NAME: Memo

GRANTLEIGH COLLEGE

MATHEMATICS PAPER 2

GRADE 11

22 NOVEMBER 2016

Time: 2 Hours

Examiner: Z. Ebersohn

Moderator: J. Austin, S, Dzingwa

Marks: 100

SD / JA / ZE

PLEASE READ THE INSTRUCTIONS CAREFULLY

1. Write your name and surname on top of this front page.
2. Circle your teacher's name.
3. **Answer all the questions on the Question Paper.**
4. This paper consists of 16 pages. Please make sure your question paper is complete.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Round off your answers to ONE decimal digit where necessary.
7. All the necessary working details must be clearly shown.
8. It is in your own interest to write legibly and to present your work neatly.
9. Diagrams are not necessary drawn to scale.

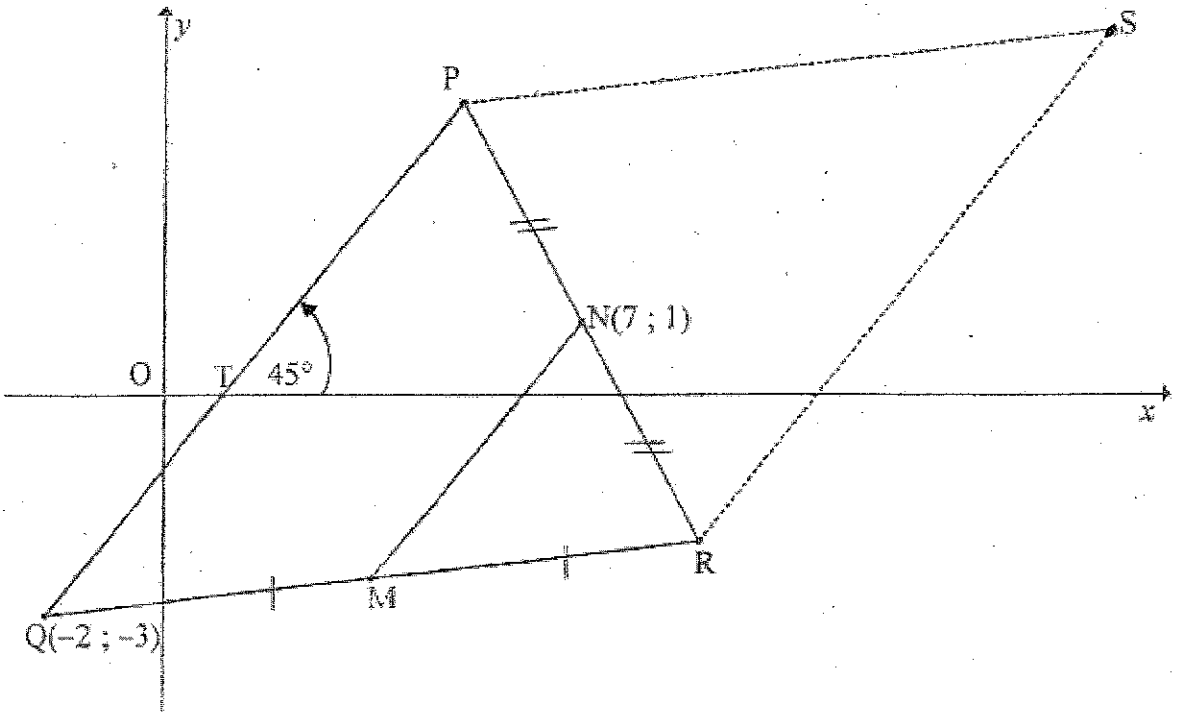
DO NOT WRITE IN THIS GRID

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	TOTAL
9	13	14	8	22	19	10	5	100

SECTION A

QUESTION 1

In the diagram below, the line joining $Q(-2;-3)$ and P makes an angle of $45°$ with the x-axis. $QP = 7\sqrt{2}$ units. $N(7;1)$ in the midpoint of PR and M is the midpoint of QR.



