\sin \theta = 0,5$ Ref $\angle = 30^\circ$ $\therefore \theta = 30^\circ$ or $\theta = 150^\circ$	$\checkmark \sin \theta = 0,5$ $\checkmark 30^\circ$ $\checkmark 150^\circ$ (3)

5.3.1	$LHS = \frac{8}{\sin^2 A} - \frac{4}{1 + \cos A}$ $= \frac{8}{1 - \cos^2 A} - \frac{4}{1 + \cos A}$ $= \frac{8}{(1 - \cos A)(1 + \cos A)} - \frac{4}{1 + \cos A}$ $= \frac{8 - 4(1 - \cos A)}{(1 - \cos A)(1 + \cos A)}$ $= \frac{8 - 4 + 4 \cos A}{(1 - \cos A)(1 + \cos A)}$ $= \frac{4(1 + \cos A)}{(1 - \cos A)(1 + \cos A)}$ $= \frac{4}{1 - \cos A} = RHS$	<p>✓ $\sin^2 A = 1 - \cos^2 A$</p> <p>✓ factorising</p> <p>✓ addition</p> <p>✓ simplification</p> <p>✓ factorising (5)</p>
5.3.2	<p>Identity is undefined when $\sin^2 A = 0$. That is when $\sin A = 0$ or $\cos A = \pm 1$</p> <p>$\therefore A = 0^\circ$ or $A = 180^\circ$ or $A = 360^\circ$.</p>	<p>✓✓✓ each value (3)</p>
5.4	$8 \cos^2 x - 2 \cos x - 1 = 0$ $(4 \cos x + 1)(2 \cos x - 1) = 0$ $\cos x = -\frac{1}{4} \text{ or } \cos x = \frac{1}{2}$ $\therefore x = 104,48^\circ + k \cdot 360^\circ; k \in Z \text{ or } x = 60^\circ + k \cdot 360^\circ; k \in Z$ $x = 255,52^\circ + k \cdot 360^\circ; k \in Z \quad x = 300^\circ + k \cdot 360^\circ; k \in Z$	<p>✓ factorising</p> <p>✓ values of $\cos x$</p> <p>✓ $104,48^\circ$ or $255,52^\circ$</p> <p>✓ 60° or 300°</p> <p>✓ $+ 360^\circ \cdot k$</p> <p>✓ $k \in Z$ (6)</p> <p>[26]</p>

QUESTION 6

6.1	$p = -45^\circ$ $q = -1$	✓ value of p ✓ value of q (2)
6.2	B(157,5° ; -0,38)	✓ value of x ✓ value of y (2)
6.3	$f(x) < g(x)$ when $-180^\circ \leq x < -22,5^\circ$ or $157,5^\circ < x \leq 180^\circ$	✓ $-180^\circ \leq x < -22,5^\circ$ ✓ $157,5^\circ < x \leq 180^\circ$ (2)
6.4.1	$h(x) = \cos(x - 45^\circ + 30^\circ)$ $= \cos(x - 15^\circ)$	✓ $+30^\circ$ ✓ simplest form (2)
6.4.2	$x = -135^\circ - 30^\circ = -165^\circ$	✓ -165° (1) [9]

QUESTION 7

7.1	<p>Draw $BD \perp AC$</p> <p>In $\triangle ABD$:</p> $\sin A = \frac{BD}{c} \therefore BD = c \cdot \sin A$ <p>In $\triangle CBD$:</p> $\sin C = \frac{BD}{a} \therefore BD = a \cdot \sin C$ $\therefore c \cdot \sin A = a \cdot \sin C$ $\therefore \frac{\sin A}{a} = \frac{\sin C}{c}$	 <p>✓ construction ✓ $\sin A$ ✓ making BD the subject ✓ $\sin C$ ✓ $c \cdot \sin A = a \cdot \sin C$ (5)</p>
7.2.1	$\frac{\sin R}{r} = \frac{\sin P}{p}$ $\frac{\sin R}{27,2} = \frac{\sin 132^\circ}{73,2}$ $\sin R = \frac{27,2 \times \sin 132^\circ}{73,2}$ $= 0,276\dots$ $\hat{R} = 16,03^\circ$	<p>✓ substitution into correct formula ✓ making $\sin R$ the subject ✓ $16,03^\circ$ (3)</p>

