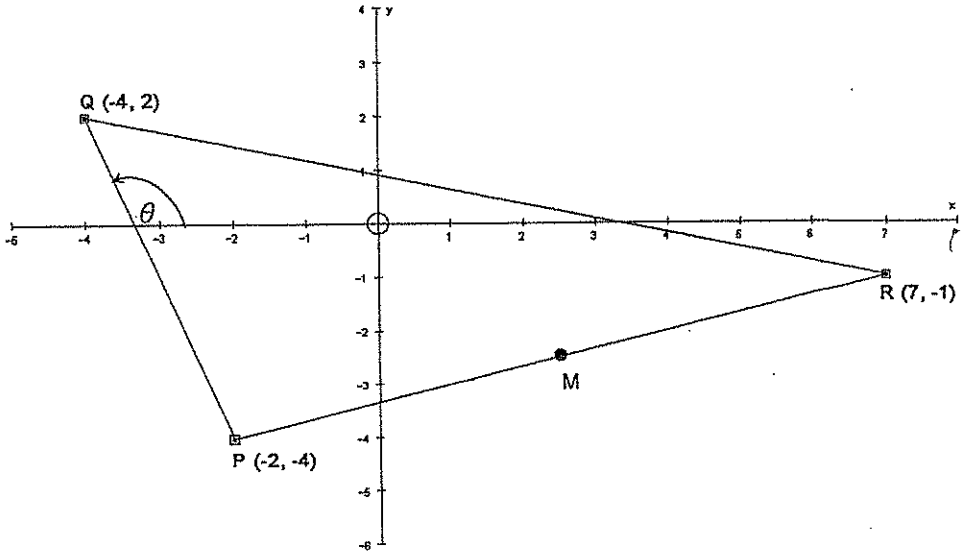
Grade 11 Nov 2015

SECTION A

Question 1 [16]

P(-2;-4), Q(-4;2) and R(7;-1) are vertices of Δ PQR in a Cartesian plane as shown below. θ is the angle of inclination of PQ.



1.1 Determine the distance of QR (Round off answer to 1 decimal place). (2)

$QR = \sqrt{(2-(-1))^2+(-4-7)^2}$ ✓m

$= 11,4$ ✓ca

1.2 Calculate the size of θ to the nearest degree. (5)

$M_{QP} = \frac{-4-2}{-2+4}$ ✓m

$= -3$ ✓a

$-3 = \tan\theta$ ✓m

$\theta = 180° - 71,57°$ ✓ca

$\theta = 108°$ ✓ca

1.3 Determine the equation of a straight line that is perpendicular to PR and that passes through point M, the midpoint of PR. (7)

$M\left(\frac{-2+7}{2}; \frac{-4-1}{2}\right)$ ✓m

$M\left(\frac{5}{2}; \frac{-5}{2}\right)$ ✓a

$M_{PR} = \frac{-4-(-1)}{-2-7}$ ✓m

$= \frac{-3}{-9}$

$= \frac{1}{3}$ ✓a

$y = -3x + C$ ✓ca

$\frac{-5}{2} = -3\left(\frac{5}{2}\right) + C$ ✓

$y = -3x + 5$ ✓ca

1.4 Prove that Δ PQR is right-angled. (2)

$M_{PR} = \frac{1}{3}$ $M_{QP} = -3$ ✓ca

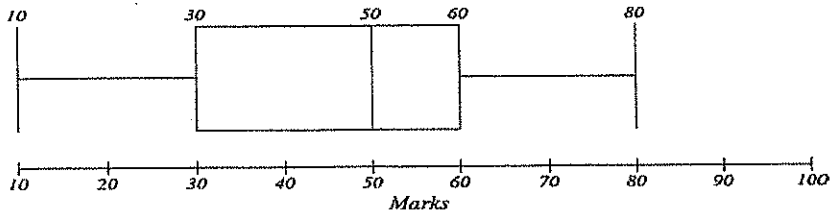
$\frac{1}{3} \cdot -3 = -1$ ✓m

∴ Δ PQR is right-angled

Question 2 [10]

Two mathematics classes, 11S and 11E, are in competition to see which class preformed best in the recent trigonometry test. Below is a box and whisker diagram illustrating 11E's results as well as a table of 11 S's results. Both classes have 30 learners in them.
(Marks are given as a %).

