

RONDEBOSCH BOYS' HIGH SCHOOL



Mathematics Paper 1

13 November 2015

Grade 11

MARKS: 150

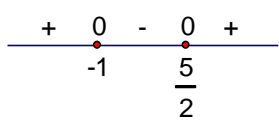
TIME: 3 hours

MEMORANDUM

Examiner: S Carletti

Moderator: P Ghignone

This memorandum consists of 9 pages

QUESTION 1				
1.1.1	$(2x + 1)(3x - 4) = 0$ $\therefore x = -\frac{1}{2}$ or $x = \frac{4}{3}$	$\checkmark -\frac{1}{2}$ $\checkmark \frac{4}{3}$ (2)	R	
1.1.2	$9^{2x-1} = \frac{3^x}{3}$ $\therefore 3^{4x-2} = 3^{x-1}$ $\therefore 4x - 2 = x - 1$ $\therefore 3x = 1$ $\therefore x = \frac{1}{3}$	\checkmark Prime factors \checkmark Equating indices \checkmark Answer (3)	R	
1.1.3	$-3x^2 + 4x = -2$ $\therefore -3x^2 + 4x + 2 = 0$ $\therefore x = \frac{-4 \pm \sqrt{4^2 - 4 \times -3 \times 2}}{2 \times -3}$ $\therefore x = -0,39$ or $x = 1,72$	\checkmark Std form \checkmark Formula $\checkmark \checkmark$ Answers – rounding (4)	R	
1.1.4	$\sqrt{2x - 3} + 3 = x$ $\therefore \sqrt{2x - 3} = x - 3$ $\therefore 2x - 3 = x^2 - 6x + 9$ $\therefore x^2 - 8x + 12 = 0$ $\therefore (x - 6)(x - 2) = 0$ $\therefore x = 6$ or $x = 2$ $\therefore x = 6$	\checkmark Std form \checkmark Squaring \checkmark Factors \checkmark Answers \checkmark Choice of answer (5)	R	
1.2.1	$2x^2 - 3x \geq 5$ $\therefore 2x^2 - 3x - 5 \geq 0$ $\therefore (2x - 5)(x + 1) \geq 0$ $\therefore x \leq -1$ or $x \geq \frac{5}{2}$		\checkmark Std form \checkmark Factors $\checkmark \checkmark$ Answers (4)	R
1.2.2	$-x + 5 < 8$ $\therefore -x < 3$ $\therefore x > -3$ $\therefore -3 < x \leq -1$ or $x \geq \frac{5}{2}$	$\checkmark x > -3$ $\checkmark \checkmark$ Answers (3)	C	
1.3	$2x - y = 7$ $\therefore y = 2x - 7$ $x^2 - xy = 12$ $\therefore x^2 - x(2x - 7) = 12$ $\therefore x^2 - 2x^2 + 7x = 12$ $\therefore x^2 - 7x + 12 = 0$ $\therefore (x - 4)(x - 3) = 0$ $\therefore x = 4$ or $x = 3$ $\therefore y = 1$ or $y = -1$	$\checkmark y =$ \checkmark Subs \checkmark Multiply \checkmark Factors $\checkmark x$ values $\checkmark y$ values (6)	R	

