RK

ROEDEAN SCHOOL (SA)

INSPIRING A LIFE OF SIGNIFICANCE

ANTE - MATRIC

NOVEMBER 2014

MATHEMATICS: PAPER II

Time: 3 hours	**150 marks**
Reading Time: 10 minutes	**Examiner: R. Karam**
	Moderator: K. Hulme

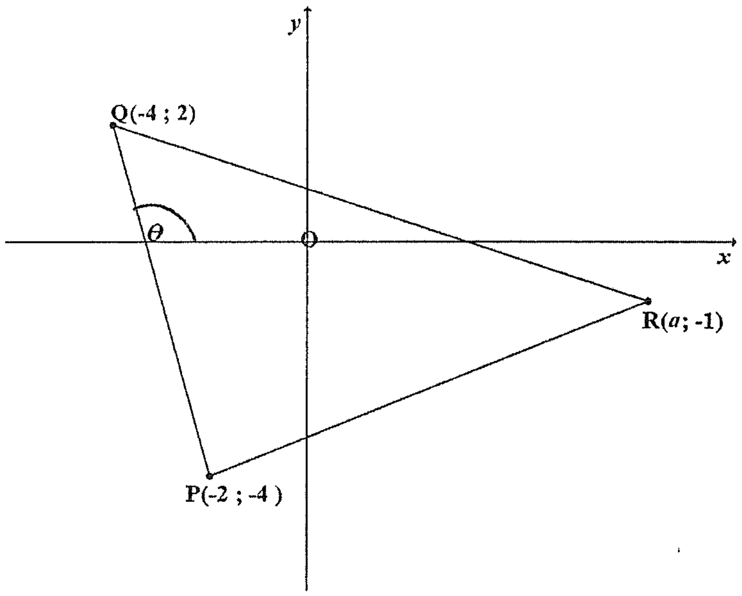
PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 28 pages (25 printed pages and 3 pages of lines for extra work if required), as well as, an Information Sheet.
2. Please check that your question paper is complete.
3. Read the questions carefully.
4. **Answer ALL the questions on the question paper and hand this in at the end of the examination.**
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. All necessary working details must be clearly shown.
7. Round off your answers to one decimal digit where necessary, unless otherwise stated.
8. Ensure that your calculator is in **DEGREE** mode.
9. It is in your own interest to write legibly and to present your work neatly.
10. The last pages can be used for additional working, if necessary. If this space is used, make sure that you indicate clearly which question is being answered.
11. No diagrams are drawn to scale.

SECTION A

QUESTION 1

Refer to the sketch below:



In the diagram, P(–2 ; –4); Q(–4 ; 2); and R(a; –1) are the vertices of Δ QPR.

(a) Determine the gradient of the line PQ. (2)

$m_{PQ} = \frac{-4-2}{-2+4}$ ✓M

$= \frac{-6}{2}$

$= -3$ ✓A

(b) Calculate the size of θ to the nearest degree. (2)

$\tan \theta = -3$

$\theta = -71{,}56^\circ + 180^\circ$ ✓A ✓m

$= 108{,}43^\circ$

$= 108^\circ$ ✓A

(c) Determine the gradient of the line PR, if QP $\perp$ PR. (1)

$m_{PR} = +\frac{1}{3}$ ✓A

(c) Determine the gradient of the line PR, if QP $\perp$ PR. (1)

$m_{PR} = +\frac{1}{3}$ ✓A

(d) Hence, determine the value of a. (3)

$m_{PR} = \frac{-1+4}{a+2} = \frac{1}{3}$ ✓M

$\therefore 9 = a+2$ ✓A

$7 = a$ ✓CA

(e) Calculate the area of Δ QPR, given that $a = 7$ (5)

$PQ = \sqrt{(-2+4)^2+(-4-2)^2}$ ✓M $PR = \sqrt{(-2-7)^2+(-4+1)^2}$

$= \sqrt{4+36}$ $= \sqrt{81+(9)}$

$= \sqrt{40}$ $= \sqrt{90}$

$= 2\sqrt{10}$ ✓A $= 3\sqrt{10}$ ✓A

$Area = \frac{1}{2}(2\sqrt{10})(3\sqrt{10})$ ✓M

$= 3(10)$

$= 30\ units^2$ ✓A

(f) Determine the coordinates of the midpoint M of QR, given that $R(7,-1)$ (1)

$M\left(\frac{-4+7}{2}; \frac{2-1}{2}\right)$

$M\left(\frac{3}{2}; \frac{1}{2}\right)$ ✓

(g) Hence, determine the equation of the line MN passing through M and parallel to PR. (3)

$M\left(\frac{3}{2}; \frac{1}{2}\right)$ $m_{PR} = \frac{1}{3}$ $MN \parallel PR \therefore m_{MN} = \frac{1}{3}$ ✓sub

$y - y_1 = m(x - x_1)$

$y - \frac{1}{2} = \frac{1}{3}\left(x - \frac{3}{2}\right)$ ✓M

$y = \frac{1}{3}x - \frac{1}{2} + \frac{1}{2}$

$y = \frac{1}{3}x$ ✓A

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QUESTION 2

The frequency table below represents the marks out of a maximum of 150 marks, obtained by a group of Grade 11 students in a Mathematics examination.

