

KING DAVID HIGH SCHOOL LINKSFIELD



MATHEMATICS PAPER 2

GRADE 11 FINAL EXAMINATION

NOVEMBER 2015

Total: 125 marks

Reading Time: 10 minutes

Writing Time: $2\frac{1}{2}$ hours

NAME:

Memo

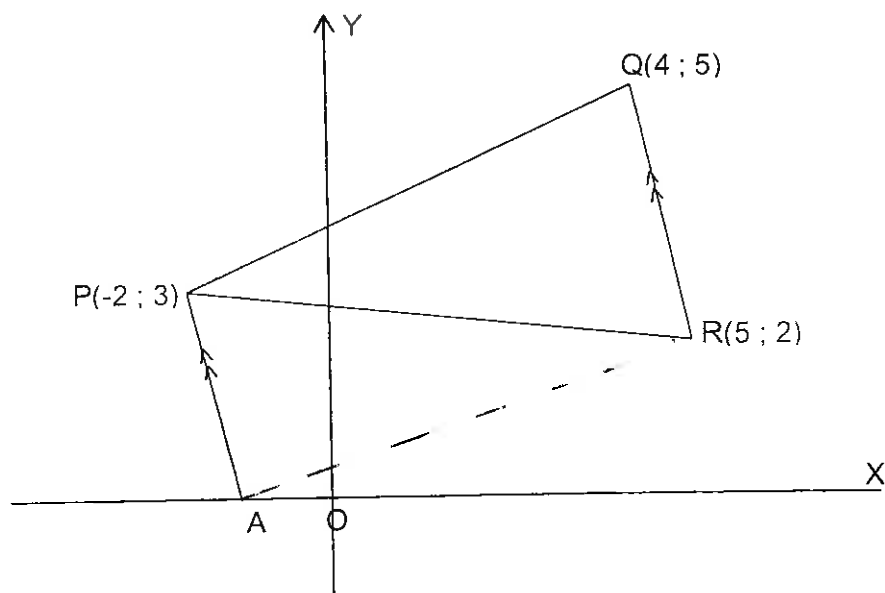
Please read the following instructions carefully:

1. This paper contains 16 pages (including this cover), and a data sheet.
Check that your paper is complete.
2. **Answer all the questions in this booklet.**
3. Pay careful attention to time management and mark allocation.
4. Write legibly and not in pencil. You may use either a blue or black pen only.
5. Non – programmable calculators may be used unless otherwise instructed.
6. Unless otherwise stated, all answers must be given to **TWO** decimal places where appropriate.
7. All necessary calculations must be clearly shown. You will **NOT** receive full credit if you write down only the answers and show no working out.

1 [10]	2 [6]	3 [14]	4 [5]	5 [5]	6 [10]	7 [6]	8 [8]
9 [5]	10 [4]	11 [7]	12 [15]	13 [7]	14 [11]	15 [5]	16 [7]

EXAM TOTAL	
_____	= %
125	

Question 1



In the diagram, which is not drawn to scale, $P(-2; 3)$, $Q(4; 5)$ and $R(5; 2)$ are the vertices of a triangle. $PA \parallel QR$ with A on OX . Determine:

a) the gradient of QR (2)

$$m_{QR} = \frac{3}{-1}$$

b) the equation of PA (3)

$$y - 3 = -3(x + 2)$$

$$y = -3x - 3$$

c) the intercept of PA with the x -axis, (2)