1.4	$x = \frac{12 \pm \sqrt{114 - 38k}}{2k - 3}$ $114 - 38k = 0 \text{ (since roots are equal)}$ $\therefore k = 3$ $\therefore x = 4$ $\therefore g(x) = a(x - 4)^2$ $\therefore g(x) = ax^2 - 8ax + 16a$ $\therefore 50 = a(-1)^2 - 8a(-1) + 16a$ $\therefore a = 2$ $\therefore g(x) = 2x^2 - 16x + 32$	$\checkmark \Delta = 0$ $\checkmark k = 3$ $\checkmark g(x)$ \checkmark Subst \checkmark Answer (5)	PS
1.5	$4(x - 5a)(x - 7a) = 0$ $\therefore 4x^2 - 48ax + 140a^2 = 0$ $-48a = -24$ $\therefore a = \frac{1}{2}$ $\therefore 140a^2 = 35$ $\therefore p = 35$ Roots are $\frac{5}{2}$ and $\frac{7}{2}$ OR $x = \frac{24 \pm \sqrt{24^2 - 16p}}{8}$ $\therefore x = 3 \pm \frac{1}{2} \sqrt{36 - p}$ $\therefore \frac{3 - \frac{1}{2} \sqrt{36 - p}}{3 + \frac{1}{2} \sqrt{36 - p}} = \frac{5}{7}$ $\therefore p = 35$ Roots are $\frac{5}{2}$ and $\frac{7}{2}$	\checkmark Equation \checkmark Value of a \checkmark Value of p \checkmark Roots \checkmark Formula \checkmark Ratio \checkmark Value of p \checkmark Roots (4) [36]	PS

QUESTION 2			
2.1	$\frac{3^{2x}-4}{2^x \cdot 3^{x-2x+1}}$ $= \frac{(3^x-2)(3^x+2)}{2^x(3^x-2)}$ $= \frac{3^x+2}{2^x}$	<ul style="list-style-type: none"> ✓ Num ✓ Denom ✓ Answer 	(3) R
2.2	$\frac{15+7\sqrt{3}}{9+5\sqrt{3}} \times \frac{9-5\sqrt{3}}{9-5\sqrt{3}}$ $= \frac{135-12\sqrt{3}-105}{81-75}$ $= \frac{30-12\sqrt{3}}{6}$ $= 5 - 2\sqrt{3} \text{ cm}$ <p>OR</p> $(9 + 5\sqrt{3})(m + n\sqrt{3}) = 15 + 7\sqrt{3}$ $\therefore 9m + 5m\sqrt{3} + 9n\sqrt{3} + 15n = 15 + 7\sqrt{3}$ $\therefore 9m + 15n = 15 \text{ and } 5m + 9n = 7$ $\therefore m = 5 \text{ and } n = -2$ $\therefore \text{side} = 5 - 2\sqrt{3} \text{ cm}$	<ul style="list-style-type: none"> ✓ Division ✓ Conjugate ✓ Num ✓ Denom ✓ Answer ✓ Mult ✓ Simpl ✓ Equating ✓ Solving ✓ Answer 	(5) C
2.3	$f(x) = x^{(x+1)}(x+2)^{(x+3)}$ $f(0) + f(-1) + f(-2)$ $= 0^1 \times 2^3 + (-1)^0 \times 1^2 + (-2)^{-1} \times 0^1$ $= 0 + 1 + 0$ $= 1$	<ul style="list-style-type: none"> ✓ Subst ✓✓ Subtract per mistake ✓ Answer 	(4) [12] PS