7.2.2	$\hat{Q} = 180^\circ - 132^\circ - 16,03^\circ = 31,97^\circ$ $\text{area of PQR} = \frac{1}{2} pr \cdot \sin Q$ $= \frac{1}{2} (73,2)(27,2) \cdot \sin 31,97^\circ$ $= 527,10 \text{ cm}^2$	$\checkmark \hat{Q} = 31,97^\circ$ $\checkmark \text{ substitution into correct formula}$ $\checkmark 527,1$ <p style="text-align: right;">(3)</p>
7.3.1	$P\hat{S}Q = 180^\circ - (a + b)$ <p>In $\triangle PSQ$:</p> $\frac{SQ}{\sin P} = \frac{PQ}{\sin P\hat{S}Q}$ $\frac{SQ}{\sin a} = \frac{h}{\sin[180^\circ - (a + b)]}$ $\frac{SQ}{\sin a} = \frac{h}{\sin(a + b)}$ $SQ = \frac{h \sin a}{\sin(a + b)}$	$\checkmark P\hat{S}Q = 180^\circ - (a + b)$ $\checkmark \sin[180^\circ - (a + b)] = \sin(a + b)$ $\checkmark \text{ making SQ the subject}$ <p style="text-align: right;">(3)</p>
7.3.2	$S\hat{Q}R = 90^\circ - b$ <p>In $\triangle RSQ$:</p> $\frac{RS}{SQ} = \sin S\hat{Q}R$ $RS = SQ \cdot \sin(90^\circ - b)$ $= \frac{h \sin a}{\sin(a + b)} \cdot \cos b$ $= \frac{h \sin a \cdot \cos b}{\sin(a + b)}$	$\checkmark S\hat{Q}R = 90^\circ - b$ $\checkmark \text{ use sine ratio correctly}$ $\checkmark \sin(90^\circ - b) = \cos b$ <p style="text-align: right;">(3)</p> <p style="text-align: right;">[17]</p>

QUESTION 8

	<p>Volume of hemisphere</p> $= \frac{1}{2} \left[\frac{4}{3} \pi r^3 \right]$ $= \frac{2}{3} \pi (3)^3$ $= 18\pi \text{ cm}^3$ <p>Volume of conical hole</p> $= \frac{1}{3} \pi r^2 h$ $= \frac{1}{3} \pi (1,5)^2 \left(\frac{8}{9} \right)$ $= \frac{2}{3} \pi \text{ cm}^3$ $\therefore \frac{\text{volume of metal A}}{\text{volume of metal B}} = \frac{17 \frac{1}{3} \pi}{\frac{2}{3} \pi} = \frac{26}{1}$ <p>Ratio of volume metal A : Volume metal B = 26 : 1</p>	<p>✓ substitution into correct formula</p> <p>✓ 18 π</p> <p>✓ substitution into correct formula</p> <p>✓ $\frac{2}{3} \pi$</p> <p>✓ $17 \frac{1}{3} \pi$</p> <p>✓ ratio 26 : 1</p> <p style="text-align: right;">(6) [6]</p>
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QUESTION 9

9.1	...bisects the chord.	✓ answer (1)
9.2.1	<p>OE = 10 cm ... O midpoint of DE</p> <p>OC = OE – CE</p> <p> = 10 – 2</p> <p> = 8 cm</p>	<p>✓ OE = 10</p> <p>✓ OC = 8</p> <p>(2)</p>
9.2.2	<p>In ΔCOQ:</p> <p>$QC^2 = OQ^2 - OC^2$... Theorem of Pythagoras</p> <p> = $(10)^2 - (8)^2$</p> <p> = 36</p> <p>QC = 6 cm</p> <p>∴ PQ = 2QC ... line drawn from centre ⊥ to chord bisects chord</p> <p>PQ = 12 cm</p>	<p>✓ Using Theorem of Pythagoras</p> <p>✓ QC = 6</p> <p>✓ PQ = 12 (S)</p> <p>✓ reason</p> <p>(4) [7]</p>