Box and Whisker Diagram for 11 E:



Marks for learners in 11S:

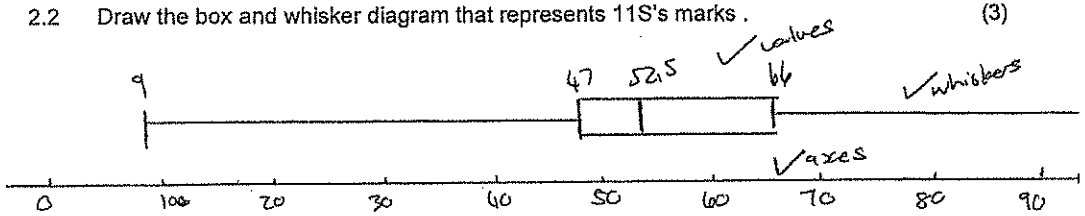
9	15	23	36	37	45
46	47	47	48	48	49
49	50	52	53	55	60
61	62	65	66	68	69
70	78	80	97	97	98

2.1 Write down the five-number summary for 11S. (5)

$L = 9$ ✓a; $H = 98$ ✓a, Median $= 52.5$ ✓a, $Q_1 = 47$ ✓a

$Q_3 = 68$ ✓a

2.2 Draw the box and whisker diagram that represents 11S's marks. (3)



2.3 Determine which class did better in the trigonometry test, and give a reason for your conclusion. (2)

11 S did better ✓a

any answer/reason that makes sense

50% of 11 S got above 52,5% – 98%. ∴ higher results

∴ positively skewed.

Question 3 [25]

3.1 Given: **sin 34° = *t***. Without using a calculator, determine the value of the following in terms of *t*.

3.1.1 cos 56° (2)

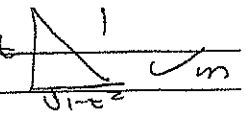
$\sin 34^\circ$ ✓a

$= t$ ✓a

3.1.2 cos 34° (3)

$\sqrt{1-t^2}$ ✓a

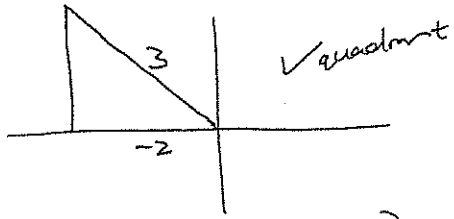
$\frac{\sqrt{1-t^2}}{1}$ ✓a



3.2 If $3\cos x + 2 = 0$ and $x \in [0°; 180°]$.

Without the use of a calculator and using a diagram, find the value of:

$9\sin x.\cos x + 2\tan x$ (Leave your answer in simplest surd form) (4)



$3^2 - (-2)^2$ (pythag)

$= \sqrt{5}$ ✓m

$9\left(\frac{\sqrt{5}}{3}.\frac{-2}{3}\right) + 2\left(\frac{\sqrt{5}}{-2}\right)$ ✓cq

$= -2\sqrt{5} - \sqrt{5}$

$= -3\sqrt{5}$ ✓cq

3.4 Prove, without the use of a calculator: $\frac{\sin 190^\circ \cos 225^\circ \tan 390^\circ}{\cos 100^\circ \sin 135^\circ} = -\frac{1}{\sqrt{3}}$ (6)

$= -\sin 10^\circ \cdot -\cos 45^\circ \cdot \tan 30^\circ$ ✓a ✓a ✓a

$-\cos 80^\circ \cdot \sin 45^\circ$ ✓a

$= -\tan 30^\circ$

$= -\frac{1}{\sqrt{3}}$ ✓CA

3.5 Prove that: $\frac{1-2\sin^2\theta}{\sin\theta\cos\theta} = \frac{\cos\theta}{\sin\theta} - \tan\theta$ (5)

$RHS = \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}$ ✓a

$= \frac{\cos^2\theta - \sin^2\theta}{\sin\theta \cdot \cos\theta}$ ✓a ✓m

$LHS = \frac{1-2\sin^2\theta}{\sin\theta \cdot \cos\theta}$

$= \frac{\sin^2\theta + \cos^2\theta - 2\sin^2\theta}{\sin\theta \cdot \cos\theta}$ ✓a