Determine:

a) The gradient of PQ. (2)(2)

$m_{PQ} = \tan 45°$

$m_{PQ} = +1$ ✓✓ 2

b) The equation of MN. (3)

$m_{MN} = 1$ ✓ MN ∥ PQ midpt th.

$\therefore y - 1 = 1(x - 7)$ ✓

$y = x - 6$ ✓ 3

c) The length of MN. (2)

$MN = \frac{1}{2}QP$ ✓ mdpt th.

$MN = \frac{7}{2}\sqrt{2}$. ✓ 2

d) The coordinates of S such that PQRS, in this order, is a parallelogram. (2)

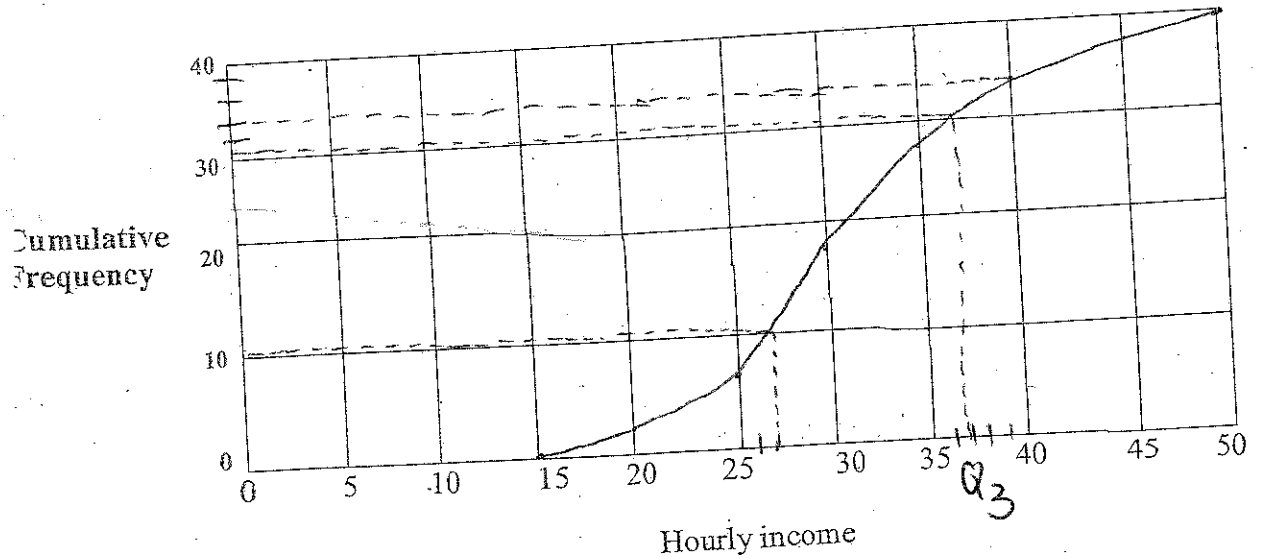
$\frac{x-2}{2} = 7$. $\frac{y-3}{2} = 1$ S(16, 5) ✓ ✓ 2

$x = 16$ $y = 5$.

[9]

QUESTION 2

a) Consider the cumulative frequency curve shown below. It depicts the hourly income (in Rands) of a group of students. Use the graph to answer the questions.



1) Determine the approximate value of the interquartile range. (3)

$Q_3 - Q_1 = 37 - 27$ ✓ ✓ 3

$= 10$ ✓

2) Determine the 83rd percentile. (2)

$83\% \times 40 = T_{33}$ ✓

83rd Percentile = 40 ✓ ✓ 2

b) A group of 30 learners each randomly rolled two dice and the sum of the values on the faces was recorded. The data is shown in the frequency table below.
Give all anwers correct to three decimal places.