$$0 = -3x - 3$$

$$x = -1 \quad A(-1; 0)$$

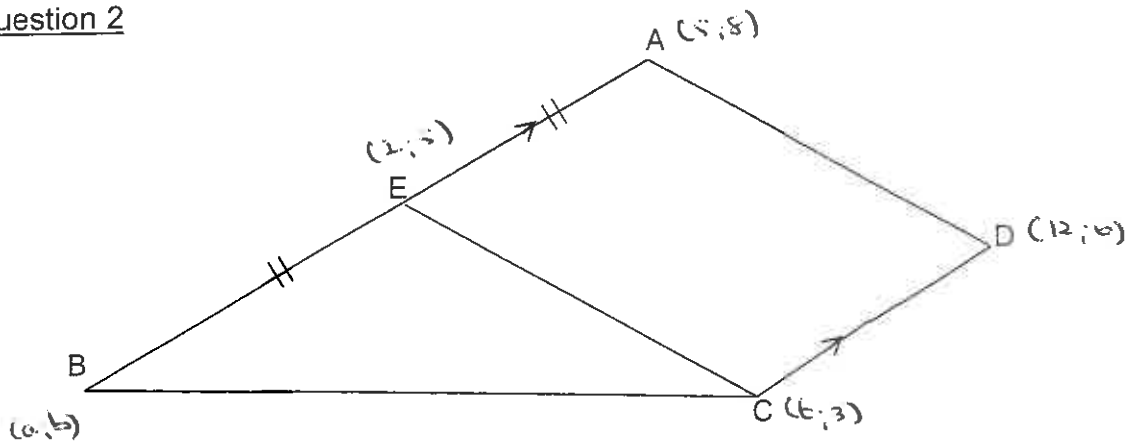
d) prove that $\hat{QRA} = 90^\circ$, (3)

$$m_{RA} = \frac{2}{6}$$

$$= \frac{1}{3}$$

$$\therefore m_{QR} \cdot m_{RA} = -1$$

Question 2



In the diagram, which is not drawn to scale, the co-ordinates of the points are: $A(5; 8)$, $B(a; b)$, $C(t; 3)$, $D(12; 6)$ and $E(2; 5)$. E is the midpoint of AB . Show the necessary calculations and

- a) determine the values of a and b . (2)

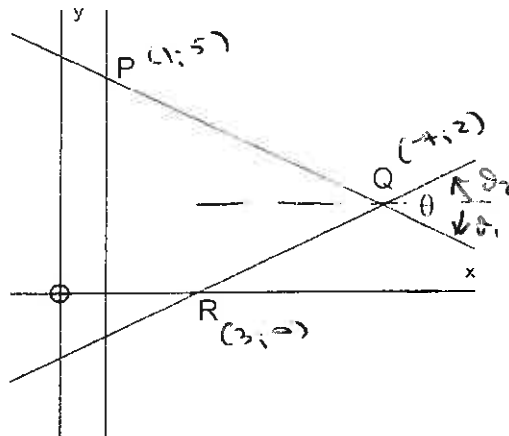
$$\begin{aligned} \text{Ans: } & x \rightarrow -3, y \rightarrow -3 \\ & B(-1; 2) \end{aligned}$$

- b) determine the value of t if $AE \parallel DC$ (4)

$$\begin{aligned} \frac{3}{3} &= \frac{3}{12-t} \\ 12-t &= 3 \\ t &= 9 \end{aligned}$$

P.T.O. QUESTION 3

Question 3



The diagram shows a triangle whose sides have equations

[A] $2y + x = 11$, [B] $2y - x + 3 = 0$ and [C] $x = 1$.

a) Write down the letter which corresponds to the equation of PQ. (1)

A

b) Determine the size of angle θ . (5)

$$m_{PQ} = \tan \theta, \quad m_{QR} = \tan \theta_2 = \frac{1}{2}$$

$$\tan \theta_1 = -\frac{1}{2}$$

$$\theta_2 = 26,56^\circ$$

$$\theta_1 = -26,56^\circ$$

$$\therefore \theta = 53,13^\circ$$

c) Calculate the co-ordinates of P and R. (3)

$$P(1; 5)$$

$$R(3; 0)$$

$$\text{Subs } x=1 \text{ into } 2y+x=11$$

- d) If Q is the point (7 ; 2) calculate the area of ΔPQR . (5)

$$PQ = \sqrt{(8-7)^2 + (6-2)^2}$$

$$= \sqrt{45}$$

$$RQ = \sqrt{(7-3)^2 + (2-0)^2}$$

$$= \sqrt{20}$$

$$\therefore \Delta = \frac{1}{2} \sqrt{45} \sqrt{20} \sin 52,13^\circ$$

$$= 11,84 \text{ u}^2$$

QUESTION 4

If $\cos 34^\circ = p$, write down the following in terms of p:

- (a) $\cos 214^\circ$ (2)

$$= -\cos 34^\circ$$

$$= -p$$

- (b) $\tan(-34^\circ)$ (3)

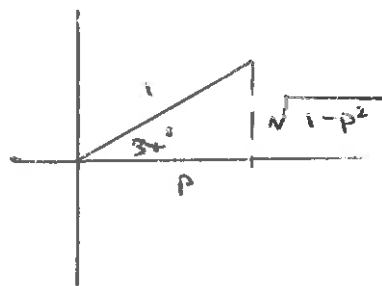
$$= -\tan 34^\circ$$

$$= -\frac{\sqrt{1-p^2}}{p}$$

OR

$$\sin 34^\circ = \sqrt{1-p^2}$$

$$\therefore \frac{\sin 34^\circ}{\cos 34^\circ} = -\frac{\sqrt{1-p^2}}{p}$$



P.T.O. QUESTION 5

Question 5

Determine the general solution, correct to one decimal place if $2 \cos(x - 30^\circ) + \frac{1}{3} = 0$

$$\cos(x - 30^\circ) = -\frac{1}{6} \quad (5)$$

$$\text{Let } A = (x - 30^\circ)$$

$$\therefore A = 199,6^\circ + 360k, \quad k \in \mathbb{Z}$$

$$x - 30^\circ = 199,6^\circ + 360k$$


$$\text{OR } x - 30^\circ = -99,6^\circ + 360k$$

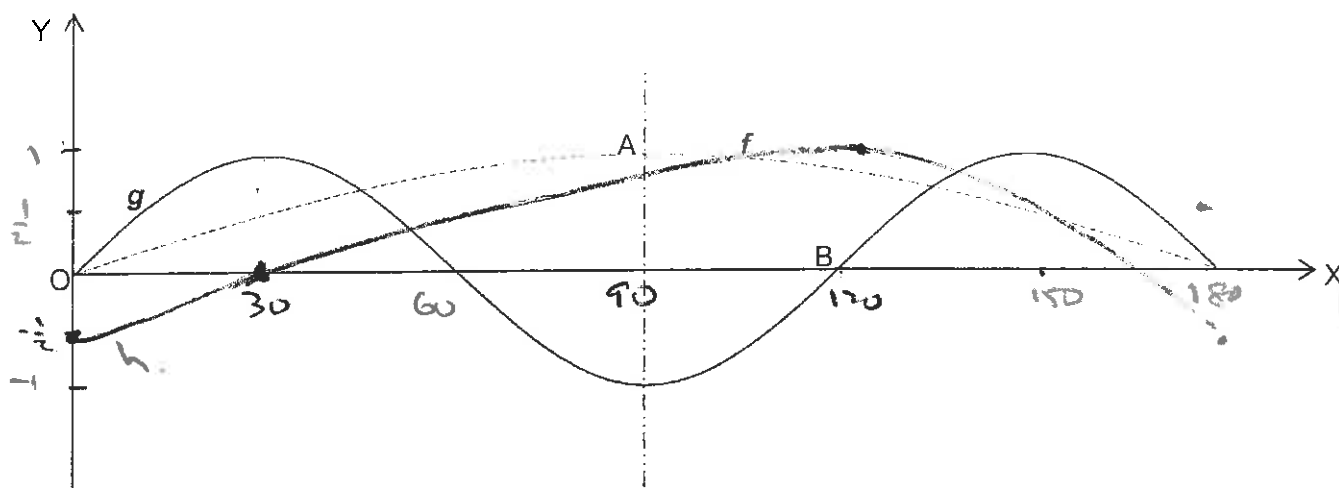
$$x = 129,6^\circ + 360k$$

$$x = -69,6^\circ + 360k$$

Question 6

The dotted curve () represents a part of the graph of $f(x) = \sin x$.

The solid curve () represents part of $g(x) = \sin(mx)$.