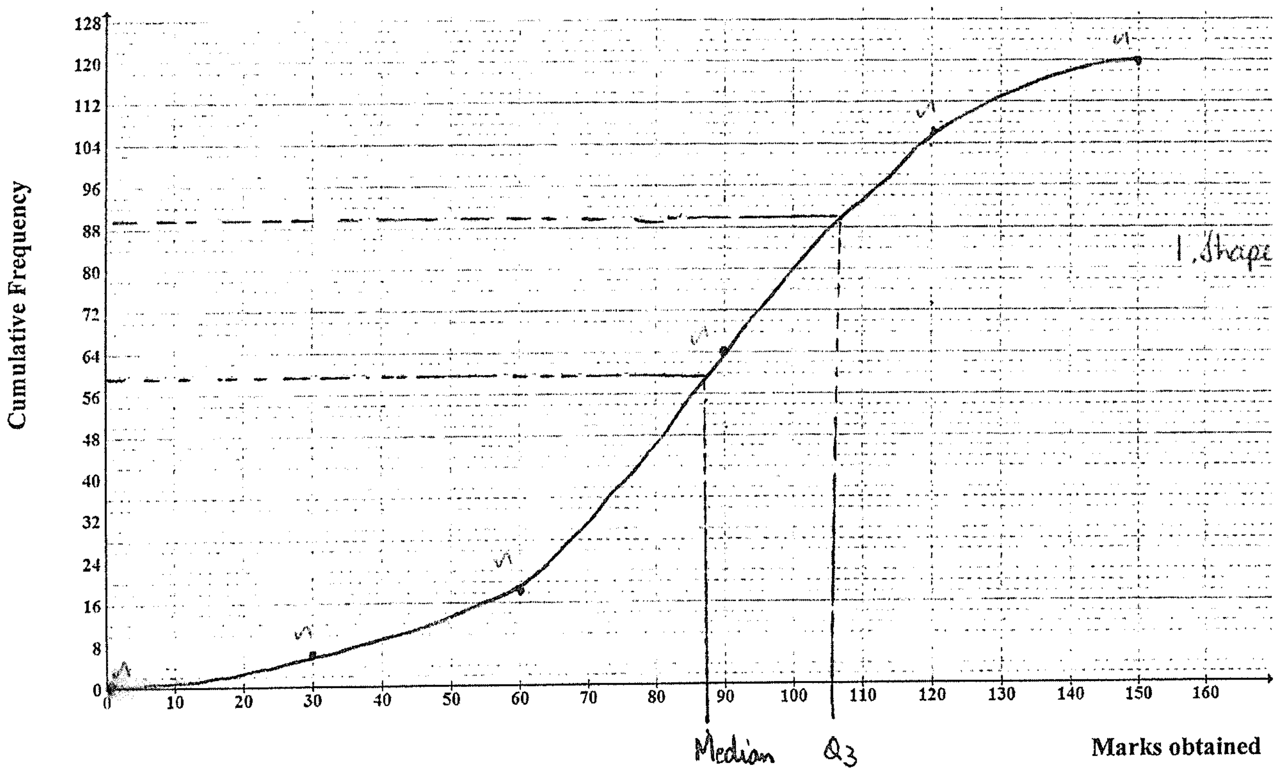
Marks Obtained	Frequency f	Cumulative Frequency	
$0 < x \leq 30$	6	6	(30, 6)
$30 < x \leq 60$	12	18 ✓✓	(60, 18)
$60 < x \leq 90$	46	64 ✓✓	(90, 64)
$90 < x \leq 120$	42	106 ✓✓	(120, 106)
$120 < x \leq 150$	14	120 ✓✓ A	(150, 120)

(a) Use the table to complete the cumulative frequency column. (2)

(b) Use the table to determine the medial class. (1)

$60 < x \leq 90$ ✓ A

(c) On the grid below, draw an Ogive, using the information from the table above. (4)



(d) Use the Ogive to determine the:

(i) median. (show where you took your reading) (1)

± 88 ✓A

(−1 for no readings)

(ii) the third quartile. (show where you took your reading) (1)

$\pm 106.$ ✓A

(e) Complete the table below: (3)

Marks Obtained	Midpoint x_i	Frequency f	$f \times x_i$
$0 < x \leq 30$	15	6	90 ✓A
$30 < x \leq 60$	45	12	540 ✓A
$60 < x \leq 90$	75	46	3 450 ✓A
$90 < x \leq 120$	105	42	4410 ✓A
$120 < x \leq 150$	135	14	1890 ✓A
			$\Sigma(f \times x_i) =$ 10380 A

(3)A

(f) (i) Using the information from the table above, determine the estimated mean. (2)

Est Mean: $\bar{x} = \frac{10380}{120}$ ✓M

$= 86,5$ ✓A

(ii) Explain briefly why it is called the estimated mean. (2)

· not actual values ✓

· midpt interval ✓

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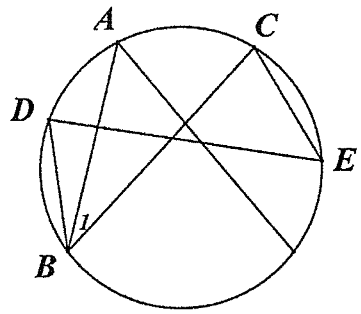
QUESTION 3

Circle the correct solution only.

(a)



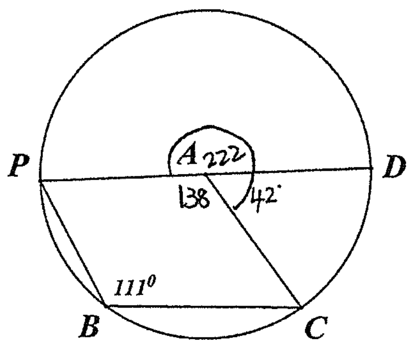
A, B, C, D, and E are points on the circumference of the circle.

Which statement is true? (1)

A. $\hat{A} = \hat{C}$ B. $\hat{D} = \hat{C}$

C. $\hat{A} = \hat{D}$ D. $\hat{B}_1 = \hat{E}$

(b)



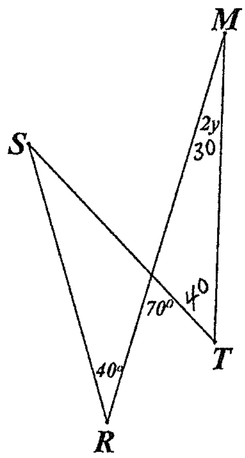
A is the centre of the circle, PAD is a straight line, and $\hat{B} = 111°$.