$= \frac{\cos^2\theta - \sin^2\theta}{\sin\theta \cdot \cos\theta}$ ✓CA

$= RHS$

OR

$= \frac{1-\sin^2\theta-\sin^2\theta}{\sin\theta \cdot \cos\theta}$ ✓a

$= \frac{1-2\sin^2\theta}{\sin\theta \cdot \cos\theta}$ ✓CA

$= LHS$

3.6 Solve for θ if $\theta \in [-90°; 90°]$ and it is given that $\tan(2\theta - 10°) = -3{,}28$ (5)

RA = 73,04° ✓a

$k \in Z$ ✓a

$2\theta - 10^\circ = 180^\circ - 73{,}04^\circ + k.180^\circ$ ✓a

$\theta = 58{,}5^\circ + k.90^\circ$ ✓CA

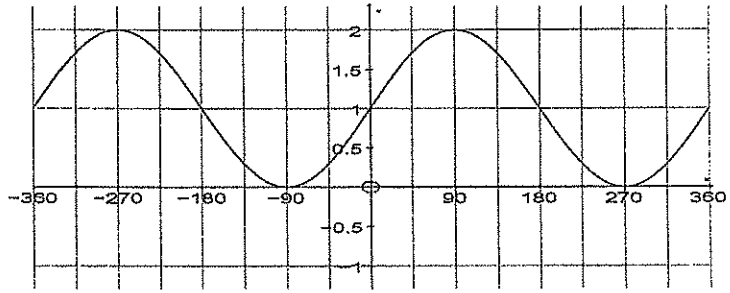
$\theta \in \{58{,}5^\circ; -31{,}5^\circ\}$ ✓CA

Question 4 [9]

Circle → Bold

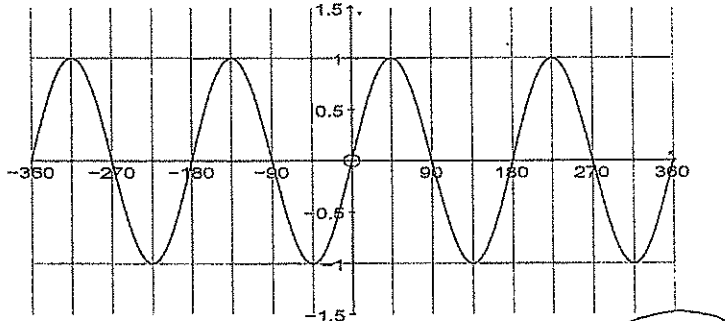
~~Choose~~ the correct answer for the following questions:

