Q1 – Q5.1.5 Linda 52 marks

Q5.1.5 – Q11 Margie 51 marks

GRADE 11 EXAMINATION

NOVEMBER 2016

*4.2

DURBAN GIRLS' COLLEGE

MATHEMATICS PAPER 2

Q12 – 15 Tiffany 47 marks

Time: 3 hours

150 marks

Examiners: Miss Eastes; Mrs Rixon

Gr11 VR Changes

Moderator: Mrs. Thorne, Mrs. Dwyer

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. Read the questions carefully. Answer all the questions on the Question Paper.
2. You may use an approved, non-programmable, and non-graphical calculator, unless otherwise stated.
3. Round off your answers to **ONE DECIMAL PLACE**, where necessary unless otherwise indicated. All the necessary working details must be clearly shown.
4. It is in your own interest to write legibly and to present your work neatly.
5. Diagrams are not drawn to scale.
6. Please note that there is an information sheet provided.
7. Give reasons for all geometry statements.

* Correct numbering for 16 + 17.

Name: ______________________________

Teacher: ______________________________

Marking Grid (for Educators' use only)

Section A:

	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8
Marks Earned								
Total Marks	14	10	7	7	27	3	5	5

Section B:

	Q9	Q10	Q11	Q12	Q13	Q14	Q15	Q16	Q17	TOTAL
Marks Earned										
Total Marks	11	7	7	4	7	11	9	9	7	150

...on A:

...estion 1: [14]

1.1 A triangle has the following vertices: $A(0;2)$ $B(4;5)$ $C(4;-4)$

1.1.1 Find the equation of the line through B and C. (1)

$x = 4$ ✓a

1.1.2 Find the midpoint of AC. (2)

$M(2,-1)$ ✓m ✓a

1.1.3 Find the gradient of AC. (2)

$\frac{2+4}{-4}$ ✓m $= -\frac{3}{2}$ ✓a

1.1.4 Find the equation of the perpendicular bisector of AC. (3)

$y = \frac{2}{3}x + c$ ✓ca

$-1 = \frac{2}{3}(2) + c$ ✓ca (sub 1.1.2)

$c = -\frac{7}{3}$

$y = \frac{2}{3}x - \frac{7}{3}$ ✓ca

1.2 $P(5;t)$; $Q(6;10)$; $R(-5;-12)$ are 3 points on the Cartesian Plane.

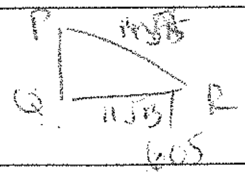
Find the value of t if:

1.2.1 the points are collinear. (3)

$m_{QR} = \frac{22}{11} = 2$ ✓a

$m_{PQ} = \frac{t-10}{-1} = 2$ ✓m

$t = 8$ ✓ca



1.2.2 the triangle PQR is a right angled triangle with $\hat{Q} = 90°$. (3)

$m_{PQ} \cdot m_{QR} = -1$

$\left(\frac{t-10}{-1}\right)(2) = -1$ ✓m ✓ca OR / (sub. into pythag using dist)

$t = 10\frac{1}{2}$ ✓ca

* $m_{PQ} / m_{QR} \perp m_{PR}$ (1/3)

Question 2: [10]

2.1 If $\sin 37° = a$, express each of the following in terms of a.

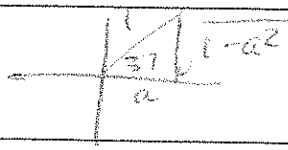
2.1.1 $\cos 53°$

$= \sin 37°$ ✓a

$= a$ ✓ca

(2)

2.1.2 $\cos 323°$

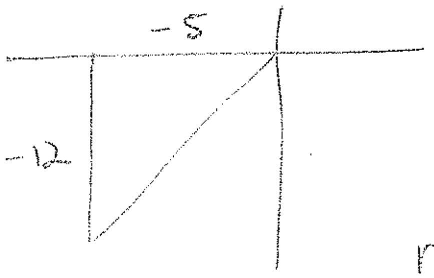


$= \cos 37°$ ✓a

$= \sqrt{1-a^2}$ ✓m ✓a

(3)

2.2 If $5\tan A = 12$ and $90° < A < 360°$.
Using a diagram calculate $3cos^2A$ without using a calculator.



✓a quadrant
✓a values

$r = \sqrt{12^2 + 5^2}$
$= 13$ ✓pythag

$3\left(\frac{-5}{\sqrt{13}}\right)^2$ ✓ca

$= 3\left(\frac{25}{169}\right)$

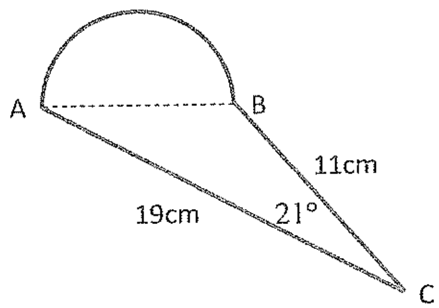
$= \frac{75}{13}$ ✓ca $\frac{75}{169}$

(5)

[7]

Question 3

The diagram below represents a semi-circle and a triangle.
All working details should be shown.