Sum of the values on uppermost faces	Frequency
2	0
3	3
4	2
5	4
6	4
7	8
8	3
9	2
10	2
11	1
12	1

1) Calculate the mean of the data. (2)

$\bar{x} = 7,129$ ✓✓ $= \frac{221}{31}$ 2

2) Determine the median of the data. (2)

7 ✓✓ 2

3) Determine the standard deviation of the data using your calculator. (1)

SD = 2,265 ✓

4) Determine the number of times that the sum of the recorded values of the dice is within ONE standard deviation from the mean. Show all calculations. (3)

$(\bar{x} - SD ; \bar{x} + SD)$

$(7,129 - 2,265 ; 7,129 + 2,265)$

$(4,864 ; 9,394)$

3+2+2+1+1 3

= 9 Times ✓

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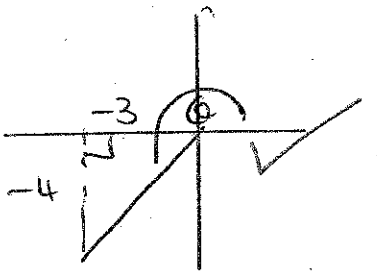
QUESTION 3

a) If $3\tan\theta - 4 = 0$ *and* $\cos\theta < 0$, calculate **without** the use of a calculator, and with the aid of a diagram, the value of:

$10\sin\theta - 25\cos^2\theta$ (5)



$\tan\theta = \frac{4}{3}$ $h = 5$

$10\left(\frac{-4}{5}\right) - 25\left(\frac{-3}{5}\right)^2$

$= -8 - 9$

$= -17$ 5

b) 1) Solve for θ if $\sin 2\theta = -0,56$ and $0° < \theta < 180°$. (4)

Ref $\angle = 34,1°$ ✓

$2\theta = 180° + 34,1°$ ✓

$\theta = 107,1$ ✓✓ 4.

2) Simplify the following **without** a calculator showing all steps.

$$\frac{\sin(450° - x)\tan(180° + x)\sin(90° - x)}{\cos(x - 360°)\sin(90° + x)\tan(-x)}$$ (5)

$90 - x$

$= \cos x \cdot \tan x \cos x$ ✓ ✓ ✓ 5.

$\cos x \cdot \cos x (-\tan x)$

$= -1$ ✓

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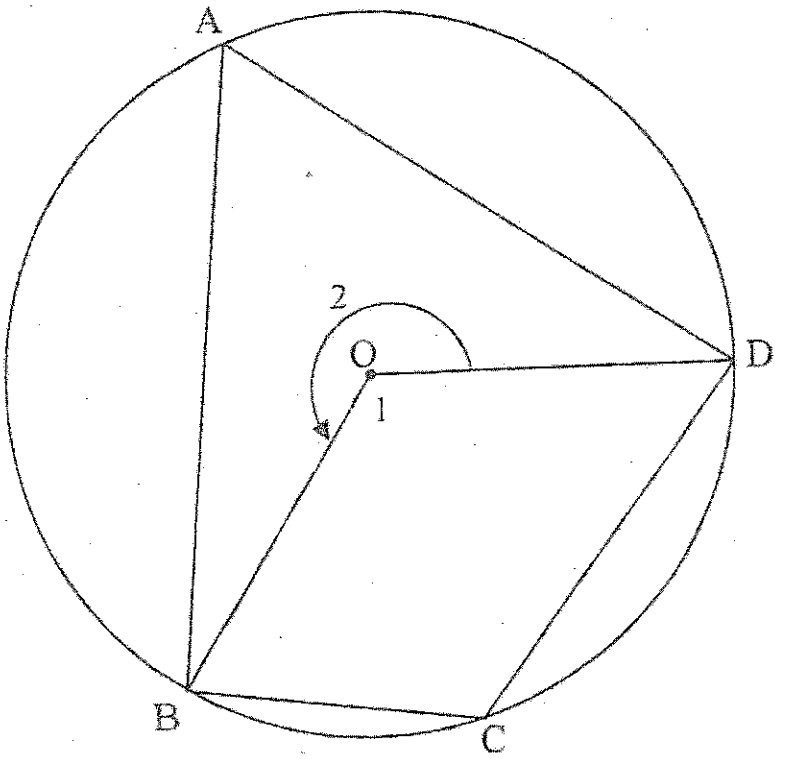
QUESTION 4

a) Complete the following statement:

The angle subtended by a chord at the centre of a circle is twice the angle subtended by the same chord at the circumference of the circle. (1)

b) Use the following diagram to prove that $\hat{A}+\hat{C}=180°$. (3)



$\hat{O}_2 = 2\hat{C}$ ✓ ∠ in centre = 2× ∠ on circ.