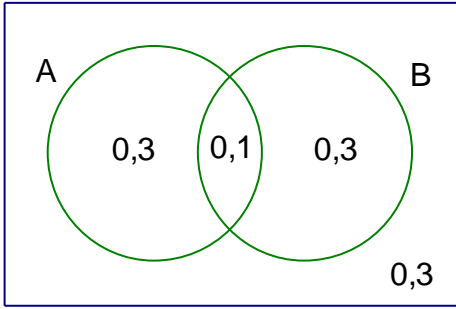
QUESTION 3			
3.1	$10 - 3y \qquad \qquad \qquad 7 \qquad \qquad \qquad 15 \qquad \qquad \qquad 8y + 1$ $\qquad \qquad \qquad 3y - 3 \qquad \qquad \qquad 8 \qquad \qquad \qquad 8y - 14$ $\qquad \qquad \qquad \qquad \qquad \qquad 11 - 3y \qquad \qquad \qquad 8y - 22$ $\therefore 11 - 3y = 8y - 22$ $\therefore -11y = -33$ $\therefore y = 3$	✓ First diffs ✓ $11 - 3y$ ✓ $8y - 22$ ✓ Answer (4)	R
3.2.1	5; 11; 19; 29	✓ 5; 11 ✓ 19; 29 (2)	R
3.2.2	$a = 1$ $3a + b = 6 \quad \therefore b = 3$ $a + b + c = 5 \quad \therefore c = 1$ $\therefore T_n = n^2 + 3n + 1$ OR $T_n = (n + 1)^2 + n$	✓ a ✓ b ✓ c ✓ T_n ✓✓ $(n + 1)^2$ ✓✓ $+n$ (4)	R
3.2.3	$T_{10} = 10^2 + 3 \times 10 + 1$ $\therefore T_{10} = 131$	✓ Subst ✓ Answer (2)	R
3.2.4	$n^2 + 3n + 1 = 600$ $\therefore n^2 + 3n - 599 = 0$ $\therefore n = \frac{-3 \pm \sqrt{3^2 - 4 \times -599}}{2}$ $\therefore n = 23,02$ or $n = -26,02$ \therefore can make the 23 rd shape T_n for bottom row = $2n + 1$ \therefore 47 blocks on the bottom row	✓ Equating ✓ Formula ✓ 23 rd shape ✓ T_n ✓ Answer (5)	C
3.3	$p = 1$ $q = -2$ T_n for first differences = $-2n + 3$ $\therefore r = -2(99) + 3$ $\therefore r = -195$	✓✓ p ✓ q ✓ T_n ✓ Subst ✓ r (6) [23]	PS

QUESTION 4			
4.1	$1 + i = \left(1 + \frac{0,087}{12}\right)^{12}$ $\therefore i = 9,06\%$	✓ Subst ✓✓ Answer (3)	R
4.2	$A = 249999(1 - 0,18)^3$ $\therefore A = R137841,45$	✓ Subst ✓✓ Answer (3)	R
4.3	$A = 10000(1 + 0,08)^{10} + 40000(1 + 0,08)^7 + 80000(1 + 0,08)^2$ $\therefore A = R183454,22$	✓ 1 st term ✓ 2 nd term ✓ 3 rd term ✓✓ Answer (5)	R
4.4	$18440,95 = 10000 \left(1 + \frac{0,1}{12}\right)^{24} \left(1 + \frac{x\%}{4}\right)^{12}$ $\therefore x = 14\%$	✓ Subst ✓✓ Answer (3) [14]	C

QUESTION 5			
$f(x) = \frac{2}{x-1} - 3 \text{ and } g(x) = \frac{1}{2}x - \frac{7}{2}$			
5.1	$x = 1$ and $y = -3$	✓ $x = 1$ ✓ $y = -3$ (2)	R
5.2	$y \in \mathbb{R}; y \neq -3$	✓ $y \in \mathbb{R}$ ✓ $y \neq -3$ (2)	R
5.3	$\frac{2}{x-1} - 3 = 0$ $\therefore \frac{2}{x-1} = 3$ $\therefore 2 = 3x - 3$ $\therefore 3x = 5$ $\therefore x = \frac{5}{3}$	✓ = 0 ✓ Simpl ✓ Answer (3)	R
5.4	$Q(-1; -4)$	✓ $x = -1$ ✓ $y = -4$ (2) [9]	R

QUESTION 6			
6.1	$f(0) - g(0)$ $= 16 - 7$ $= 9$	✓ $16 - 7$ ✓ Answer (2)	C
6.2	$f(x) = -x^2 - 6x + 16$ $= -(x^2 + 6x - 16)$ $= -(x + 3)^2 - 25$ $= -(x + 3)^2 + 25$	✓ Comm fac ✓ Compl sq ✓ Answer (3)	R
6.3	$(-3; 25)$	✓ $x = -3$ ✓ $y = 25$ (2)	R
6.4		$f(x)$: ✓✓ x ints ✓ y int ✓ TP $g(x)$: ✓ x int ✓ y int (6)	R
6.5.1	$-9 \leq x \leq 1$	✓ Answer (1)	R
6.5.2	$x \leq -8$ or $-\frac{7}{2} \leq x \leq 2$	✓ 1 st answer ✓ 2 nd answer (2)	R
6.5.3	$x \leq -8$ or $0 \leq x \leq 2$	✓ 1 st answer ✓ 2 nd answer (2)	R
6.6	$y = -2x - 7$	✓ $-2x$ ✓ -7 (2)	C
6.7	$y = (x - 3)^2 - 25$ OR $y = x^2 - 6x - 16$	✓ positive ✓ $(x - 3)^2$ ✓ -25 ✓ x^2 ✓ $-6x$ ✓ -16 (3) [23]	C