Determine the magnitude of $C\hat{A}D$. (2)

A. 69° B. 62°

C. 59° D. 42°

(c)



S, R, T and M lie on the circumference of a circle.
Determine the numerical value of y. (2)

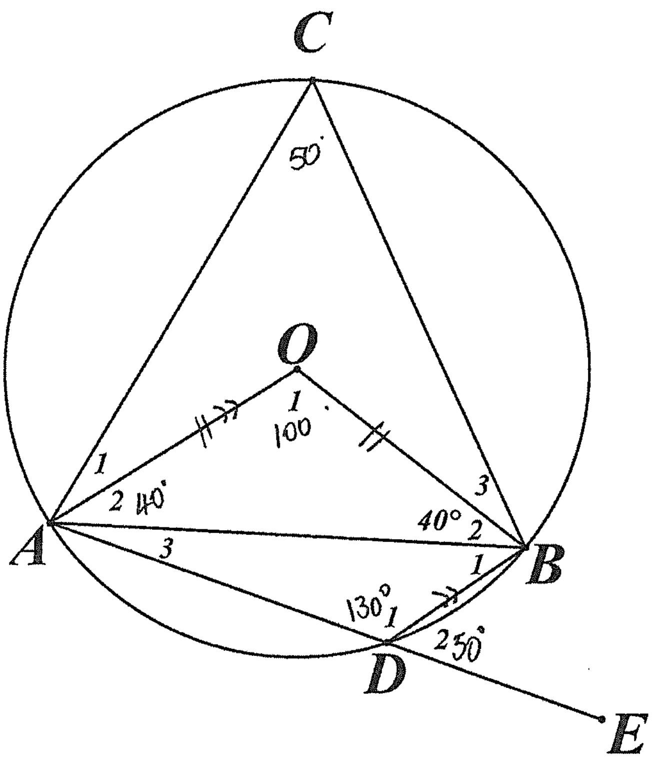
A. 35° B. 30°

C. 20° D. 15°

[5]

QUESTION 4

Refer to the figure below:



O is the centre of the circle CADB. ADE is a straight line. $\hat{B}_2 = 40°$.

Determine the following, **stating all necessary reasons**:

(a) $\hat{C}$. (4)

OA = OB ✓ radii ✓

$\hat{B}_2 = \hat{A}_2 = 40°$ ✓ ~~isos~~ (∠ opp eq sides) ✓

∴ $\hat{O}_1 = 100°$ ✓ 3∠'s Δ ✓

∴ $\hat{C} = 50°$ ✓ ∠ centre = 2∠ circum ✓

(b) $\hat{D}_2$. (1)

$\hat{D}_2 = 50°$ ✓✓ ext < = opp int ✓✓

(c) $\hat{A}_3$ if AO // DB. (2)

$\hat{A}_2 + \hat{A}_3 = 50°$ ✓✓ co-int <'s; AO ∥ DB ✓✓ ✓✓

$\therefore \hat{A}_3 = 10°$ ✓✓ ($\hat{A}_2 = 40$ proved)

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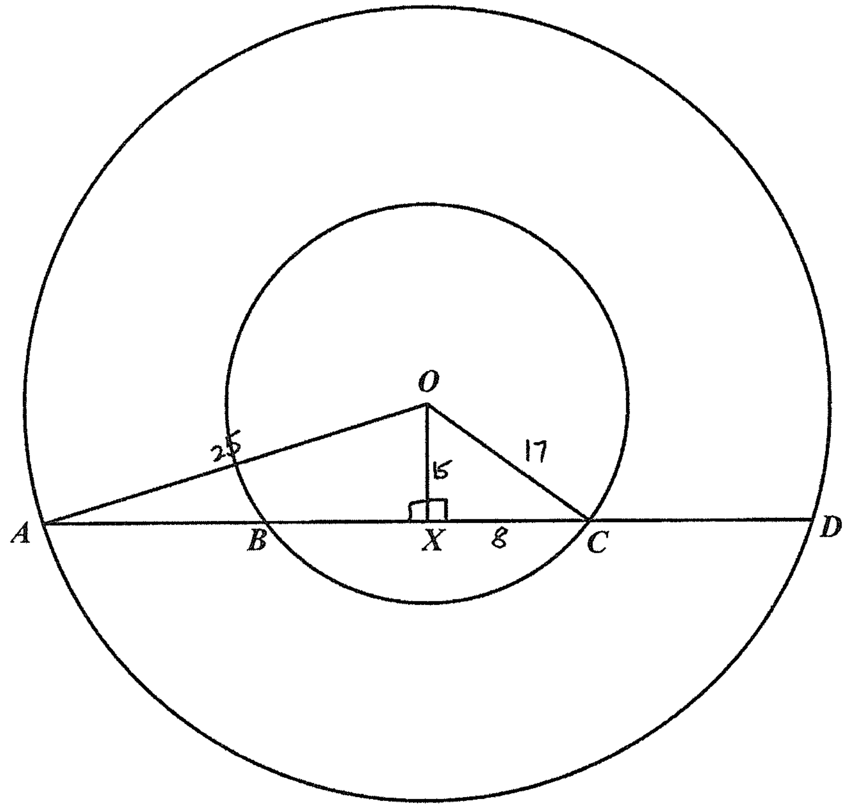
QUESTION 5

Refer to the diagram below:



In the diagram, O is the centre of two concentric circles.
ABCD is a straight line that intersects te circle as shown.
OX ⊥ AD; OA = 25 *cm*; OC = 17 *cm*; OX = 15 *cm*.