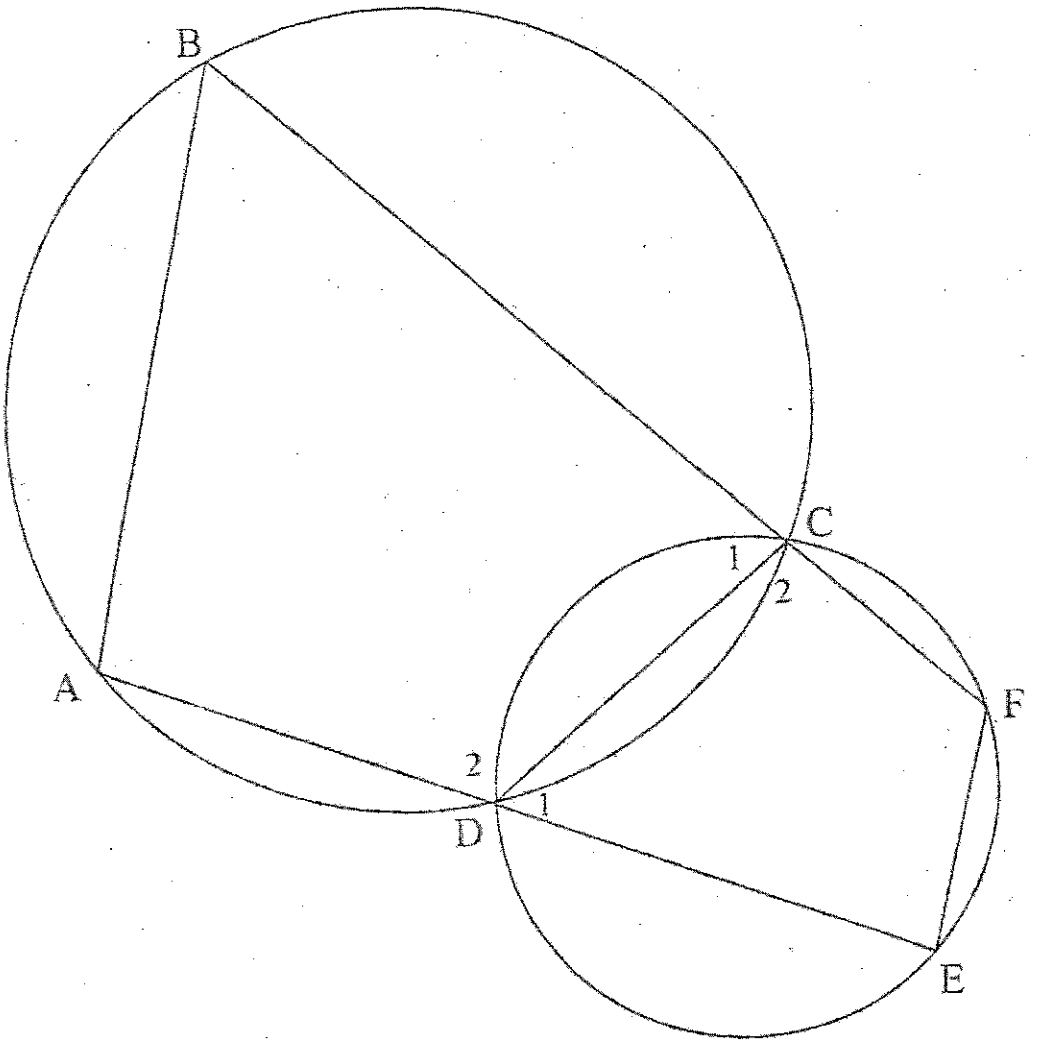
$\hat{O}_1 = 2\hat{A}$ ✓ ∠ in centre = 2× ∠ on circ.