QUESTION 7			
7.1	$f(x) = \left(\frac{1}{2}\right)^x$ $4f(x+1) = \sqrt{2}$ $\therefore 4\left(\frac{1}{2}\right)^{x+1} = \sqrt{2}$ $\therefore 2^2 \times 2^{-x-1} = 2^{1/2}$ $\therefore 2 - x - 1 = \frac{1}{2}$ $\therefore x = \frac{1}{2}$	✓ Subst ✓ Prime fact ✓ Equating ✓ Answer (4)	C
7.2	$x + 3 = -x - 7$ $\therefore 2x = -10$ $\therefore x = -5$ $(-5; -2)$ $\therefore p = 5$ and $q = -2$	✓ Equating ✓ $x = -5$ ✓ $p = 5$ ✓ $q = -2$ (4)	C
7.3.1	$P(a; 0)$ and $Q(0; b)$	✓ Both answers (1)	R
7.3.2	$\sqrt{a^2 + b^2} = \sqrt{45}$ $\therefore a^2 + b^2 = 45$ $\frac{b-0}{0-a} = -\frac{1}{2}$ $\therefore a = 2b$ $\therefore 4b^2 + b^2 = 45$ $\therefore b^2 = 9$ $\therefore b = 3$ and $a = 6$ (positive constants)	✓ $a^2 + b^2 = 45$ ✓ $a = 2b$ ✓ $b = 3$ ✓ $a = 6$ (4)	P

QUESTION 8																			
8.1	$0,4 \times 0,6 + 0,6 \times 0,3 = 0,42$ or $\frac{21}{50}$	✓ 1 st term ✓ 2 nd term ✓ Answer (3)	R																
8.2.1	<table border="1"> <thead> <tr> <th></th> <th>Fruit tree</th> <th>Other tree</th> <th>Total</th> </tr> </thead> <tbody> <tr> <td>Bird's nest</td> <td>2</td> <td>4</td> <td>6</td> </tr> <tr> <td>No nest</td> <td>5</td> <td>9</td> <td>14</td> </tr> <tr> <td>Total</td> <td>7</td> <td>13</td> <td>20</td> </tr> </tbody> </table>		Fruit tree	Other tree	Total	Bird's nest	2	4	6	No nest	5	9	14	Total	7	13	20	✓✓ Subtract 1 for each mistake (2)	R
	Fruit tree	Other tree	Total																
Bird's nest	2	4	6																
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8.2.2	$\frac{9}{20}$ or 0,45	✓✓ Answer (2)	R																
8.2.3	$\frac{2}{7}$ or 0,29	✓✓ Answer (2)	C																
8.3	 <p>$\therefore P(A') = 0,6$</p>	✓ 0,1 ✓✓ 0,3 on either side ✓ 0,3 outside ✓ Answer (5)	C																
8.4.1	$k + 0,3 = 0,8$ $\therefore k = 0,5$	✓ Equation ✓ Answer (2)	R																
8.4.2	$0,8 = 0,3 + k - 0,3 \times k$ $\therefore 0,7k = 0,5$ $\therefore k = \frac{5}{7}$ or 0,71	✓ Equation ✓ $0,7k$ ✓ 0,5 ✓ Answer (4) [20]	C																

Total marks: 150