3.1 Determine the length of AB (2)

$AB^2 = 19^2 + 11^2 - 2.19.11.\cos 21^\circ$ ✓m

$AB = 9,6$ ✓a

3.2 Determine the area of the shape. (5)

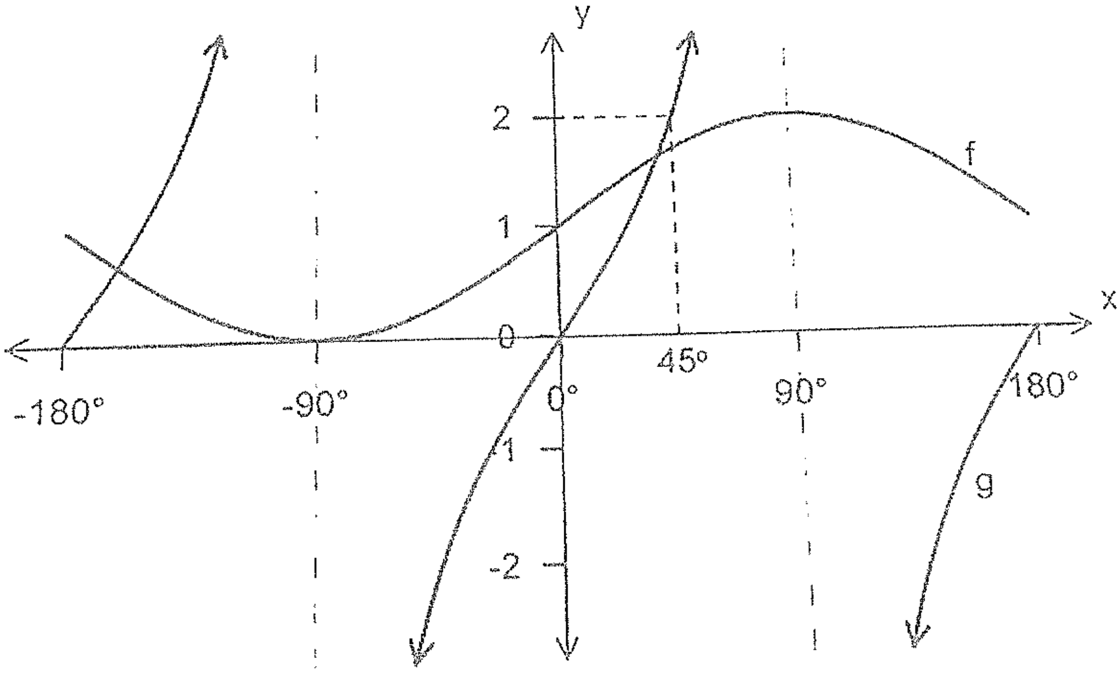
✓m ✓ca ✓a ✓rd

$\frac{1}{2}.19.11.\sin 21^\circ + \frac{1}{2}.\pi.\left(\frac{9,6}{2}\right)^2$

$= 73,6\ cm^2$ ✓ca

uestion 4

Given $f(x) = a.sinx + q$ and $g(x) = d.tan\, b.x$



4.1 Give the values of a, q, d and b.

a = 1 ✓a

q = 1 ✓a

d = 2 ✓a

b = 1 ✓a

(4)

4.2 For which values of x will $f(x) > 0$, given that $x \in [180°; 180°]$ [-180°; 180°]

$x \in \mathbb{R}$ ✓a , $x \neq -90°$ ✓a (2)

4.3 $f(x)$ is reflected around the x – axis. Give the new equation. (1)

$y = -\sin x - 1$ ✓eq

...stion 5:

[29]

5.1 8 Students wrote a Maths test and their results are listed below:

35%	26%	49%	58%	57%	87%	71%	51%

5.1.1 Determine the mean

$\bar{x} = 54,3$ ✓a ✓a

(2)