(a) What is the domain of f as given in the diagram? (1)

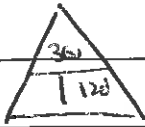
$$[0^\circ; 180^\circ]$$

(b) Write down the co-ordinates of A and B. (3)

$$A(90; 1)$$

$$B(120; 0)$$

- (c) The equation of g is given by $g(x) = \sin(mx)$. Determine the value of m . (2)



$$\therefore m = 3$$

$$y = \sin 3x$$

- (d) On the same set of axes sketch the graph of $h(x) = \sin(x - 30^\circ)$.
Label all important points clearly. (4)

Question 7

Prove that

$$\frac{\cos(-x)}{\sin 90^\circ - \sin x} - \tan x = \frac{1}{\sin(90^\circ - x)} \quad (6)$$

$$\text{LHS} = \frac{\cos x}{1 - \sin x} - \frac{\sin x}{\cos x}$$

$$\text{RHS} = \frac{1}{\cos x}$$

$$= \frac{\cos^2 x - \sin x(1 - \sin x)}{\cos x(1 - \sin x)}$$

$$\cos x(1 - \sin x)$$

$$= \frac{\cos^2 x + \sin^2 x - \sin x}{\cos x(1 - \sin x)}$$

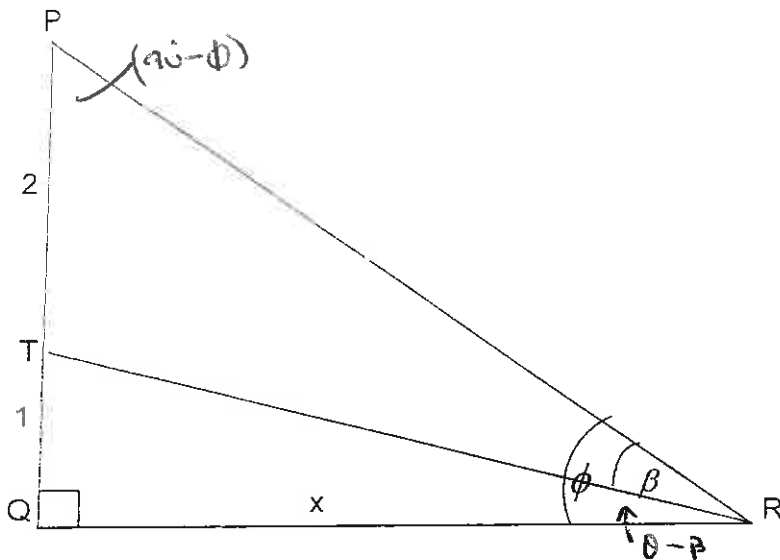
$$\cos x(1 - \sin x)$$

$$= \frac{(1 - \sin x)}{\cos x(1 - \sin x)}$$

$$\cos x(1 - \sin x)$$

$$= \text{RHS}$$

Question 8



In the diagram PT is a chalkboard 2 metres high, 1 metre above eye level QR of the learner at R . $QR = x$ metres. The angle of elevation of the top of the board, from R is ϕ . $\hat{PRT} = \beta$

a) Show that $TR = \frac{x}{\cos(\phi - \beta)}$ (2)

In $\triangle TRP$:

$$\frac{x}{\cos(\phi - \beta)} = TR$$

b) Express \hat{P} in terms of ϕ (1)

$$(90 - \phi)$$

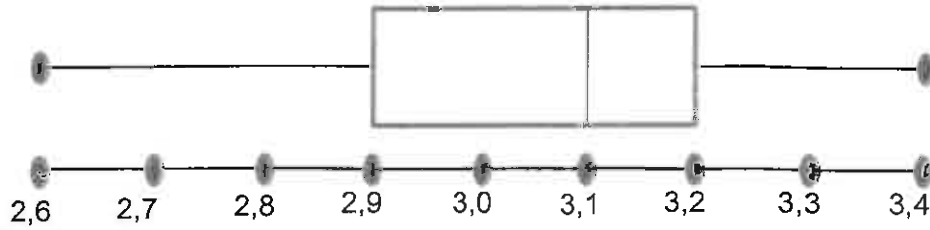
c) Using $\triangle PTR$, find TR . HENCE show that $x = \frac{2 \cos(\phi - \beta) \cdot \cos \phi}{\sin \beta}$ (5)

$$\frac{TR}{\sin(90 - \phi)} = \frac{2}{\sin \beta}$$

$$TR = \frac{2 \cos \phi}{\sin \beta}$$

$$\therefore \frac{x}{\cos(\phi - \beta)} = \frac{2 \cos \phi}{\sin \beta} \quad \therefore x = \frac{2 \cos \phi \cdot \cos(\phi - \beta)}{\sin \beta}$$

Question 9



The box-and-whisker diagram represents the mass of **60 babies** born at a public hospital in a one-week period.