4.1 The **amplitude** of the graph below is: (1)



A: 2 B: 4 C: 1: D: 1,5 ✓a

4.2 The **period** of the graph below is: (1)

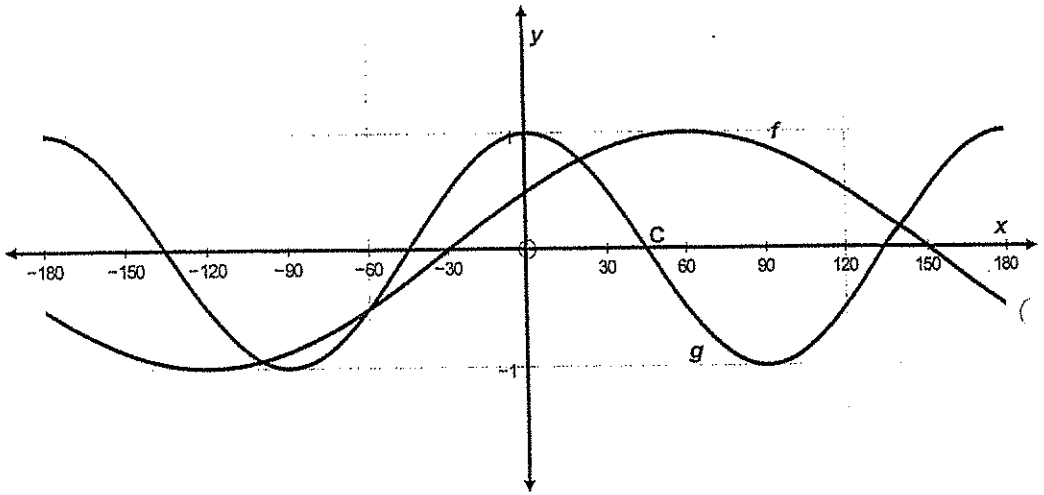


A: 360° B: 90° C: 270° D: 180° ✓a

4.3 The maximum value of $y = -2\sin\theta - 1$ is: (1)

A: -3 B: 1 C: -1 D: 3 ✓a

4.4 The graphs below represent the functions $g(x) = cosax$ and $f(x) = sin(x + b)$ where **a** and **b** are constants and $x \in [-180°; 180°]$.



4.4.1 Determine the values of **a** and **b**. (2)

$a = 2$ ✓a $\quad b = 30°$ ✓a

4.4.2 What is the range of $g(x)$? (1)

$y \in [-1; 1]$ ✓a

4.4.3 Write down the coordinates of C. (1)

$\frac{90°}{2} = 45°$ $\quad C(45°; 0)$ ✓a

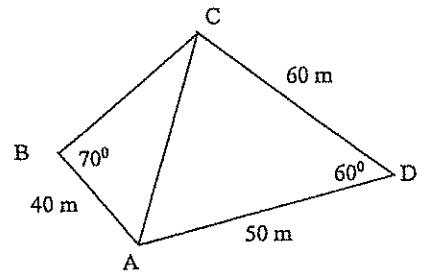
4.4.4 For which x-values will $f(x) > 0$, if $x \in [-180°; 180°]$? (2)

✓a ✓a

$-30° < x < 150°$

Question 5 [8]

ABCD is a small field with AB = 40 m, AD = 50 m and CD = 60m. $A\hat{B}C = 70°$ and $A\hat{D}C = 60°$.



5.1 Determine the size of $B\hat{C}D$. (8)

$AC^2 = 60^2 + 50^2 - 2.60.50.\cos 60°$ ✓a ✓m

$AC = 56$ m ✓cg

$\frac{\sin A\hat{C}D}{50} = \frac{\sin 60°}{56}$ ✓m

$A\hat{C}D = 50,6°$ ✓cg $\quad 51,05172...$

$\frac{\sin B\hat{C}A}{40} = \frac{\sin 70°}{56}$ ✓cg

$B\hat{C}A = 42,2°$ ✓cg $\quad 42,4615$

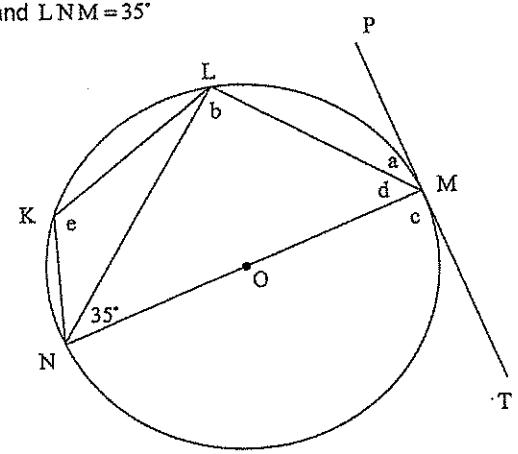
$\hat{C} = 92,8°$ ✓cg

$(93,5°)$

Question 6 [5]

Refer to the diagram. PT is a tangent to circle LMNK with centre O.

