RONDEBOSCH BOYS' HIGH SCHOOL

Mathematics Paper 2

30 November 2015

Grade 11

MARKS: 150

TIME: 3 hours

MEMORANDUM

Examiner: P Ghignone

Moderator: S Carletti

QUESTION 1

1.1 $\bar{x} = 69{,}5$ kg ✓✓ $\sigma = 8{,}88$ kg ✓✓ (4)R

1.2 $[69{,}5 - 8{,}88;\ 69{,}5 + 8{,}88] = [60{,}62$ ✓$; 78{,}38$ ✓$]$
10 learners ✓ (3)R

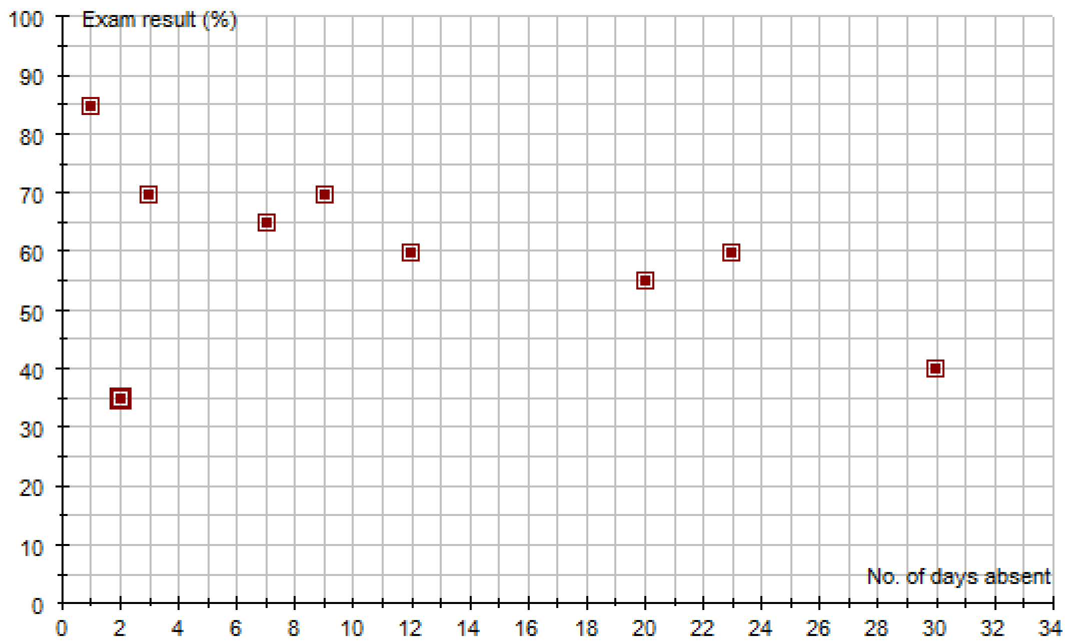
1.3 Total mass of Group 2 $= 18 \times 72 = 1296$
Total mass of Group 1 $= 973$ ✓ (both totals)
$\frac{1296-973}{4} = \frac{323}{4}$ ✓ $= 80{,}75$ kg ✓ (3)P

[10]

QUESTION 2

2.1 ✓✓✓

2.5 L✓



(3+1)R

2.2 $y = A + Bx$ $A = 68{,}02$ ✓ $B = -0{,}67$ ✓ (3)R
$= 68{,}02 - 0{,}67x$ ✓

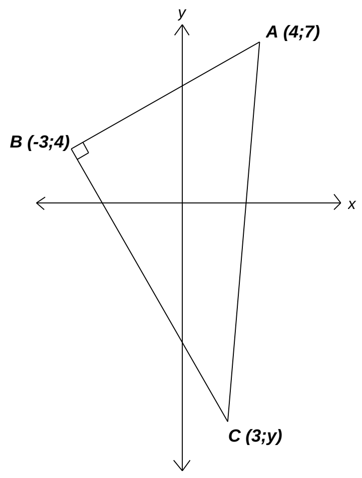
2.3 $r = -0.45$ ✓✓ (2)R

2.4 Weak negative correlation ✓ (1)R

2.6 B would decrease ✓✓ (become more negative) (2)P

[12]