But $\hat{O}_1 + \hat{O}_2 = 360°$ rev ✓

$\therefore 2\hat{A} + 2\hat{C} = 360°$ 3

$\hat{A} + \hat{C} = 180°$

c) In the diagram below, CD is a common chord of two circles. Straight lines ADE and BCF are drawn. Chords AB and EF are drawn.



4

Prove EF ∥ AB. (4)

$\hat{A} = \hat{C}_2$ ✓ ext ∠ of cyd. quad.

$\hat{E} = \hat{C}_1$ ✓ ext ∠ of cyd. quad.

$\hat{C}_1 + \hat{C}_2 = 180°$ ✓ SN Line.

$\therefore \hat{A} + \hat{E} = 180°$ ✓

EF = AB ✓ corr ∠'s

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SECTION B

QUESTION 5

a) Given that $\sin 23° = k$, determine in its simplest form, the value of the following in terms of k.

1) $\sin 203°$ (1)

$\sin(180° + 23°)$

$= -k$ ✓ 1

2) $\cos 23°$ (2)

$= \sqrt{1 - k^2}$ ✓✓ 2

3) $\cos 103°$ $113°$ +2 (2)

b) Prove the following identity:

$$\frac{1}{1-\sin x} - \frac{1}{1+\sin x} = \frac{2\tan x}{\cos x} \qquad (5)$$

LH $= \dfrac{1+\sin x-(1-\sin x)}{(1-\sin x)(1+\sin x)}$ ✓

$= \dfrac{2\sin x}{1-\sin^2 x}$ ✓ ✓

$= \dfrac{2\sin x}{\cos^2 x}$ ✓ $= \dfrac{2\tan x}{\cos x}$ ✓ = RH

c) 1) Complete the identity: $\cos^2\theta =$ $1-\sin^2\theta$ ✓ (1)

2) Determine the general solution of the following equation:

$$1-\sin\theta-2\cos^2\theta = 0 \qquad (6)$$

$1-\sin\theta-2(1-\sin^2\theta)=0$ ✓

$1-\sin\theta-2+2\sin^2\theta=0$

$2\sin^2\theta-\sin\theta-1=0$ ✓

$(2\sin\theta+1)(\sin\theta-1)=0$ ✓

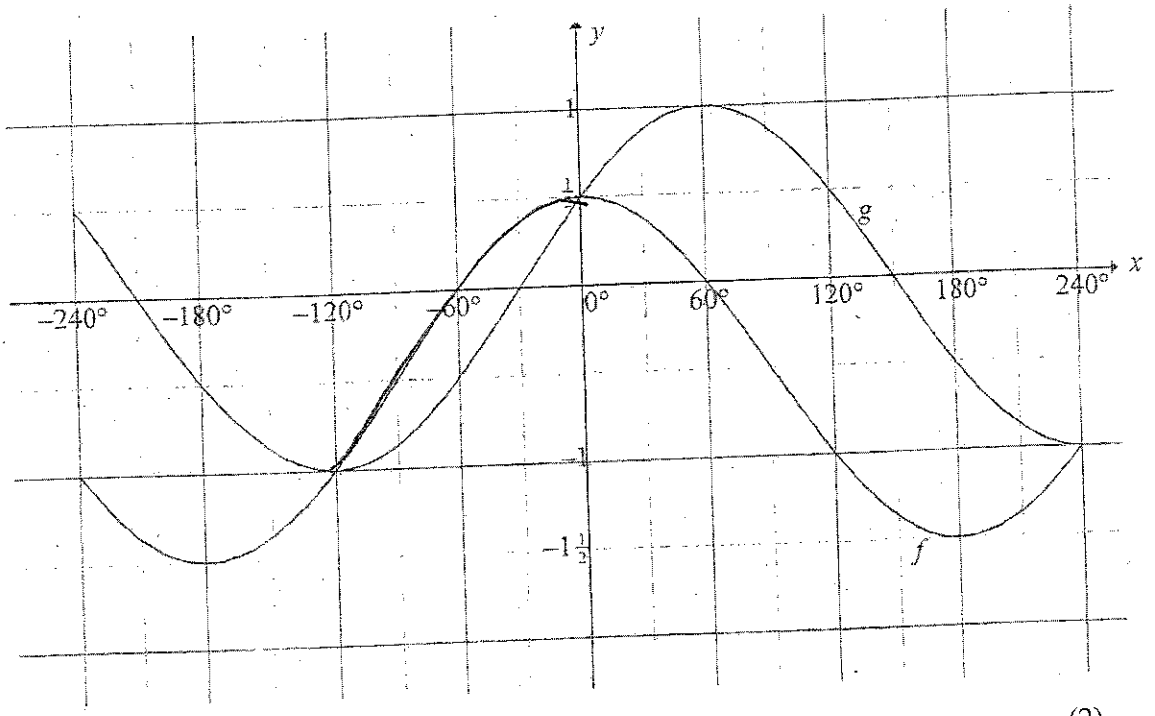
$\sin\theta = -\frac{1}{2}$ or $\sin\theta = 1$ ✓