NOM is a straight line and $L\hat{N}M = 35°$



Find, with reasons, the size of the angles marked (a) to (e).
Fill in your answers in the table below:

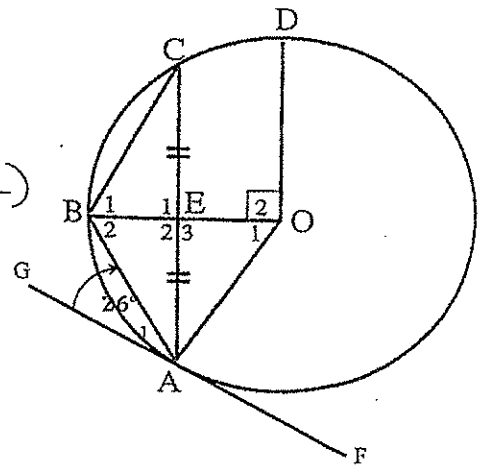
Angle	Answer	Reason
a	$35°$	tan chord theorem ✓q
b	$90°$	∠ in semi ⊙ ✓q
c	$90°$	rad ⊥ tangent ✓q
d	$55°$	int ∠'s of Δ ✓q
e	125	opp ∠ of cyclic quad ✓q

(5)

Question 7[10]

In the circle centre O, BO ⊥ OD, AE = EC and $\hat{A}_1 = 26°$. GAF is a tangent to the circle.

Calculate the size of the following angles giving reasons for your answers:



7.1 $\hat{O}_1$ (4) ✓q ✓q

$\hat{C} = 26°$ (tan chord theorem)

$\hat{O}_1 = 52°$ (∠ at centre) ✓q ✓q

7.2 $\hat{B}_1$ (4)

CE = EA (given)

$\hat{E}_1 = 90°$ ✓q (midpoint chord) ✓q

$\hat{B}_1 = 180° - 90° - 26°$ (Int ∠'s of Δ) ✓q

$= 64°$ ✓q

7.3 Prove that CA // DO (2)

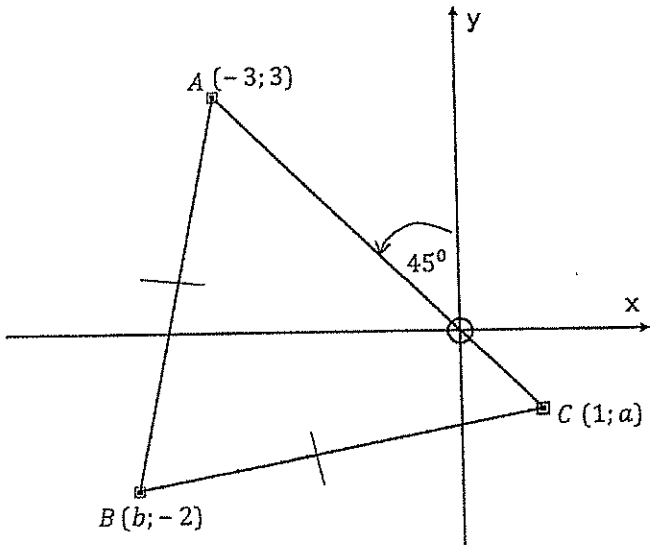
$\hat{E}_1 = \hat{O}_2 = 90°$ (proven) ✓q

∴ CA // DO (corre ∠'s are =) ✓q

SECTION B

Question 8 [9]

Given ΔABC, with $A(-3;3)$, $B(b;-2)$, $C(1;a)$ and $Y\hat{O}A = 45^0$.



8.1 Show that $C(1;-1)$. (4)

tan 135° ✓m

= −1 ✓a

$\frac{a-3}{1+3} = -1$ ✓ca

$a - 3 = -4$ ✓ca

$a = -1$

∴ C(1, −1)