QUESTION 3



3.1.1 $m_{AB} = \frac{7-4}{4+3} \checkmark = \frac{3}{7} \checkmark$ (2)R

3.1.2 $m_{BC} = -\frac{7}{3} \checkmark$ (1)R

3.1.3
$$\frac{4-y}{-3-3} = -\frac{7}{3} \checkmark$$
$$12 - 3y = 42 \checkmark$$
$$-30 = 3y$$
$$y = -10 \checkmark$$

OR $4 = -\frac{7}{3}(-3) + c \checkmark$

$\therefore c = -3$

$\therefore y = -\frac{7}{3}(3) - 3 \checkmark = -10 \checkmark$ (3)R

3.1.4 $AB = \sqrt{(4+3)^2 + (7-4)^2} = \sqrt{58} \checkmark\checkmark$ (4)C

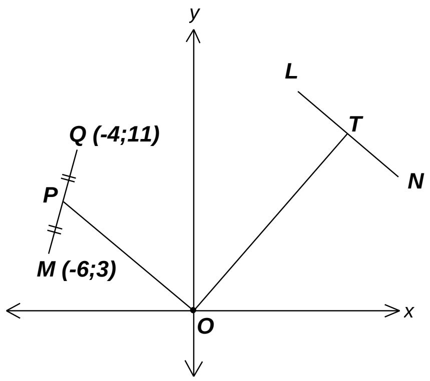
$A = \frac{1}{2}\left(2\sqrt{58}\right).AB \checkmark$

$= \frac{1}{2}\left(2\sqrt{58}\sqrt{58}\right)$

$= 58 \text{ units}^2 \checkmark$

OR

$A = \frac{1}{2}(2\sqrt{58}.)(\sqrt{58}. \checkmark\checkmark)\sin 90° \checkmark = 58 \checkmark$



3.2.1 NB No working 0/3

$$x + \frac{3}{2}x = 15 \checkmark$$
$$2{,}5x = 15$$
$$x = 6 \checkmark$$
$\therefore y = 9 \checkmark$

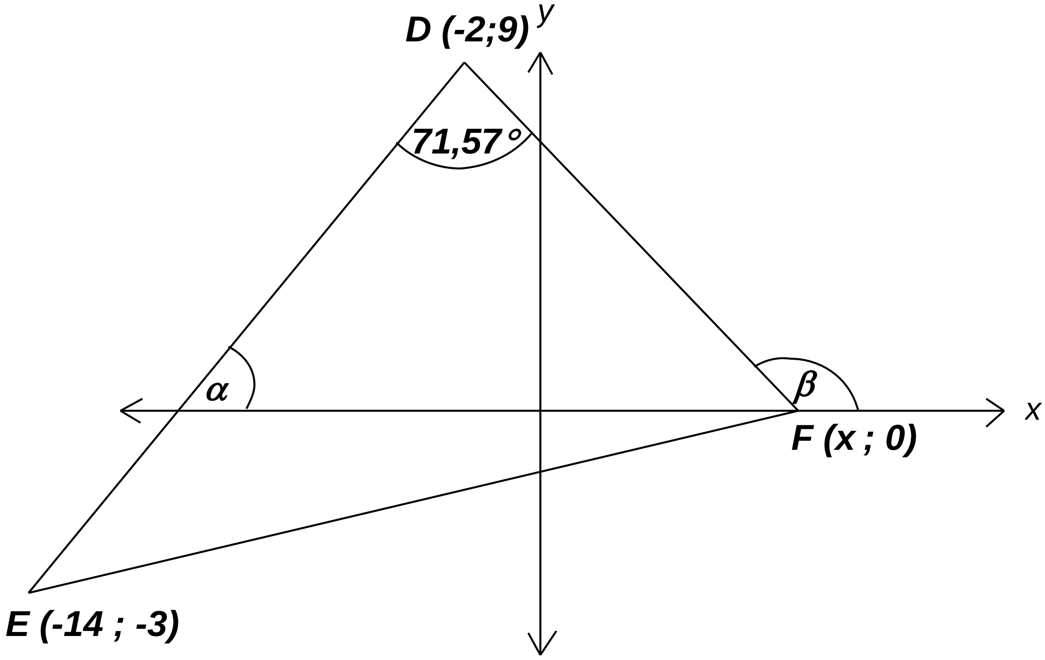
$\therefore T(6;9)$ (3)R

3.2.2 $P(-5;7) \checkmark\checkmark$ and $T(6;9)$

$PT = \sqrt{(-5-6)^2 + (7-9)^2} \checkmark = \sqrt{125} = 5\sqrt{5} \checkmark$ (11,18) if dec 3/4 (4)C