QUESTION 10

<p>10.1</p>	<div style="text-align: center;"> </div> <p>Construction: Produce DO to E Proof: In $\triangle OBD$: $O\hat{B}D = O\hat{D}B$... $OD = OB = r$ $E\hat{O}B = 2 \times O\hat{D}B$... exterior angle of triangle In $\triangle AOD$: $O\hat{A}D = O\hat{D}A$... $OA = OD = r$ $E\hat{O}A = 2 \times O\hat{D}A$... exterior angle of triangle $A\hat{O}B = E\hat{O}B + E\hat{O}A$ $= 2 \times O\hat{D}B + 2 \times O\hat{D}A$ $= 2(O\hat{D}B + O\hat{D}A)$ $= 2A\hat{D}B$</p>	<p>✓ construction</p> <p>✓ $O\hat{B}D = O\hat{D}B$ ✓ $E\hat{O}B = 2 \times O\hat{D}B$ (S/R)</p> <p>✓ $E\hat{O}A = 2 \times O\hat{D}A$ (S/R) ✓ $A\hat{O}B = E\hat{O}B + E\hat{O}A$</p> <p style="text-align: right;">(5)</p>
<p>10.2.1(a)</p>	<p>$\hat{M} = 76^\circ$... \angle at centre = $2(\angle$ at circumference)</p>	<p>✓ 76° ✓ reason</p> <p style="text-align: right;">(2)</p>
<p>10.2.1(b)</p>	<p>$\hat{T}_2 = 38^\circ$... ext \angle of cyc quad KTAB</p>	<p>✓ 38° ✓ reason</p> <p style="text-align: right;">(2)</p>
<p>10.2.1(c)</p>	<p>$\hat{C} = 38^\circ$... ext \angle of cyclic quad or \angle^s in same segment</p>	<p>✓ 38° ✓ reason</p> <p style="text-align: right;">(2)</p>
<p>10.2.1(d)</p>	<p>$\hat{C}\hat{A}N = \hat{C} = 38^\circ$... $NA = NC$ $\hat{K}_4 = 38^\circ$... ext \angle of cyclic quad CATK</p>	<p>✓ $\hat{C}\hat{A}N = 38^\circ$ (S/R) ✓ $\hat{K}_4 = 38^\circ$</p> <p style="text-align: right;">(2)</p>
<p>10.2.2</p>	<p>$\therefore \hat{K}_4 = \hat{T}_2$ $\therefore NK = NT$... base \angle^s equal</p>	<p>✓ statement ✓ reason</p> <p style="text-align: right;">(2)</p>
<p>10.2.3</p>	<p>$\hat{N} = 180^\circ - (38^\circ + 38^\circ)$... \angle^s of $\triangle KNT$ $= 104^\circ$ $\hat{N} + \hat{K}\hat{M}A = 104^\circ + 76^\circ = 180^\circ$ \therefore AMKN is cyclic quad ... opposite $\angle^s = 180^\circ$</p>	<p>✓ $\hat{N} = 104^\circ$ (S/R) ✓ $\hat{N} + \hat{K}\hat{M}A = 180^\circ$ ✓ reason</p> <p style="text-align: right;">(3) [18]</p>

QUESTION 11

11.1 equal to the angle subtended by the same chord in the alternate segment.	✓ alternate segment (1)
11.2.1	$\hat{A}_1 = \hat{C}_2 = x$...tangent chord theorem $\hat{C}_2 = \hat{G}_2 = x$...tangent chord theorem $\therefore \hat{A}_1 = \hat{G}_2 = x$ $\therefore BCG \parallel EA$...alternate $\angle^s =$	✓ $\hat{A}_1 = \hat{C}_2 = x$ ✓ reason ✓ $\hat{C}_2 = \hat{G}_2 = x$ ✓ reason ✓ conclusion with reason (5)
11.2.2	$\hat{E}_1 = \hat{C}_3 = y$...alternate \angle^s ; $BG \parallel EA$ $\hat{F}_1 = \hat{C}_3 = y$...ext \angle of cyclic quad CDFG $\therefore \hat{E}_1 = \hat{F}_1 = y$ $\therefore EA$ is a tangent ...converse tangent-chord theorem	✓ $\hat{E}_1 = \hat{C}_3 = y$ (S/R) ✓ $\hat{F}_1 = \hat{C}_3 = y$ (S) ✓ reason ✓ $\hat{E}_1 = \hat{F}_1 = y$ ✓ reason (5)
11.2.3	$\hat{B} = \hat{CAE}$...tangent-chord theorem $\hat{C}_1 = \hat{CAE}$... alternate \angle^s ; $BG \parallel EA$ $\hat{C}_1 = \hat{B}$ $\therefore AB = AC$...base $\angle^s =$	✓ $\hat{CAE} = \hat{B}$ ✓ reason ✓ $\hat{C}_1 = \hat{CAE}$ (S/R) ✓ reason (4) [15]

TOTAL: 150