- a) Is the data set skewed to the left or right or is it symmetrical? (1)

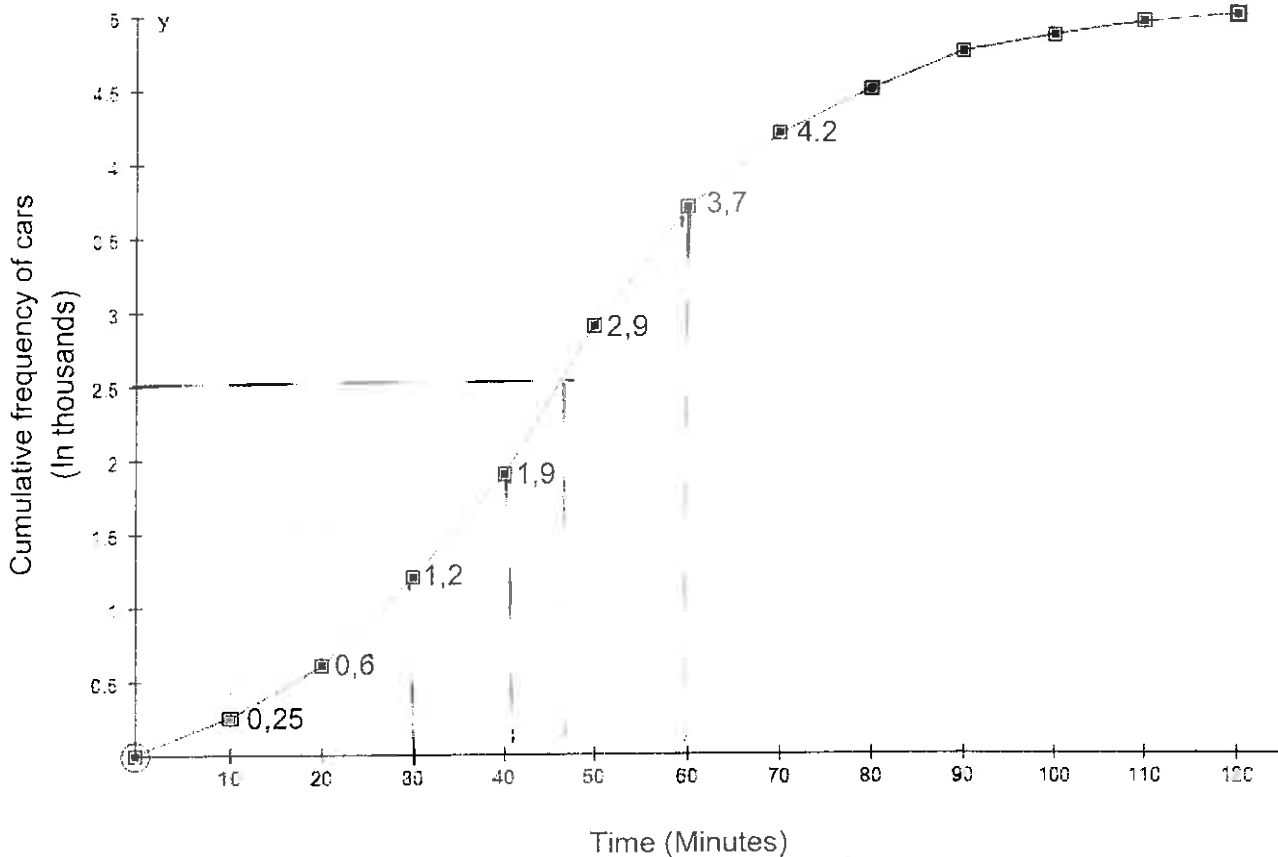
To the left

- b) The frequency table for the data is shown below. Write down (in the relevant spaces) the values of A, B, C and D. (4)

Mass interval (in kg)	Frequency
$2,6 \leq x < 2,7$	(A) = 2
$2,7 \leq x < 2,8$	6
$2,8 \leq x < 2,9$	7
$2,9 \leq x < 3,0$	7
$3,0 \leq x < 3,1$	(B) = 8
$3,1 \leq x < 3,2$	(C) = 15
$3,2 \leq x < 3,3$	(D) = 13
$3,3 \leq x < 3,4$	2

P.T.O. QUESTION 10

Question 10



The ogive above represents the number of cars passing through a toll gate in a 2 hour period on a Sunday, starting at 13h00.