[17]

QUESTION 4



4.1 $\tan\alpha = m_{DE}$ (2)R

$= \frac{9+3}{-2+14}$

$= 1$ ✓

$\therefore \alpha = 45°$ ✓

4.2 $\beta = 71{,}57° + 45°$ (ext ∠ of Δ) ✓ R (4)C

$= 116{,}57°$ ✓

$m_{DF} = \tan 116{,}57°$ ✓

$= -2{,}00$ ✓ *NB MUST SHOW WORKING*

4.3 $\frac{9-0}{-2-x} = -\frac{2}{1}$ ✓ (2)R

$9 = 4 + 2x$

$x = \frac{5}{2}$ ✓

$\Longrightarrow F\left(\frac{5}{2}; 0\right)$

4.4 $P\left(-\frac{19}{2}\right.$ ✓$; -12$ ✓$\left.\right)$ (2)R

4.5 $OP = \sqrt{\left(-\frac{19}{2}\right)^2 + (-12)^2}$ ✓ $= 15{,}31$ ✓

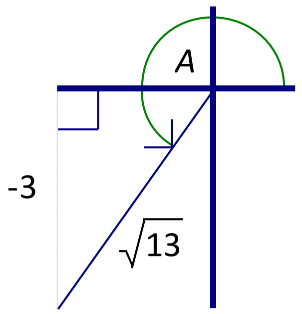
$\therefore P$ lies outside the circle because $OX > r$ ✓(*must have reason*) (3)P

[If $P\left(\frac{29}{2}; 0\right)$, then $OP = \frac{29}{2} < r$, so P lies inside the circle.]

[13]

QUESITON 5

5.1.1 $\sin A = -\frac{3}{\sqrt{13}}$ ✓ so A is in quadrant III ✓ (2)R



5.1.2 a) $\tan^2 A + 1 = \left(\frac{-3}{-2✓}\right)^2$ ✓ $+ 1 = \frac{13}{4}$ ✓ (3)R

b) $(\sin A + \cos A)^2 = \left(-\frac{3}{\sqrt{13}} + \left(-\frac{2}{\sqrt{13}}\right)✓\right)^2$ ✓ (3)R

$= \left(-\frac{3}{\sqrt{13}} - \frac{2}{\sqrt{13}}\right)^2$

$= \left(-\frac{5}{\sqrt{13}}\right)^2$

$= \frac{25}{13}$ ✓

5.2.1 $\frac{\cos(90°+x)}{\tan(180°-x).\cos(360°-x)} = \frac{-\sin x ✓}{-\tan x ✓ .\cos x ✓}$ (5)R

$= \frac{\sin x}{\frac{\sin x}{\cos x}✓ . \frac{\cos x}{1}}$

$= 1$ ✓

5.2.2 $\frac{3-3\cos^2 50°}{\sin^2 230°} = \frac{3(1-\cos^2 50°)✓}{(-\sin 50°)^2}$ (4)R

$= \frac{3\sin^2 50° ✓}{\sin^2 50° ✓}$

$= 3$ ✓

5.3 LHS: $\frac{\cos x}{1+\sin x}+\tan x$ (5)C

$= \frac{\cos x}{1+\sin x}+\frac{\sin x}{\cos x}$ ✓ $\tan x=\frac{\sin x}{\cos x}$