8.2 If it is given that $AB = BC$, determine the value of b. (5)

✓a ✓m

$(3+2)^2 + (-3-b)^2 = (-1+2)^2 + (1-b)^2$ ✓a

$5^2 + 9 + 6b + b^2 = 1 + 1 - 2b + b^2$ ✓ca

$25 + 9 - 6b = 2 - 2b$

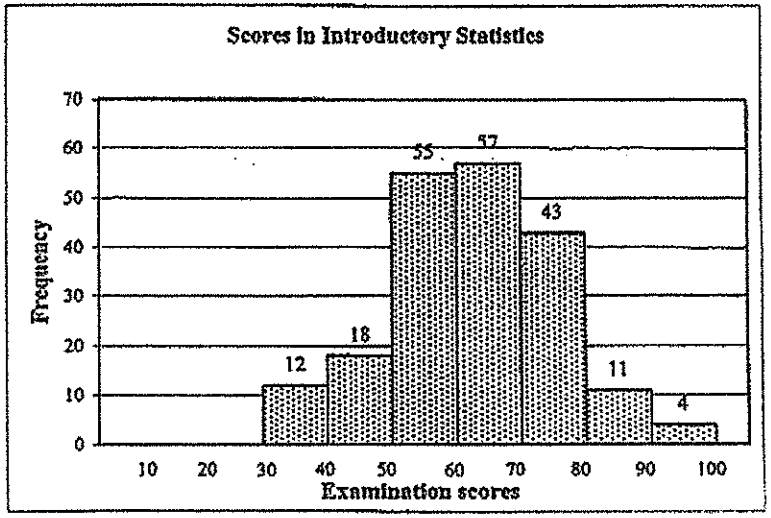
$32 = 4b$

$8 = b$ ✓ca

[9]

Question 9 [10]

The Histogram below shows the distribution of examination scores of 200 learners in Introductory Statistics, to the nearest percent.

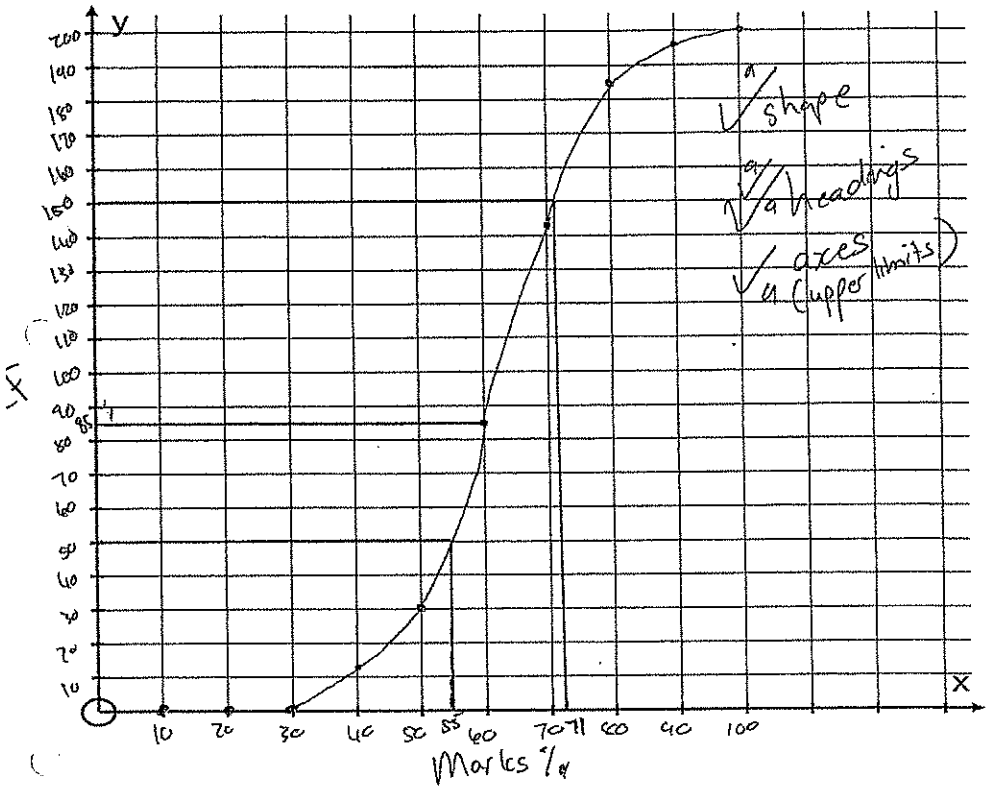


9.1 Complete the cumulative frequency table for the above data provided below. (2)

Scores	Frequency	Cumulative Frequency
$0 \le x < 10$	0	0
$10 \le x < 20$	0	0
$20 \le x < 30$	0	0
$30 \le x < 40$	12	12
$40 \le x < 50$	18	30
$50 \le x < 60$	55	85
$60 \le x < 70$	57	142
$70 \le x < 80$	43	185
$80 \le x < 90$	11	196
$90 \le x < 100$	4	200