(a) Determine, with reasons, the length of AC. (5)

In ΔOXC

$XC^2 = 17^2 - 15^2$ Pythag

$XC = 8$ cm

In ΔOAX

$AX^2 = 25^2 - 15^2$ Pythag

$AX = 20$ cm

$\therefore AC = 28$ cm.

(b) Prove, with reasons, that AB = CD. (3)

BX = XC ⊥, centre, bisects chd

AX = XD ⊥; centre, bisects chd.

∴ AB+BX = XC+CD

BX = XC proved

∴ AB = CD

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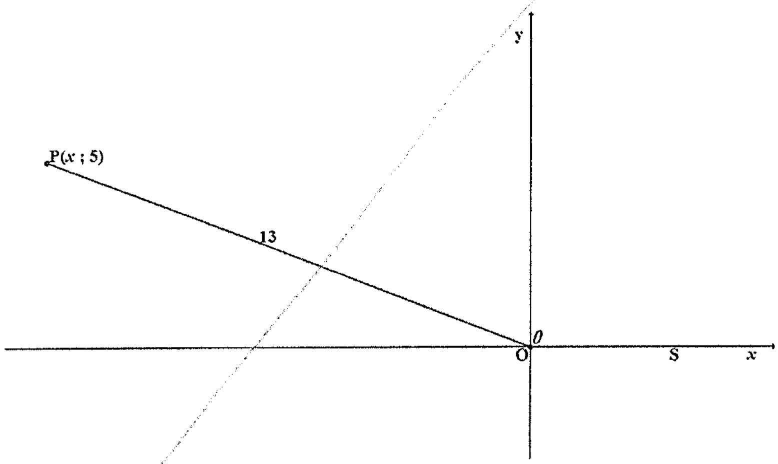
QUESTION 6

In the diagram below, P is the point $(x\,;5)$.

$\hat{POS} = \theta$. OP = 13 units.



Determine without the use of a calculator:

(a) the value of x. (2)

$x^2+5^2=13^2$ Pythag

$x^2 = 144$

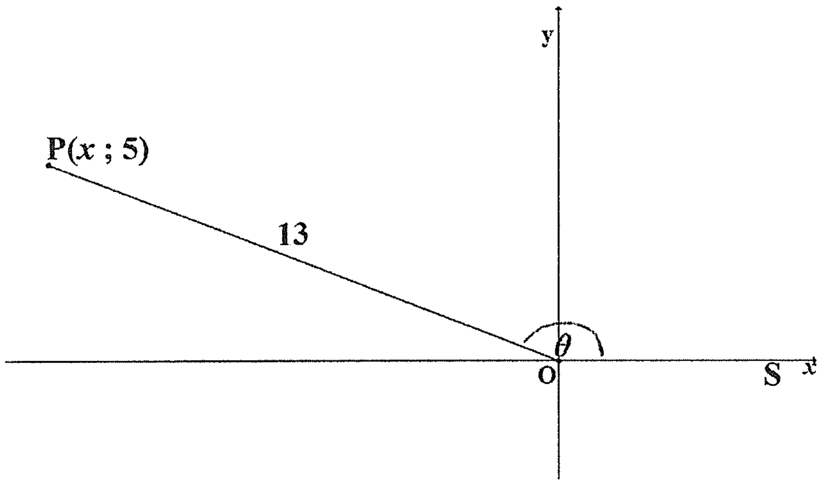
$x = \pm 12$

$\therefore x = -12$ (Quad 2)

QUESTION 6

In the diagram below, P is the point $(x\,;5)$.

$P\hat{O}S = \theta$. OP = 13 units.



Determine without the use of a calculator:

(a) the value of x. (2)

$x^2 + 5^2 = 13^2$ Pythag.

$x^2 = 144$ ✓M

$x = \pm 12$

$x = -12$ ✓A (Quad 2)

(b) $\cos\theta$. (1)

$\cos\theta = \frac{-12}{13}$ ✓A

(c) $\tan(360° - \theta)$ (2)

$\tan(360 - \theta)$

$= -\tan\theta$ ✓A

$= -\left(\frac{5}{-12}\right)$ ✓

$= \frac{5}{12}$ ✓A

[5]

QUESTION 7

Simplify, without the aid of a calculator. Show **all** calculations.

(a) $\dfrac{3\cos 150^\circ .\sin 270^\circ}{\tan(-45^\circ) + \cos 600^\circ}$ (6)

$= \dfrac{3(-\cos 30)(-1)}{\tan 315^\circ + \cos 240^\circ}$ Reductions

$= \dfrac{3(-\sqrt{3}/2)(-1)}{(-\tan 45^\circ) + (-\cos 60^\circ)}$ Special ∠s

$= \dfrac{3(-\frac{\sqrt{3}}{2})(-1)}{(-1) - \frac{1}{2}}$

$= \dfrac{3\sqrt{3}}{2} \div \dfrac{-3}{2}$ ✓ m

$= \dfrac{3\sqrt{3}}{\cancel{2}} \times \dfrac{\cancel{2}}{-3}$

$= -\sqrt{3}$ ✓ A

(b) $\dfrac{\sin(180^\circ - \theta).\tan\theta.\sin(90^\circ + \theta).\sin 310^\circ}{\tan(180^\circ - \theta).\cos(-\theta).\sin(360^\circ + \theta).\cos 140^\circ}$ (6)

$= \dfrac{(\cancel{\sin\theta})(\cancel{\tan\theta})(\cancel{\cos\theta})(-\sin 50^\circ)}{(-\cancel{\tan\theta})(\cancel{\cos\theta})(\cancel{\sin\theta})(-\cos 40)}$ (✓A)

$= -\dfrac{\cancel{\sin 50}}{\cancel{\cos 40}}$ ✓ ✓
sin50

$= -1$ ✓ (✓A)