$\theta = 210° + k\cdot 360°$ ✓ $\theta = 90° + k\cdot 360°$ ✓

OR $\theta = 330° + k\cdot 360°$ ✓ $k \in \mathbb{Z}$

d) In the diagram below, the graphs of $f(x) = \cos x + q$ and $g(x) = \sin(x + p)$ are drawn on the same system of axes for $-240° \leq x \leq 240°$.

The graphs intersect at $(0°; \frac{1}{2})$, $(-120°; -1)$ and $(240°; -1)$



1) Determine the values of p and q. (2)

$p = -30°$ ✓ $q = -\frac{1}{2}$ ✓ 2

2) Determine the values of x in the given interval for which $f(x) > g(x)$. (2)

$x \in (-120°; 60°)$ ✓✓ 2

3) Determine the equation of $h(x)$ if h is the graph of f shifted 90° to the left. (1)

$h(x) = \cos(x + 90°) - \frac{1}{2}$ ✓ cl ✓

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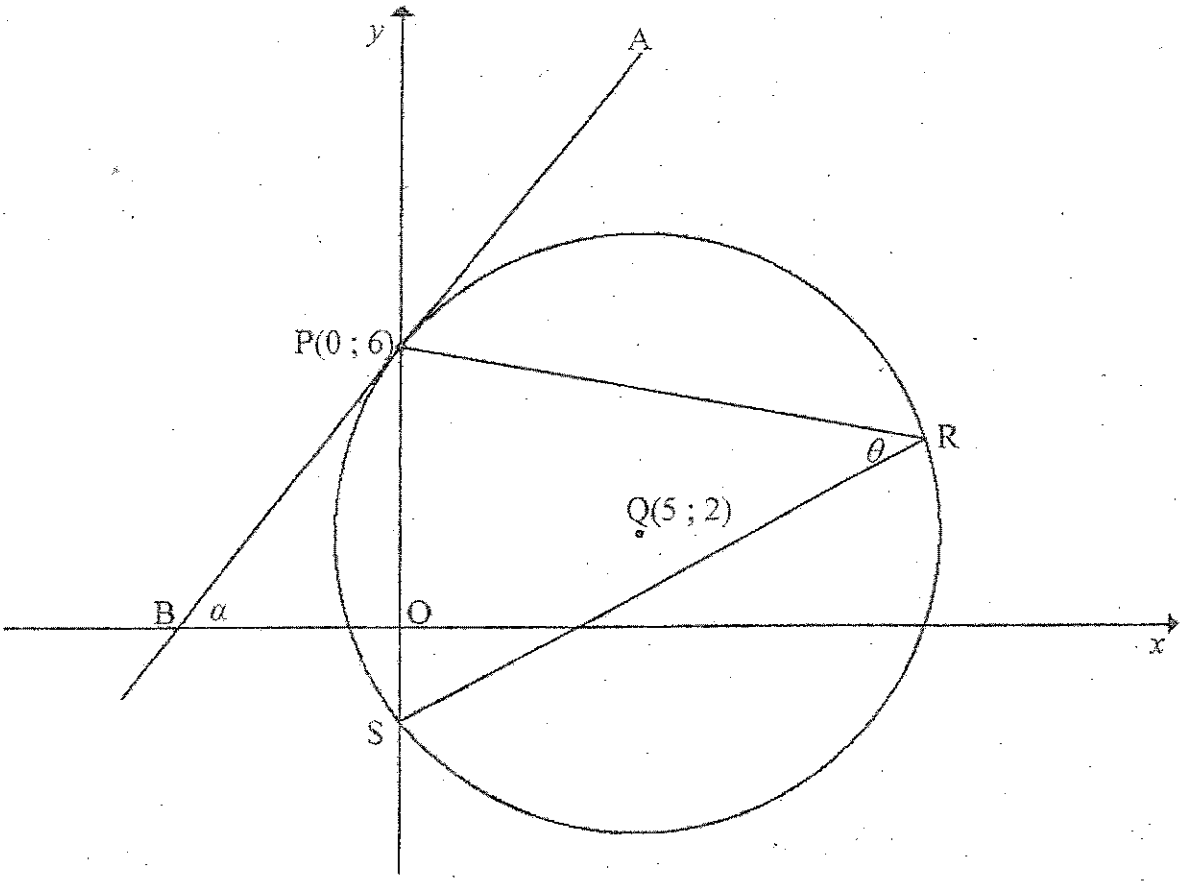
QUESTION 6