Note that the number of cars is shown in thousands.

- (a) How many cars passed through the toll gate in the first hour? (1)

3700

- (b) How many cars passed through the toll gate between 13h30 and 13h40? (1)

$1.9 - 1.2 = 700$

- (c) Estimate the time at which half the cars had passed through the toll gate (1)

$\approx 13h45$

- (d) During which ten-minute period did the most cars pass through the toll gate? (1)

13:40 - 13:50

QUESTION 11

A teacher, who teaches a small class of 7 students, claims that the class average of 65% for her class is a reasonable indication of the spread of the marks.

a) Complete the table: (2)

x	$x - \bar{x}$	$(x - \bar{x})^2$
87	18	324
66	1	1
42	-23	529
73	8	64
67	2	4
81	16	256
39	-26	676
Total		1854

b) Use a formula to calculate the standard deviation (to 2 decimal places) (2)

$$Sd = \sqrt{\frac{1854}{7}}$$

$$= 16,27$$

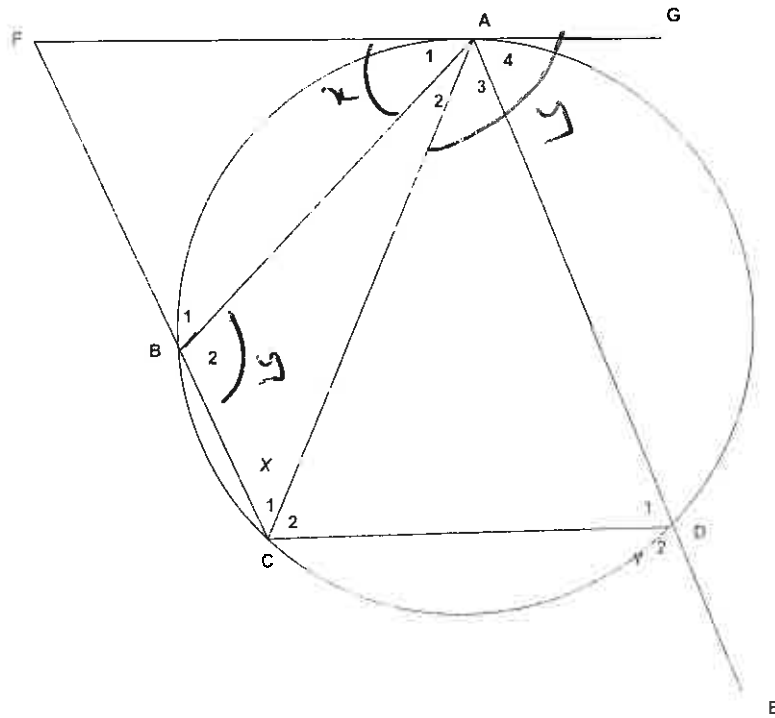
c) What does the value of the standard deviation suggest about the spread of the data? (1)

A good test

d) Is the class average a fair indication of the spread of the marks? Explain. (2)

Yes, $\approx \frac{2}{3}$ of class within 1sd

Question 12



In the figure, ABCD is a cyclic quadrilateral and FAG is a tangent to the circle at A.
 $\hat{C}_1 = x$ and $\hat{D}_2 = y$. Reasons must be given in each case.