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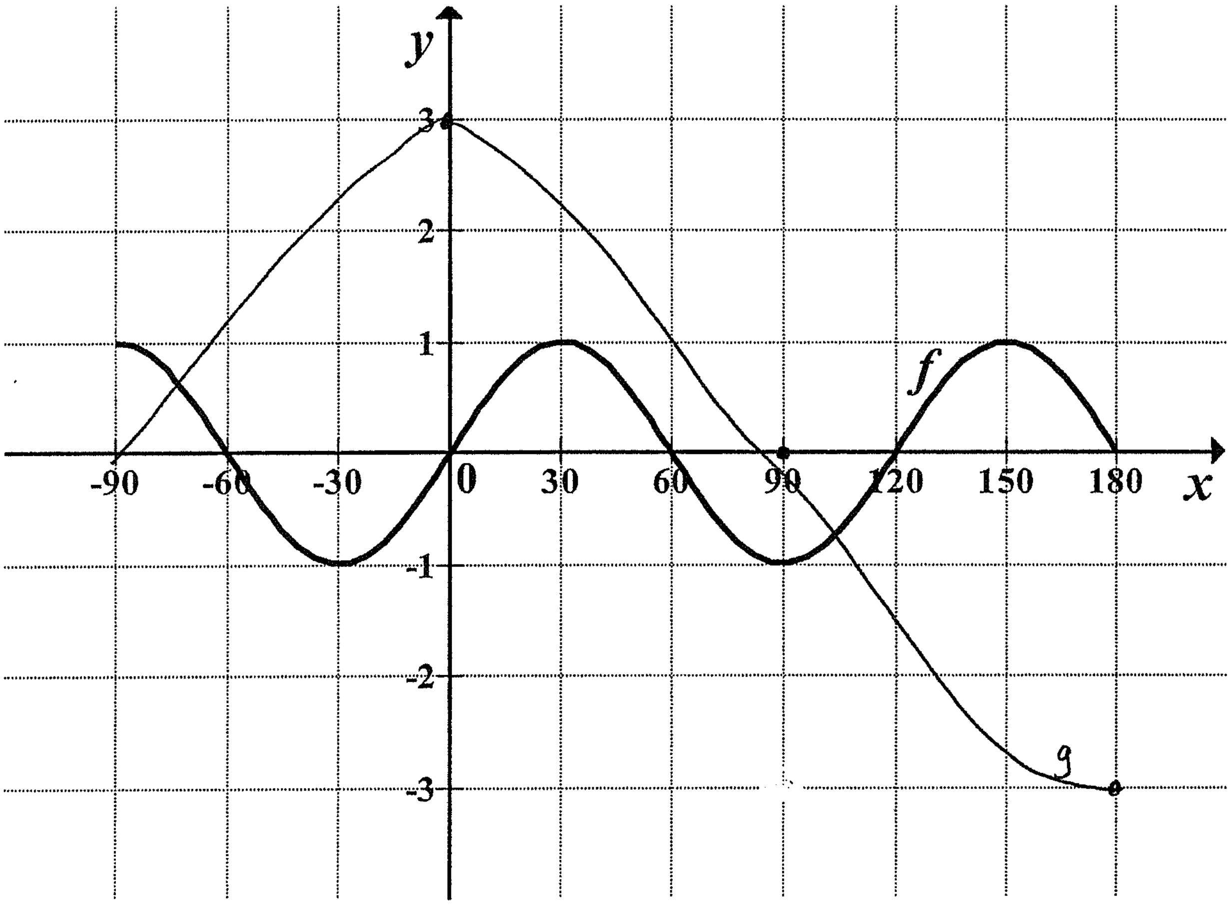
SECTION B

QUESTION 8

The graph $f(x) = \sin 3x$; $x \in [-90^0 ; 180^0]$, is drawn below.



(a) Write down the:

(i) period of f. (1)

120^0

(ii) frequency of f

(iii) amplitude of f. (1)

1

(b) Write down the value(s) of x, which satisfy the equation $\sin 3x = -1$, in the interval $x \in [-90^0 ; 180^0]$ (2)

$x = -30°$ ✓ or $x = 90°$ ✓ A

(c) Given $h(x) = f(x) - 2$, determine the maximum value of h. (1)

$h(x) = \sin 3x - 2$

$\therefore$ max $y = 1 - 2$ ✓ A

$\therefore -1 = y$

(d) Draw the graph of $g(x) = 3\cos x$ for $x \in [-90^0 ; 180^0]$ on the same system of axes, as f, on the grid provided on Page 14. (3)

(e) Use the graphs to determine the number of solutions that exist for the equation $\frac{\sin 3x}{3} - \cos x = 0$ on the interval $x \in [-90^0 ; 180^0]$. (2)

$\therefore \frac{\sin 3x}{3} = \cos x$

$\therefore \sin 3x = 3\cos x$ ✓ m

$\therefore f(x) = g(x)$ ✓ A

2 Solutions

(f) **Use the graphs** to solve for x:

(i) $f(x) \leq g(x)$ (2)

$x \in [-75° ; 105°]$ ✓ Approx

(ii) $f(x) \times g(x) \leq 0$ (4)