5.1.2 Determine the standard deviation (2)

$\sigma = 18,0$ ✓a ✓a

5.1.3 What percentage of test results lie within one standard deviation of the mean? (4)

$36,3 < x < 72,3$ ✓a
✓a

$\frac{5}{8}$ ✓ca $\times 100$

$= 62,5\%$ ✓ca

5.1.4 The teacher found that the marks were too low. He added 10 to each mark.

Write down the mean and the standard deviation of the new set of scores. (2)

$\bar{x} = 64,3$ ✓a

$SD = 18,0$ ✓a

5.1.5 Identify any possible outliers in the set of data if applicable. (4)

$UF = Q_3 + \frac{3}{2}(Q_3 - Q_1)$ and $LF = Q_1 - \frac{3}{2}(Q_3 - Q_1)$

$UF = 79 + \frac{3}{2}(79 - 37,5)$ $\quad$ $LF = 37,5 - \frac{3}{2}(79 - 37,5)$

$= 141,25$ ✓m $\quad$ $= -24,75$

$Q_3 = 79$ ✓a $\quad$ ∴ no outliers ✓ca

$Q_1 = 37,5$ ✓a

$Q_1 \quad 42$ ✓a

$Q_3 \quad 64{,}5$ ✓a

$UF = 64{,}5 + \frac{3}{2}(64{,}5 - 42)$

$= 98{,}3$ ✓m

$LF = 42 - \frac{3}{2}(64{,}5 - 42)$

$= 8{,}3$

$\therefore$ no outliers ✓ca

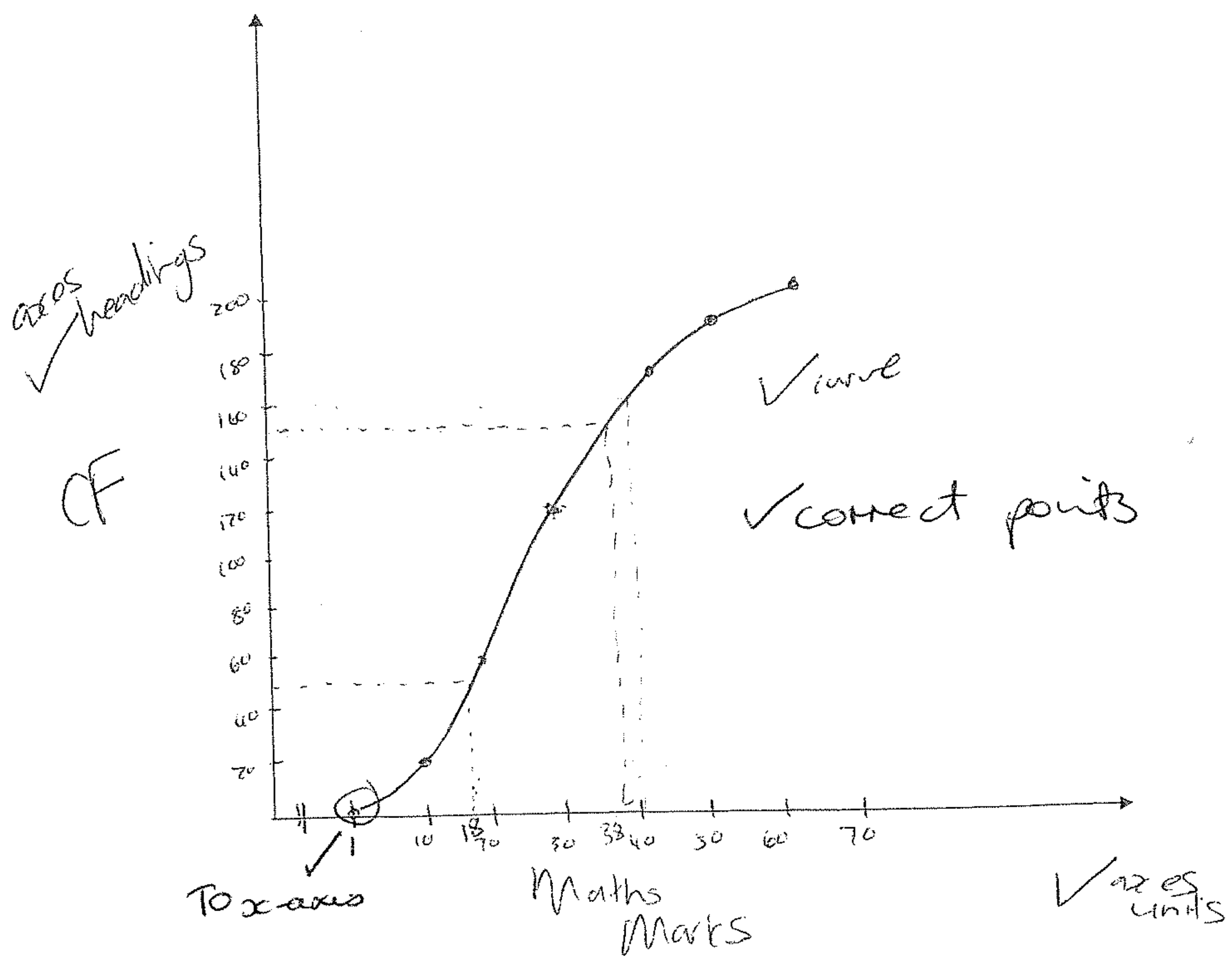
5.2 The following frequency table shows the distribution of the marks of 200 students in a Mathematics Test.