In the diagram below, $Q(5;2)$ is the centre of the circle that intersects the y-axis at $P(0;6)$ and S. The tangent APB intersects the x-axis at B and makes the angle α with the x-axis. R is a point on the circle and $P\hat{R}S = \theta$.



a) Determine the equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$. (3)

$r^2 = (5-0)^2 + (2-6)^2$

$= 41$

$(x-5)^2 + (y-2)^2 = 41$ 3

b) Calculate the coordinates of S. (3)

y int. $25 + y^2 - 4y + 4 = 41$

$y^2 - 4y - 12 = 0$

$(y-6)(y+2) = 0$

S(−2 ; 0) 3

c) Determine the equation of the tangent APB. (4)

$m_r = \frac{6-2}{0-5}$

$= \frac{-3}{5}$ ✓

$m_t = \frac{5}{3}$ ✓

$y = \frac{5}{3}x + 6$ ✓ ✓

4

d) Calculate the size of angle α. (2)

$\tan\alpha = \frac{5}{3}$ ✓

$\alpha = 59°$ ✓

2

e) Calculate the area of ΔPQS. (3)

$\text{Area} = \frac{1}{2}(6+2)(5)$ ✓

$= 20 \text{ units}^2$ ✓

3

f) Determine, with reasons, the value of θ. (4)