$x \in [-60° ; 0°]$ ✓ or $[60° ; 90°]$ ✓ or $[120° ; 180°]$ ✓

or $x = -90° ; 0°$; ✓

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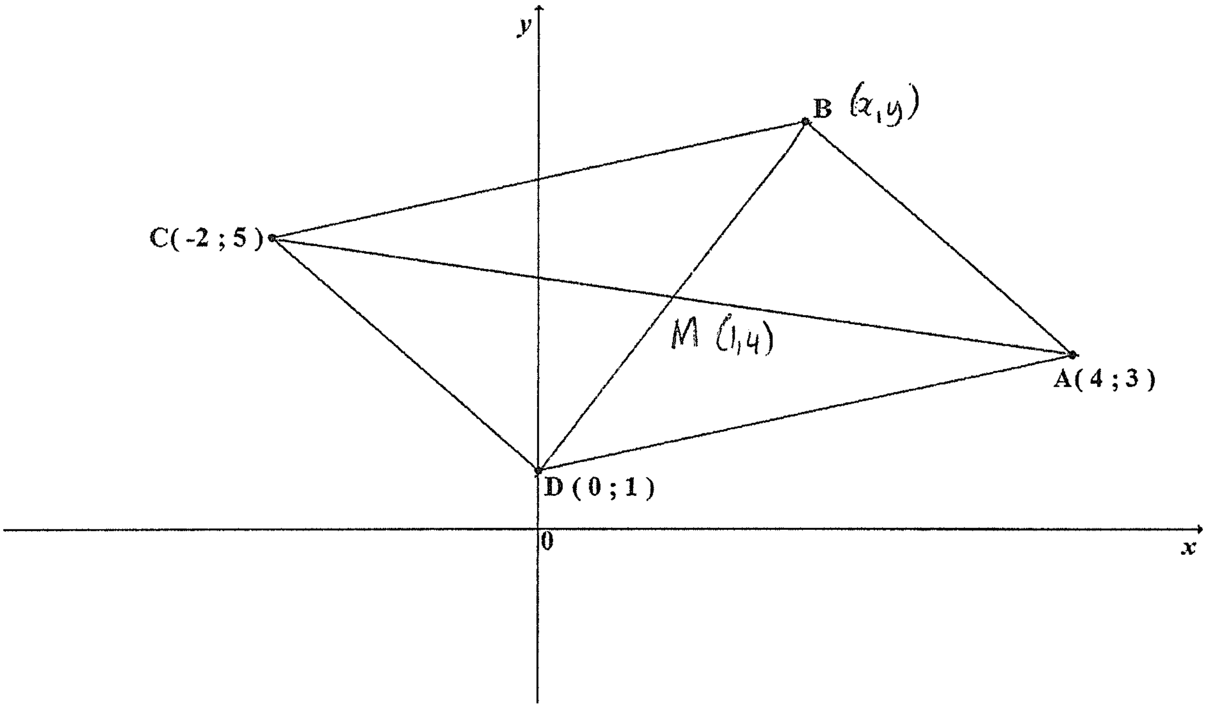
[18]

QUESTION 9

(a) Given: ABCD is a parallelogram.



(i) Determine the coordinates of M, the midpoint of AC. (2)

$M\left(\frac{-2+4}{2} ; \frac{5+3}{2}\right)$ ✓m

$M(1 ; 4)$ ✓A

(ii) Determine the coordinates of B. (2)

$B(x, y)$:

$\frac{x}{2} = 1$ OR $\frac{y+1}{2} = 4$

$x = 2$ ✓m $y + 1 = 8$

$y = 7$

$B(2, 7)$ ✓A

OR by transformation ✓m

$B(2, 7)$ ✓A

(iii) Prove that A, D and T are collinear, using analytical methods, given that T has coordinates $(-3\,;-\frac{1}{2})$ (3)

A (4; 3) D (0, 1) T $(-3; -\frac{1}{2})$

$m_{AD} = \frac{3-1}{4}$ ✓m $= \frac{1}{2}$ ✓A

$m_{DT} = \frac{1-(-\frac{1}{2})}{0-(-3)}$ ✓m $= \frac{3/2}{3} = \frac{1}{2}$ ✓A

$m_{AD} = m_{DT}$ ✓A

∴ Collinear as D is common

(iv) Determine $C\hat{B}D$ (6)

$\tan\alpha = m_{CB} = \frac{2}{4}$ ✓m $= \frac{1}{2}$

$\alpha = 26{,}6^\circ$ ✓A

$\tan\theta = \frac{6}{2}$ ✓m $= 3$

$\theta = 71{,}6^\circ$ ✓A

∴ $C\hat{B}D = 71{,}6 - 26{,}6$ ✓m $= 45^\circ$ ✓A

(b) The equation of a straight line AB is given by $y = -\frac{2}{3}x + 2$

The equation of the straight line CD is given by $3x + ry = -2$; $r \neq 0$

Determine the value(s) of r such that :

(i) CD // AB (3)

$3x + ry = -2$

$ry = -3x - 2$

$y = \frac{-3}{r}x - \frac{2}{r}$ ✓A

$m_{CD} = m_{AB}$

$\therefore -\frac{3}{r} = -\frac{2}{3}$ ✓m

$\therefore -2r = -9$

$r = 9/2$ ✓A

(ii) CD will intersect the line AB at the point (–3 ; 4). (3)

$y = -\frac{3}{r}x - \frac{2}{r}$

$4 = -\frac{3}{r}(-3) - \frac{2}{r}$ substitution ✓ m

$4 = \frac{9}{r} - \frac{2}{r}$ ✓ A

$4r = 7$

$r = \frac{7}{4}$ ✓A

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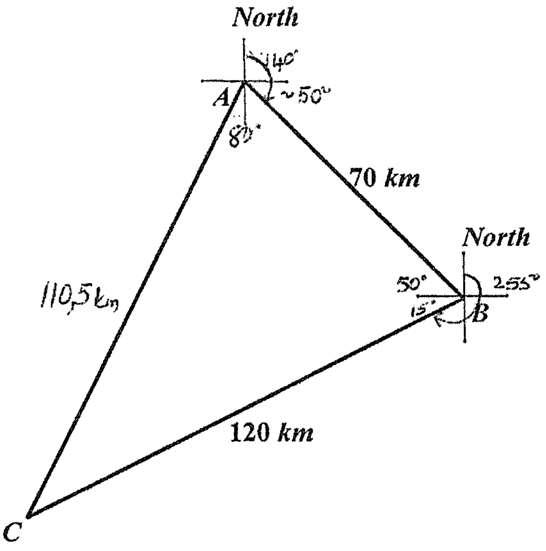
QUESTION 10



A ship leaves the Harbour A on a bearing of 140°.