Mathematics Mark	Frequency	Cumulative frequency
1 - 10	20	20
11 – 20	40	60
21 – 30	a	120
31 – 40	50	170
41 – 50	b	190
51 – 60	10	200 ✓a

5.2.1 Complete the cumulative frequency table above and subsequently, determine the values of a and b. (3)

a = 60 ✓a b = 20 ✓a

5.2.2 Draw the ogive **on the grid provided below**. (5)



5.2.3 Estimate the interquartile range from your graph. (2) ~~(4)~~

$Q_3 - Q_1$ ✓a from graph

$= 38 - 18$

$= 20$ ✓ca

5.2.4 Determine the cut-off mark for the top 20% of students for this test from your graph. (3)

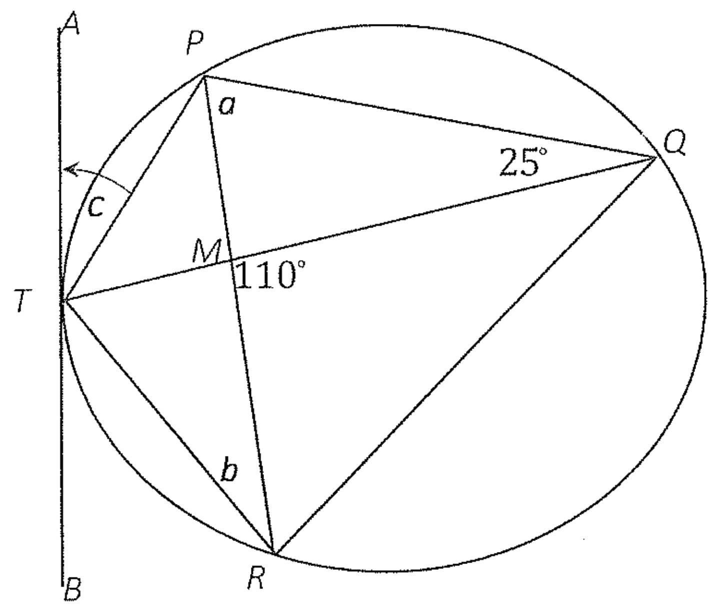
$20\% \times 200 = 40$ ✓a

$200 - 40 = 160$ ✓ca $\therefore$ 40 out of 60 ✓ca

Question 6: [3]

In the figure, *AB* is a tangent at *T* on the circle. *P*, *Q* and *R* are points on the circle. *PR* and *TQ* are drawn to intersect at M. $P\hat{Q}T = 25°$ and $Q\hat{M}R = 110°$.

Determine, with reasons, the values of the angles marked with *a*, *b* and *c*.



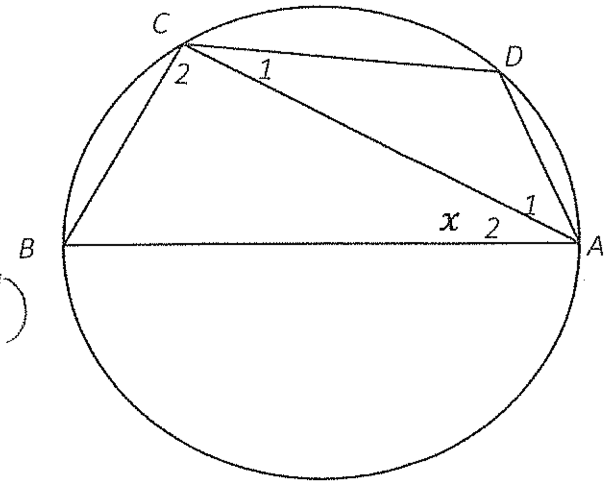
$a = 85°$ (Ext $\angle$ of $\triangle$) ✓a

$b = 25°$ ($\angle$'s in same segment) ✓a

$c = 25°$ (tan chord) ✓a

Question 7: [5]

In the diagram, *AB* is the diameter of circle *ABCD*. $\hat{A}_2 = x$.
Express, with reasons, $\hat{D}$ in terms of *x*.