$B\hat{P}S = \theta$ ✓ tan/chord theorem

$P\hat{O}B = 90°$ angle

$\theta = 90° - 59°$ ✓ ∠ in ½ ●

$= 31°$ ✓

4

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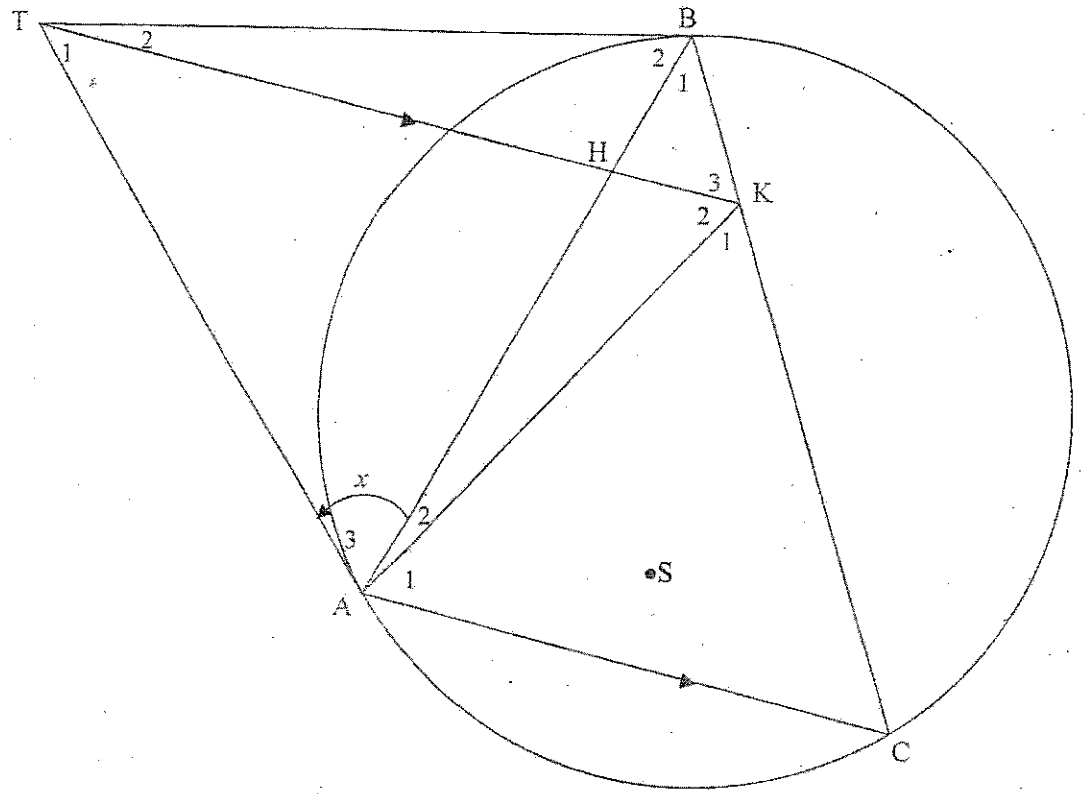
QUESTION 7

In the diagram below, ΔABC is drawn in the circle. TA and TB are tangents to the circle. The straight line THK is parallel to AC with H on BA and K on BC. AK is drawn. Let $\hat{A}_3 = x$.



a) Prove that $\hat{K}_3 = x$. (4)

$\hat{A}_3 = \hat{C} = x$ ✓ tan/chord ✓ theorem 4

$= \hat{K}_3 = x$ ✓ corresp ∠'s ✓ TK ∥ AC

b) Prove that AKBT is a cyclic quadrilateral. (2)

$\hat{K}_3 = \hat{A}_3$ ✓ (proven in a)) 2

∴ AKBT is cyclic quad. equal ∠'s subt. by chord TB ✓

c) Hence, prove that TK bisects $A\hat{K}B$. (2)

$\hat{B}_2 = \hat{A}_3 = x$ ✓ tangents from common point.

∴ $\hat{B}_2 = \hat{K}_2 = x$ ✓ ∠'s in same segm. 2

∴ $\hat{K}_3 = \hat{K}_2 = x$.

d) Prove that TA is a tangent to the circle passing through the points A, K and H. (2)

$\hat{A}_3 = \hat{K}_2$ ✓ proven.

∴ TA B tng. to AKH ∠ between tng ✓ and chord = ∠ in app segment.

[10]

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QUESTION 8

a) A set of 25 data values has a mean of p and a standard deviation of q.
Each data value is increased by c and the resulting value is then multiplied by d.
Determine the **MEAN** of this 'adjusted' data set. (2)

$(p+c)d$ ✓ 2

b) 99 numbers have an average of 101. Ninety of these numbers have an average of 100.
What is the average of the other nine numbers? (3)

Total off 99 numbers $= 101 \times 99$
$= 9\,999$ ✓

Total of 90 numbers $= 100 \times 90$ 3
$= 9000$ ✓

[5]

$\therefore$ 999 left $\therefore$ Ave $= \frac{999}{9}$ ✓ $= 111$ ✓

TOTAL: 100

5