The ship travels 70 km, before the Captain notices that they are off course.

He requests a change of direction and the ship then travels 120 km on a bearing of 255°.

(a) How far is the ship from the Harbour? Finding ∠'s (2) (4)

$AC^2 = 70^2 + 120^2 - 2(70)(120)\cos 65^\circ$ ✓m

$= 12200,\ldots$

$AC = 110,5$ km ✓A

(b) Determine the area of sea in this triangular section of sea. (2)

Area $= \frac{1}{2}(70)(120)\sin 65$ ✓m

$= 3806,5\ \text{km}^2$ ✓A

(c) What is the bearing of the ship from the harbour to the nearest degree? (4)

$$\frac{\sin A}{120} = \frac{\sin 65}{110{,}5} \quad \checkmark m$$

$$\therefore \sin A = \frac{120 \sin 65}{110{,}5} \quad \checkmark m$$

$$= 0{,}984\ldots \quad \checkmark m$$

$$A = 79{,}809\ldots \quad \checkmark A$$

$$\hat{A} = 80^\circ$$

$$\therefore \text{Bearing is } 140^\circ + 80^\circ \quad \checkmark m$$

$$= 220^\circ \quad \checkmark A$$

[10]

QUESTION 11

(a) Solve for θ: $\sin(2\theta + 10^\circ) = -0{,}4678, \quad \theta \in [-90^\circ ; 270^\circ]$ (6)

QUESTION 11

(a) Solve for θ: $\sin(2\theta + 10°) = -0{,}4678, \quad \theta \in [-90°\ ; 270°]$ (6)

✓M

$2\theta + 10 = -27{,}89\ldots + 360k$ ✓A (A)

$2\theta = -37{,}89\ldots + 360k$

$\theta = -18{,}94\ldots + 180k$ ✓A (C)

$\theta = -18{,}9°$ ✓ ✓ CA

$\theta = 161{,}1°$

$2\theta + 10 = 180 - A + 360k\ ; k \in Z$

$= 207{,}89\ldots + 360k$ ✓A (B)

$2\theta = 207{,}9° - 10 + 360k$

$2\theta = 197{,}8\ldots + 360k$

$\theta = 98{,}9° + 180k$ ✓A

$\theta = 98{,}9°$ ✓ CA

$\theta = -81{,}1°$ ✓

(b) Prove that: $\sin^4\theta - \cos^4\theta = 1 - 2\cos^2\theta$ (4)

$LHS = \sin^4\theta - \cos^4\theta$

$= (\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)$ ✓M

$= 1\,(\sin^2\theta - \cos^2\theta)^*$ ✓A

$= 1 - \cos^2\theta - \cos^2\theta$ ✓A_2

$= 1 - 2\cos^2\theta$ ✓A

$= RHS$

OR// $RHS = 1 - 2\cos^2\theta$

$= \sin^2\theta + \cos^2\theta - 2\cos^2\theta$ ✓

$= \sin^2\theta - \cos^2\theta^*$ ✓

$LHS = RHS$

(c) Determine, **without the use of a calculator**, the value of :

$$\frac{\sin 1^\circ}{\cos 1^\circ} \times \frac{\sin 2^\circ}{\cos 2^\circ} \times \frac{\sin 3^\circ}{\cos 3^\circ} \times \ldots\ldots \times \frac{\sin 88^\circ}{\cos 88^\circ} \times \frac{\sin 89^\circ}{\cos 89^\circ}$$

Show all your working. (4)

$\sin 1^\circ = \cos 89^\circ$ ✓ $\therefore \frac{\sin 1^\circ}{\cos 1^\circ} \times \frac{\sin 89^\circ}{\cos 89^\circ} = 1$

$\cos 1^\circ = \sin 89^\circ$ ✓

$\sin 2^\circ = \cos 88^\circ$

$\cos 2^\circ = \sin 88^\circ$

$\therefore 1 \times 1 \times 1 \times 1$ ✓ $\times \ldots \frac{\sin 45^\circ}{\cos 45^\circ}$ ✓