$\hat{C}_2 = 90°$ ($\angle$ in semi $\odot$) ✓a

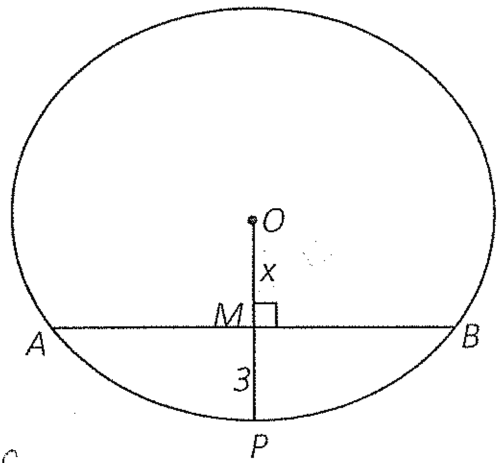
$\hat{B} = 90° - x$ ✓a ($\angle$'s of $\triangle$) ✓a

$\hat{D} = 90° + x$ ✓ca (opp $\angle$'s of cyclic quad) ✓a

Question 8: [5]

In the diagram, *AB* is a chord of a circle with centre *O*. *OMP* ⊥ *AB*, *AB* = 14, *MP* = 3 and *OM* = *x*

Determine, with reasons, the length of the radius of the circle.



$AM = 7$ (Line ⊥ from centre to chord) ✓a

$OA = x + 3$ (radii) ✓a

$7^2 + x^2 = (x+3)^2$ (Pythag) ✓M

$49 + x^2 = x^2 + 6x + 9$

$40 = 6x$

$\frac{20}{3} = x$ ✓a

$\therefore r = \frac{20}{3} + 3$

$= \frac{29}{3}$ or $9{,}7$ ✓ca

Section B:

Question 9: [8] +3 11

9.1 Prove: $$\frac{(tan^2\theta - sin^2\theta)\left(\frac{cos^2\theta + sin^2\theta}{sin^2\theta}\right)}{\frac{sin^2\theta}{cos^2\theta}} = 1$$ (5)

$$LHS = \frac{\left(\frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta\right)\left(\frac{1}{\sin^2\theta}\right)}{\frac{\sin^2\theta}{\cos^2\theta}}$$ ✓a ✓a

$$= \left(\frac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta}\right)\left(\frac{1}{\sin^2\theta}\right) \times \frac{\cos^2\theta}{\sin^2\theta}$$ ✓a ✓m

$$= \frac{\sin^2\theta(1-\cos^2\theta)}{\sin^4\theta}$$ ✓m

$$= 1$$

9.2 For which values of x will the identity in 9.1 be undefined? ~~(3)~~ (6)

$\tan^2\theta = 0$ ✓a

$\theta = 0° + k.180°$ ✓a

$\tan\theta$ at asymptotes

$\theta = 90° + k.180°$ ✓a

$\sin^2\theta = 0$ ✓a

$\theta = 0° + k.360°$ ✓ca

or

$\theta = 180° + k.360°$ ✓ca

Question 10 [7]

Without the use of a calculator, solve for α in the following:

$$sin^2\alpha = \sqrt{3}\, sin\, \alpha . \cos\alpha \quad \text{and} \quad -90° < \alpha < 180°$$