✓✓ 9

9.2 Draw an ogive of the above data on the grid below. (4)



9.4 Use the ogive to estimate the values of Quartile 1 and Quartile 3 (2)

$Q_1 = 55$ ✓ $Q_3 = 71$ ✓ 9

had to use graph (show lines)

9.3 Use the ogive to estimate how many learners scored 60% or more for the examination. (2)

$200 - 85$ ✓ m

$= 115$ ✓ 9 (not ca) has to be correct.

Question 10 [9]

10.1 Complete the identity: $\sin^2 x =$ $1 - \cos^2 x$ (1)

10.2 Hence, determine the general solutions of the following **quadratic** equation $\sin^2 x + 5\cos x = 2\cos^2 x - 1$. (8)

$1 - \cos^2 x + 5\cos x - 2\cos^2 x + 1 = 0$ ✓a ✓m

$-3\cos^2 x + 5\cos x + 2 = 0$

$3\cos^2 x - 5\cos x - 2 = 0$ ✓a

$(\cos x - 2)(3\cos x + 1) = 0$

$\cos x = 2$ ✓ca $\quad \cos\theta = -\frac{1}{3}$ ✓ca

no solution ✓a

$\theta = 109{,}47^\circ + k \cdot 360^\circ$ ✓a

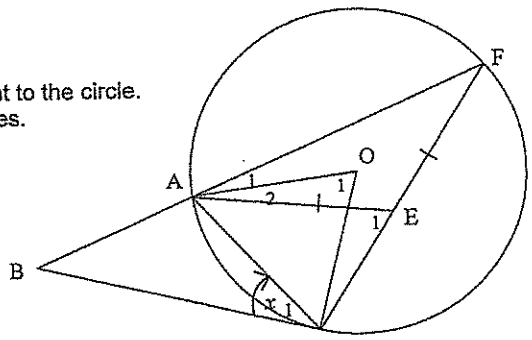
$\theta = 250{,}53^\circ + k \cdot 360^\circ$ ✓a

$k \in \mathbb{Z}$

Question 11 [10]

O is the centre of the circle.
AE = EF and BC is a tangent to the circle.
BAF and CEF are straight lines.

Let $\hat{C}_1 = x$.



11.1 Find $\hat{F}$ terms of x. (2)

$\hat{F} = x$ ✓a (tan chord theorem) ✓a

11.2 Express $\hat{O}_1$ in terms of x. (2)

$\hat{O}_1 = 2x$ ✓a (∠ at centre) ✓a

11.3 Express $\hat{E}_1$ in terms of x. (4)

$\hat{A}_1 + \hat{A}_2 = x$ ✓a (Isosceles Δ) ✓a

$\hat{E}_1 = 2x$ ✓a (Ext ∠ of Δ) ✓a

11.4 Why is OACE a cyclic quadrilateral? (2)

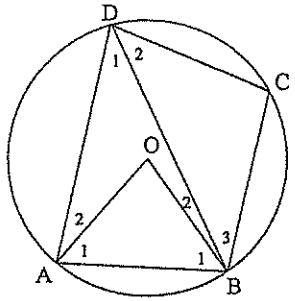
$\hat{O}_1 = \hat{E}_1 = 2x$ ✓a

∴ OACE is a cyclic quad

(converse of ∠'s in same segment) ✓a

QUESTION 12 [11]

Given ABCD is a cyclic quadrilateral.
O is the centre of the circle.
OA bisects $\hat{A}$.
$\hat{A}_1 = x$.