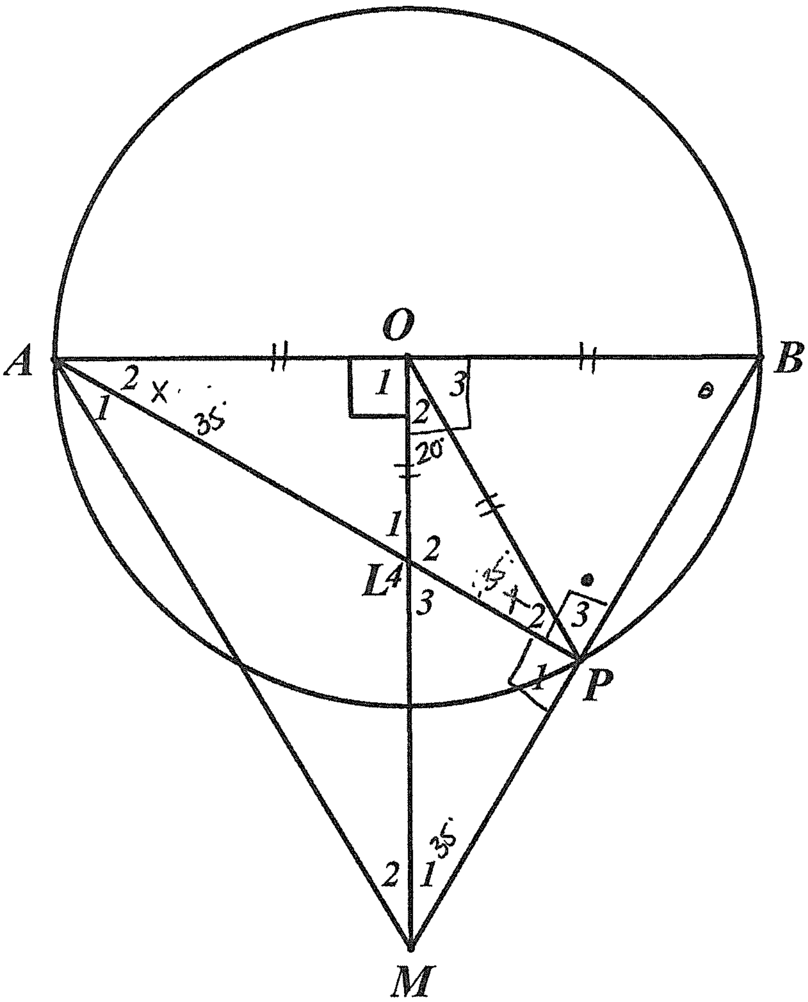
$= \frac{\sqrt{2}/2}{\sqrt{2}/2}$

$= 1$ ✓

[14]

[16]

QUESTION 14



In the diagram, O is the centre of circle ABP.

BP is produced to M, such that MO $\perp$ AB.

AP intersects OM at L.

$\hat{O}_2 = 20°$

(a) Calculate, the following, **stating all necessary reasons:**

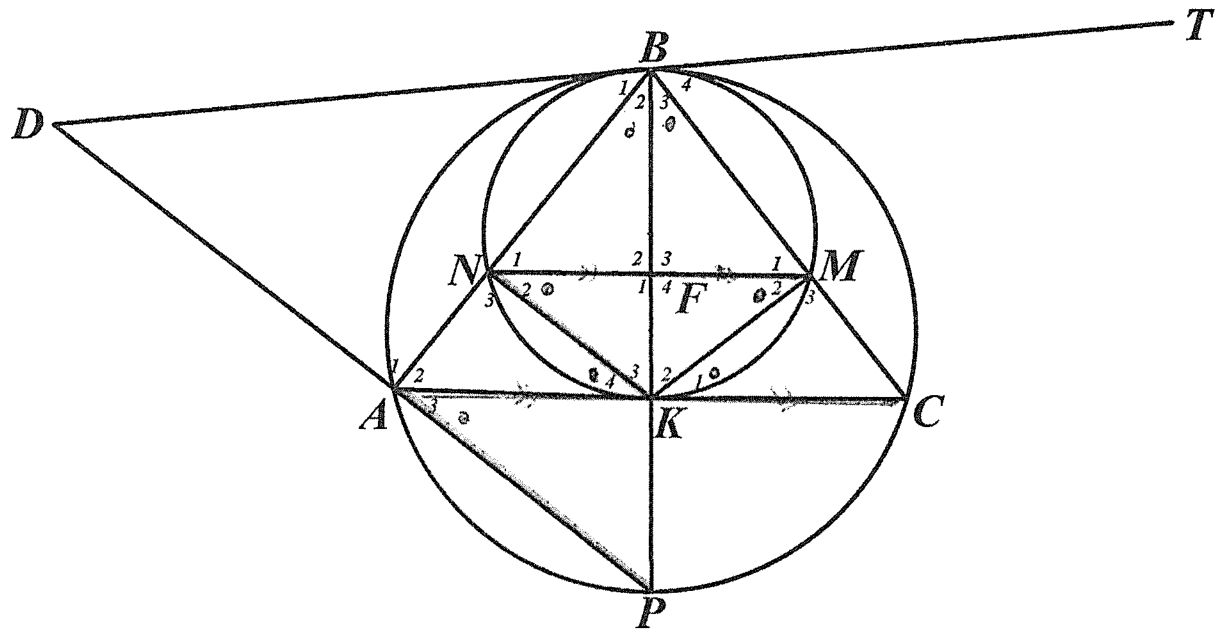
(i) $\hat{A}_2$ (2)

$OA = OP = OP$ } radii

$\therefore \hat{A}_2 = \hat{P}_2$ } ∠'s opp eq sides (<'s opp eq sides)

$\therefore \hat{A}_2 = \hat{P}_2 = 35°$ 3<'s Δ; given $\hat{O}_2 = 20°$; $\hat{O}_1 = 90°$

QUESTION 14



In the given diagram:

- DBT is a common tangent to circles BNKM and BAPC, at B.
- AKC is also a tangent to the smaller circle at K.
- MN // CA.

Prove, with reasons:

(a) ΔKMN is isosceles. (3)

$\hat{M}_2 = \hat{K}_1$ alt <s MN // AC

$\hat{K}_1 = \hat{N}_2$ tan chd (3)

$\therefore \hat{K}_1 = \hat{N}_2 = \hat{M}_2$

$\therefore$ NK = KM isos Δ (sides opp eq <s)

(b) NK // AP (4)

$\hat{N}_2 = \hat{B}_3$ ✓ <'s same seg.

$\hat{B}_3 = \hat{A}_3$ ✓ <'s same seg.

$\hat{K}_4 = \hat{N}_2$ ✓ alt <'s NM // AC

$\therefore \hat{N}_2 = \hat{K}_4 = \hat{B}_3 = \hat{A}_3$

$\therefore \hat{K}_4 = \hat{A}_3$

$\therefore$ NK // AP ✓ alt <'s eq.

(c) DP is a tangent to the circle passing through points A, B, and K. (5)

$\hat{K}_4 = \hat{M}_2 = \hat{B}_2$ tan chd

$\hat{K}_4 = \hat{A}_3$ alt <'s NK // AP

$\therefore \hat{K}_4 = \hat{A}_3 = \hat{M}_2 = \hat{B}_2$

$\therefore \hat{A}_3 = \hat{B}_2$

$\therefore$ AP is tang to ABK tan chd conv.

(5)

[12]

TOTAL: SECTION A: 70 MARKS SECTION B : 80 MARKS