$\sin^2\alpha - \sqrt{3}\sin\alpha . \cos\alpha = 0$ ✓m

$\sin\alpha(\sin\alpha - \sqrt{3}.\cos\alpha) = 0$ ✓a

$\sin\alpha = 0$

$\alpha = 0° + k.180°$ ✓a

$\frac{\sin\alpha}{\cos\alpha} = \sqrt{3}$ ✓m

$\tan\alpha = \sqrt{3}$ ✓a

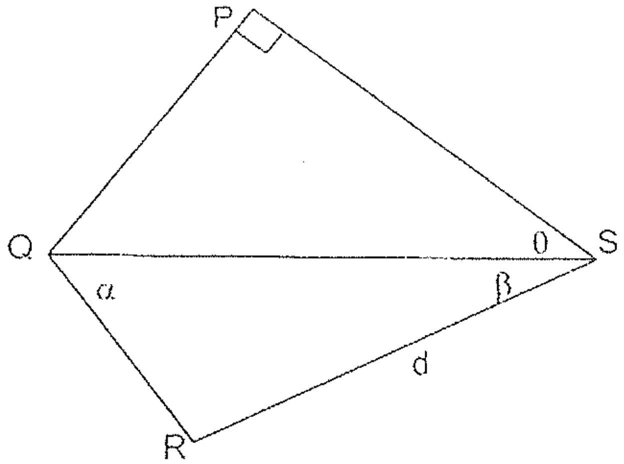
$\alpha = 60° + k.180°$ ✓a

$\alpha = \{0°, 60°, \}$ ✓ca

If divided by $\sin\alpha$, max. of 4 marks

Question 11 [7] ~~[6]~~

Use the diagram below and answer the following questions giving reasons where necessary.



11.1 Determine $\hat{R}$ in terms of α and β.

$\hat{R} = 180° - (\alpha + \beta)$ ✓ ✓ reason

(int ∠s of △)

(2)

11.2 Prove that: $PS = \dfrac{d\sin(\alpha+\beta)\cos\theta}{\sin\alpha}$

$\dfrac{\sin(\alpha+\beta)}{QS}$ ✓a $= \dfrac{\sin\alpha}{d}$ ✓m

$QS = \dfrac{d\sin(\alpha+\beta)}{\sin\alpha}$ ✓ca

$\cos\theta = \dfrac{PS}{\frac{d\sin(\alpha+\beta)}{\sin\alpha}}$ ✓m ✓ca

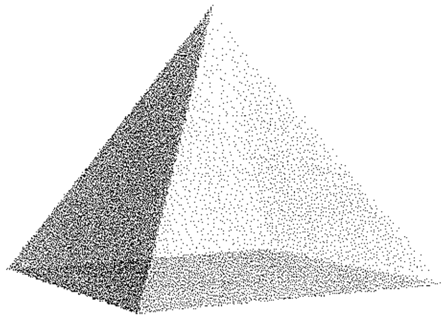
$PS = \dfrac{\cos\theta \cdot d \cdot \sin(\alpha+\beta)}{\sin\alpha}$

~~(4)~~ (5)

Question 12 [4]

A pentahedron (a solid shape with five faces) is in the form of a pyramid on a square base.

The height is twice as long as one side of its base.
Its Volume is $V = \frac{446}{3}$ cm^3



Determine the dimensions of the square base.

Volume of pyramid = $\frac{1}{3} \times area\ of\ the\ base \times Height$

$\frac{446}{3} = \frac{1}{3}x^2 \cdot 2x$ ✓a ✓a

$446 = 2x^3$ ✓ca

$x = 6.1$ cm ✓ca

Question 13: [7]

Given that $(k-3)y = (k+2)x + 8$ is a straight line.

13.1 Find the value of k if the line is horizontal.

$y = \frac{(k+2)}{(k-3)}x + \frac{8}{k-3}$

✓a m

$\frac{k+2}{k-3} = 0$ ✓m

$k = -2$ ✓a $k \neq 3$

(3)