- (a) Write down an angle equal to x (2)

$\hat{A}_1 = x$ *tan chord th*

- (b) Write down **two** angles each equal to y . (4)

\hat{B}_2 *ext. l cyclic quad*
 $\hat{A}_3 + \hat{A}_4$ *tan chord th*

- (c) Express \hat{F} in terms of x and y . (2)

$\hat{F} = y - x$ *ext. l of Δ*

- (d) Express \hat{A}_2 in terms of x and y . (2)

$180^\circ - (x + y)$ *ls on st line*

(e) Calculate y so that AC is a tangent to circle AFB. (4)

$\hat{A}_2 = F$ *conv. base chd.*

$\therefore 180 - x - y = y - x$

$\therefore 180 = 2y$

$90^\circ = y$

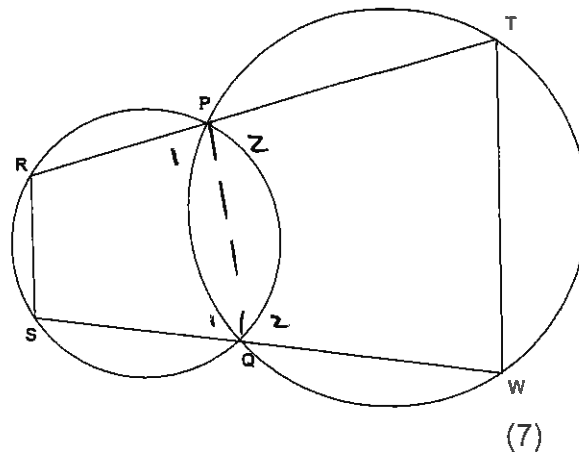
(f) What can be deduced about the line AC relative to circle ABCD if y has the value determined in (e) above? (1)

AC is a diam.

Question 13

In the figure, two circles intersect at P and Q. Double chords RPT and SQW are drawn.

Prove that $RS \parallel TW$



$\angle RPQ = \angle SQP$

$\hat{R} = \hat{Q}_2$ *ext. \angle cyclic quad*

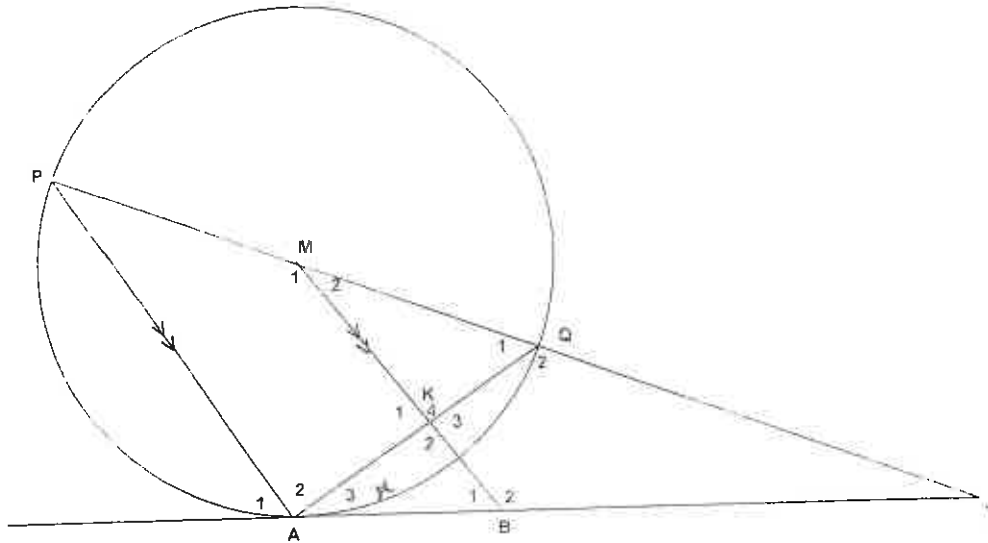
$\hat{Q}_2 + \hat{T} = 180$ *opp \angle s cyclic quad*

$\therefore \hat{R} + \hat{T} = 180$

$\therefore RS \parallel TW$ *corr. \angle s supp.*

Question 14

In the figure, M is the centre of the circle. PQ is a diameter and ABT is a tangent to the circle. PA // MB. MB cuts AQ at K.