12.1 Name TWO other angles equal to *x*, giving reasons. (3)

OA = OB (radii) ✓a
$\hat{B}_1 = x$ (Isosceles Δ) ✓a
$\hat{A}_2 = x$ (given) ✓a

12.2 Find $\hat{C}$ in terms of *x*, giving a reason. (2)

$\hat{C} = 180^\circ - 2x$ ✓a ✓a (opp ∠s of cyclic quad)

12.3 Express $\hat{D}_1$ in terms of *x*, giving reasons. (4)

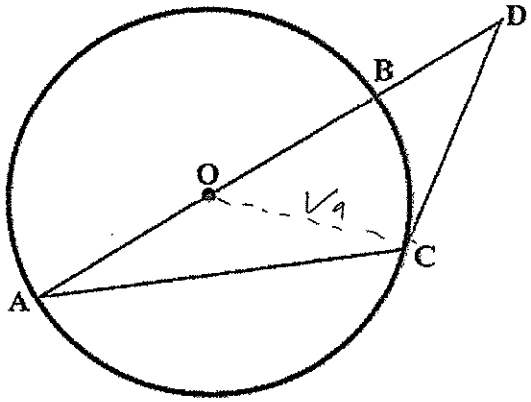
$A\hat{O}B = 180^\circ - 2x$ ✓a (Int ∠'s of Δ) ✓a
$A\hat{D}B = 90^\circ - x$ ✓a (∠ at centre) ✓a

12.4 Could AD be a tangent to the circle through B,C and D? (2)
Motivate your answer.

No, $\hat{D}_1 \neq \hat{C}$ ✓a ∴ converse of tan chord theorem does not apply ✓a

QUESTION 13 [7]

In the diagram below, a circle centre O is drawn.
- AB is a diameter of the circle and C is a point on the circle
- AB produced meets the tangent at C at D.
- AC = DC



Determine, giving reasons, the size of $\hat{A}$ (7)
HINT: you will need to construct in your own line

Let $\hat{A} = x$
AO = OC (radii) ✓a
∴ $\hat{C}_1 = x$ (Isosceles Δ) ✓a
$\hat{C}_2 = 90^\circ$ (tang ⊥ radius) ✓a

$\hat{D} = x$ (Isosceles Δ) ✓a

$x + x + 90^\circ + x = 180^\circ$ ✓a (Int ∠s of Δ)
$x = 30^\circ$
$\hat{A} = 30^\circ$ ✓ca

QUESTION 14 [5]

Five numbers: a, b, c, d and e are given, and:

- $e = c$
- $e < a < b$
- d is the maximum value
- The modal number is 5
- The difference between the fourth number and the second number is 4
- The range of the numbers is 9
- The average of the numbers is 8

Determine the values of a, b, c, d and e.

e, e, a, b, d

$b - 4 = 4$ $\quad d - e = 9$

$e = 5$ ✓a

$b = 9$ ✓a $\quad d = 14$ ✓a

$\frac{a + 9 + 14 + 5 + 5}{5} = 8$ ✓m

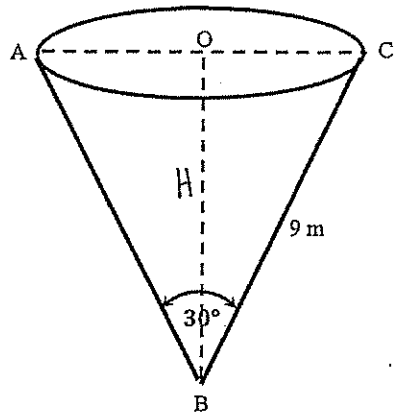
$a = 7$ ✓ca

[5]

QUESTION 15 [6]

The following regular cone is given with a side length of 9 m and the angle of $A\hat{B}C = 30°$.
O is the centre of the circle base of the cone. OB is the height of the cone.

Calculate the Volume of the cone if: $V = \frac{1}{3}\pi r^2 H$. (6)



diameter $= \sqrt{9^2 + 9^2 - 2.9.9.\cos 30°}$ ✓m

$= 4,7$m ✓a

radius $= 2,3$ ✓ca

$H = \sqrt{9^2 - (2,3)^2}$ (Pythag) ✓m

$= 8,7$m ✓ca

$V = \frac{1}{3}\pi (2,3)^2 . 8,7$

$= 48,2\ m^3$ ✓ca