13.2 Find the value of k if the line has an inclination of 135°.

$m = \tan 135°$ ✓m

$\frac{k+2}{k-3} = -1$ ✓a

$k + 2 = -k + 3$ ✓ca

$k = \frac{1}{2}$ ✓ca

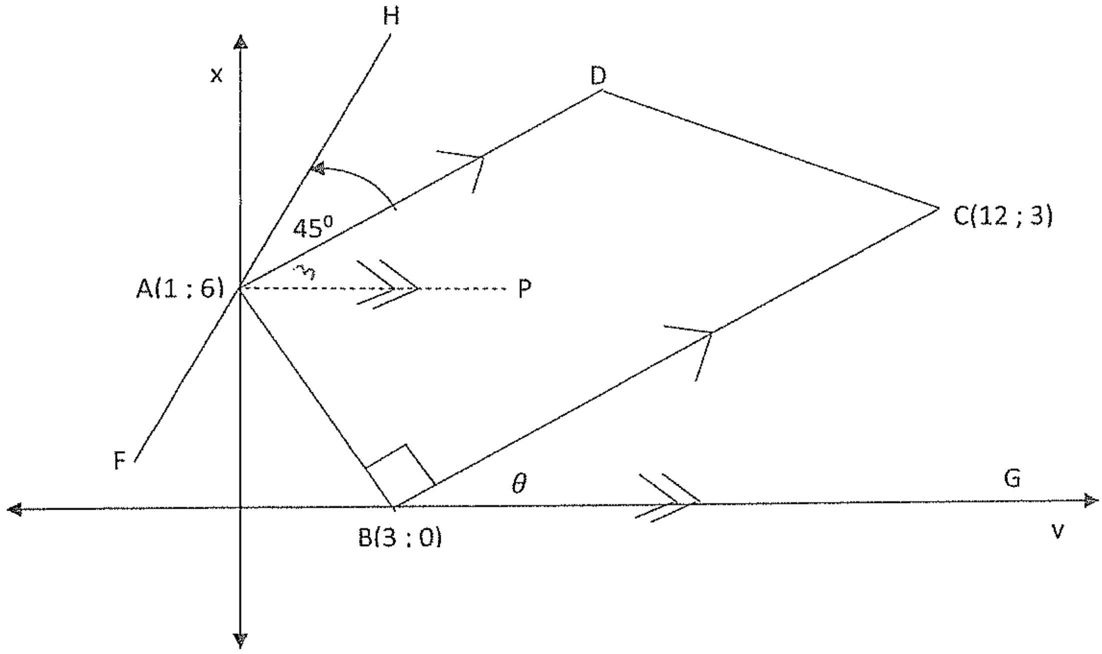
(4)

Question 14: **[12]**

$A(1\,;6)$ $B(3\,;0)$ $C(12\,;\,3)$ and D are the vertices of a trapezium with $AD \parallel BC$.

The angle of inclination of the straight line **BC** is θ, as shown in the diagram below.

A straight line FH passing through vertex A does not pass through any of the sides of the trapezium. This line makes an angle of **45°** with the side AD of the trapezium.



1~~3~~4.1 Determine the gradient of the straight line FAH.

$\tan\theta = \dfrac{3-0}{12-3}$ ✓a ✓m

$\theta = 18{,}43^\circ$ ✓ca

$\hat{A}_3 = 18{,}43^\circ$ ($\angle$ with horizontal) ✓a

✓ca

$\therefore \tan 63{,}43^\circ = m$

$m = 2$ ✓ca

(6)

13.2 Determine the co-ordinates of D such that ABCD forms a rectangle.

$M_{AC}\left(\frac{13}{2};\frac{9}{2}\right)$ ✓m

midpt. of diag. bisect (parm) ✓a

$\frac{3+x}{2}=\frac{13}{2}$ ✓m $\qquad \frac{y+0}{2}=\frac{9}{2}$ ✓a

$x=10$ ✓a $\qquad y=9$ ✓a

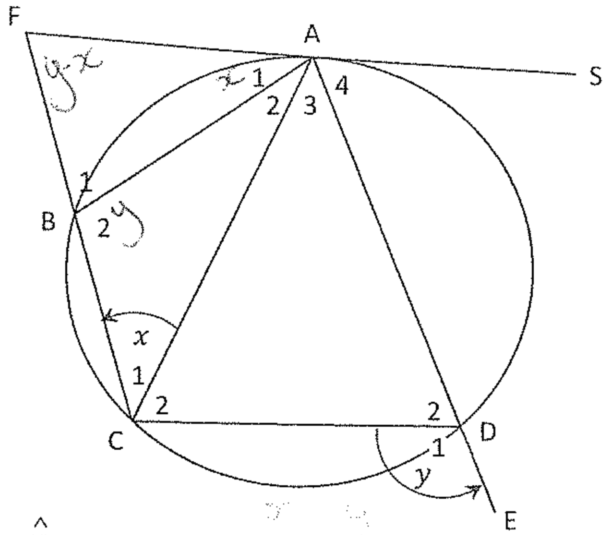
$D\,(10,9)$

(5)

Question 15: [9]

In the figure ABCD is a cyclic quadrilateral and FAS is a tangent to the circle at A.

$\hat{C}_1 = x$ and $\hat{D}_1 = y$.