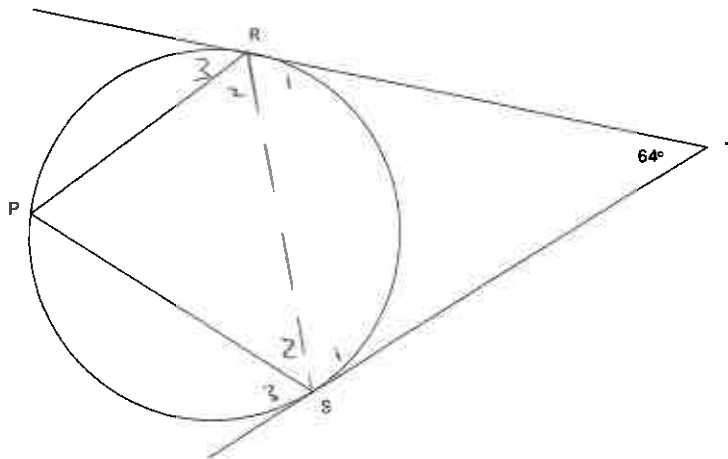
(a) Prove that $AK = KQ$ (5)

$\hat{A}_2 = 90^\circ$ ∠ subt by diam / ∠ in semi circle
 $\therefore \hat{K}_4 = 90^\circ$ corresp ∠ PA // MB
 $\therefore AK = KQ$ line from centre ⊥ chord

(b) If $\hat{A}_3 = x$, name with reasons, **two** other angles each of which equals x . Hence, or otherwise, prove that A, M, Q and B are concyclic. (6)

$\hat{P} = x$ tan chord th
 $\hat{M}_2 = x$ corresp ∠ PA // MB
 $\therefore \hat{M}_2 = \hat{A}_3$
 $\therefore AMQB$ cyclic quad conv. ∠s insc. seg

Question 15



In the figure, RT and ST are tangents to the circle at R and S respectively. $\hat{T} = 64^\circ$.
Determine with reasons the size of \hat{P} . (5)

Join RS

$$\therefore \hat{R} = \hat{S}, \text{ tangs from common pt}$$

$$\therefore \hat{R} = \hat{S} = \frac{180^\circ - 64^\circ}{2} \quad \text{ls of } \Delta$$

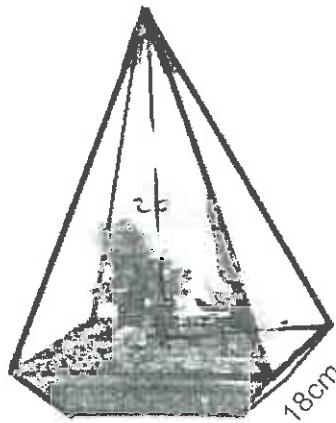
$$= 58^\circ$$

$$\therefore \hat{P} = 58^\circ \quad \text{ls chord th}$$

P.T.O QUESTION 16

Question 16

A glass flower container was formed by joining two regular pyramids, of equal height and base area, together. The lower pyramid, which holds the soil, was cut off so that it has a square base and **its volume is a quarter of the volume of the upper pyramid**. The volume of the part which holds the soil is 540 cm^3 . The length of the base of the upper pyramid is 18 cm.



- a) Calculate the height of the container. (4)

$$\text{Vol of pyramid} = 4 \times 540$$

$$= 2160 \text{ cm}^3$$

$$\text{Vol} = \frac{1}{3} (18)^2 h = 2160$$

$$h = 20 \text{ cm}$$

$$\frac{3}{4} \text{ Volume} = \frac{1}{3} \left(\sqrt{\frac{3}{4}} \cdot 18 \right)^2 \left(\sqrt{\frac{3}{4}} \cdot 20 \right)$$

$$\frac{3}{4} \text{ Height} = \sqrt{\frac{3}{4}} \cdot 20 = 18,17 \therefore \frac{1}{4} H = 1,83$$

$$\text{Total height} = 21,83 \text{ cm}$$

- b) Calculate the amount of glass required for the upper pyramid which has only **3 side panels**. (It is open in front). Leave your answer in **surd form**. (3)

$$s.l = \sqrt{(20)^2 + (9)^2}$$

$$= \sqrt{481}$$

$$\text{Area} = 3 \times \frac{1}{2} \times \sqrt{481} \times 18$$

$$= 27 \sqrt{481} \text{ cm}^2$$