15.1 Giving reasons express $\hat{F}$ and in terms of x and y. (4)

$\hat{A}_1 = x$ (tan chord) ✓a

$\hat{B}_2 = y$ (Ext ∠ of cyclic quad) ✓a

$\hat{F} = y - x$ ✓ca (Ext ∠ of △) ✓a

15.2 Express $\hat{A}_2$ in terms of x and y (2)

$\hat{A}_2 = 180° - y - x$ (int ∠s of △) ✓a

✓ca

15.3 Calculate the value of y if AC is a tangent to circle AFB. (3)

$\hat{A}_2 = y - x$ (tan chord th.) ✓a

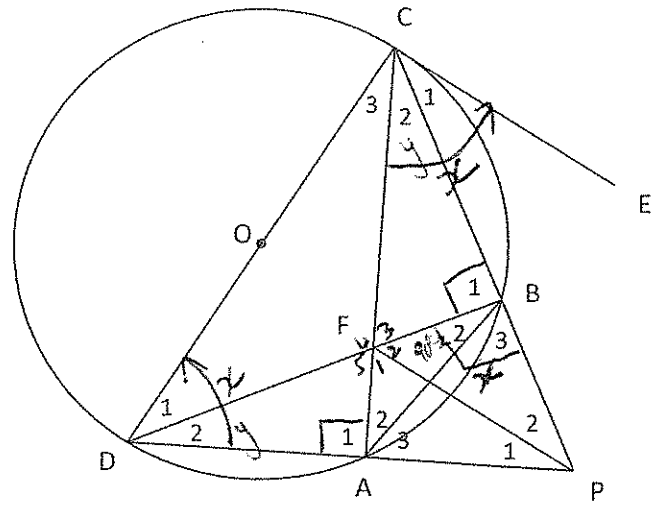
$180° - y - x = y - x$ ✓m

$180° = 2y$

$90° = y$ ✓ca

QUESTION 16 **[8]**

In the figure DC is a diameter of centre O. CE is a tangent to the circle at C. The diagonals of cyclic quadrilateral ABCD intersect at F.



15.1 Prove FAPB is a cyclic quadrilateral. (4)

$\hat{B}_1 = 90^\circ$ ($\angle$ in semi ⊙)

$\hat{A}_1 = 90^\circ$ ($\angle$ in semi ⊙)

$\hat{A}_2 + \hat{A}_3 = 180^\circ - 90^\circ$ ($\angle$'s on str line)

$\hat{A}_2 + \hat{A}_3 = 90^\circ$

$\therefore \hat{B}_1 = \hat{A}_2 + \hat{A}_3$

$\therefore$ FAPB is cyclic quad (opp $\angle$'s are suppl.)

4

15.2 If $A\hat{C}E = x$, name **three** other angles equal to x. (3)

$\hat{D}_1 + \hat{D}_2 = x$ (tan chord)

$\hat{B}_3 = x$ (Ext $\angle$ of cyclic quad)

$\hat{F}_1 = x$ ($\angle$'s in same segment)

3

15.3 What can you deduce about FP and CE? (2)

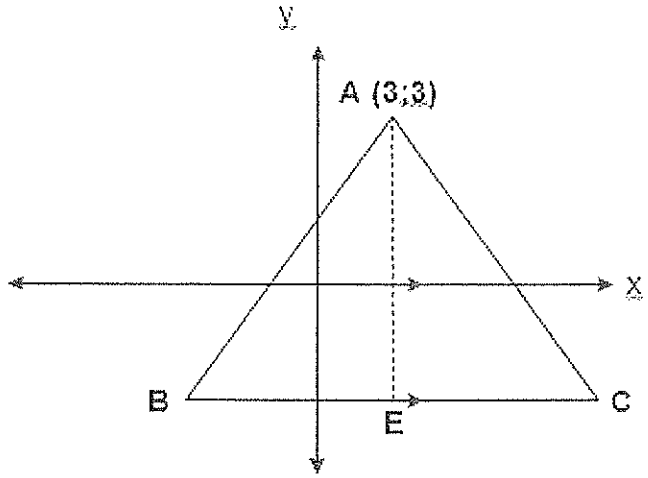
$\hat{F}_1 = \hat{C}_1 + \hat{C}_2 = x$

$\therefore$ CE ∥ FP (corre $\angle$'s are $=$)

2

Question 16: [7]

Consider the following **equilateral triangle** with sides of length 10 units.
AE is the perpendicular height of the triangle.



Determine the coordinates of the points B and C.
Give reasons where necessary. $BE = EC = 5$

$x_1 = 3 - 5 = -2$ } by symmetry
$x_2 = 8$

$AP^2 = 10^2 - 5^2$ (Pythag)
$AP = 8{,}7$ $5\sqrt{3}$
$y = -5{,}7$

$B(-2; -5{,}7)$ }
$C(8; -5{,}7)$ }