$= \frac{\cos^2 x+\sin x(1+\sin x)}{\cos x(1+\sin x)}$

$= \frac{\cos^2 x+\sin x+\sin^2 x}{\cos x.(1+\sin x)}$ ✓numerator ✓denominator

$= \frac{1+\sin x}{\cos x.(1+\sin x)}$ ✓$\cos^2 x+\sin^2 x=1$

$= \frac{1}{\cos x}=\text{RHS}$ ✓ finishing off/layout

5.4 $2\sin x.\tan x-\tan x=0$

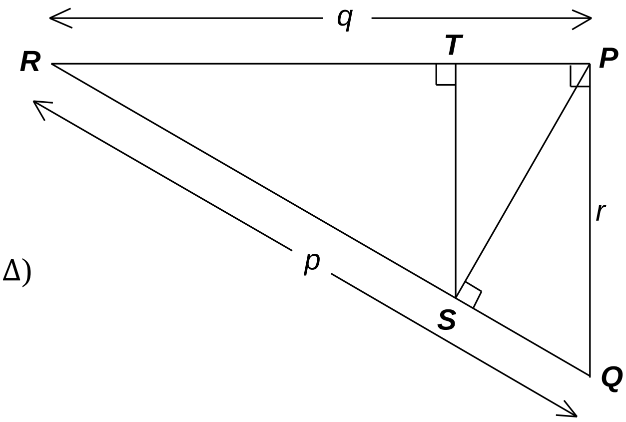
$\therefore \tan x\,(2\sin x-1)=0$ ✓

$\tan x=0$ ✓ OR $\sin x=\frac{1}{2}$ ✓

$x=n.180°$ ✓ $x=30°+n.360°$ ✓

or $x=150°+n.360°$ ✓ $n\in\mathbb{Z}$ (-1 if missing) (6)C

5.5



Let $\hat{Q}=x$.

$S\hat{P}Q=90°-x$ (∠ sum in Δ)

$\therefore T\hat{P}S=x$

In ΔPQR, $\sin Q=\frac{q}{p}$ ✓

In ΔPQS, $\sin Q=\frac{PS}{r}$ ✓ $\Rightarrow PS=r\sin Q$ ✓

In ΔPTS, $\sin T\hat{P}S=\frac{ST}{PS}$ ✓ $\Rightarrow ST=PS\sin T\hat{P}S=r.r.\sin\hat{Q}$

$\Rightarrow ST=\frac{rq^2}{p^2}$ ✓ (5)P

OR $\Delta RTS|||\Delta RPQ\ (AAA)\ (Proof)$

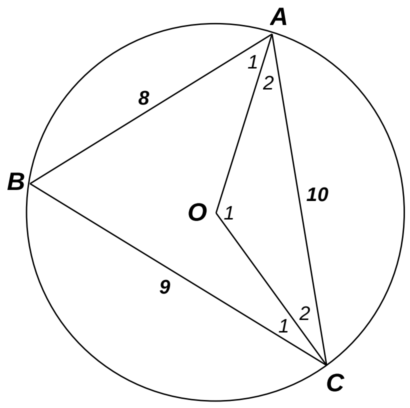
$\Delta RPQ|||\Delta RSP\ (AAA)(Proof)$ ✓*for both proofs*

$\frac{TS}{r}=\frac{RS}{p}(|||\quad)\ \therefore TS=\frac{RS.r}{p}$ ✓ $\frac{RS}{q}=\frac{q}{p}(|||)\ \therefore RS=\frac{q^2}{p}$ ✓ $\therefore TS=\frac{q^2.r}{p^2}$ ✓✓

(5)P

[33]

QUESTION 6



6.1.1 $\cos \hat{B} = \frac{(9^2+8^2-10^2)}{2(9)(8)}$ ✓f✓s (3)R

$\hat{B} = 71{,}79°$ ✓

6.1.2 Area $= \frac{1}{2}(8)(9)\sin 71{,}79°$ ✓f✓s (3)R

$= 34{,}20 \text{ units}^2$ ✓

6.1.3 $\hat{O}_1 = 143{,}58°$ (∠ at centre = 2 × ∠ at circumference) ✓

$\hat{C}_2 = \hat{A}_2$ (∠s opp equal sides)

$\hat{C}_2 = 18{,}20°$ (∠ sum in ΔAOC) ✓ (both reasons)

$\therefore \frac{AO}{\sin 18{,}21°} = \frac{10}{\sin 143{,}58°}$ ✓

$\therefore AO = 5{,}26$ units ✓

$\therefore$ diameter $= 10{,}53$ units ✓ (5)C

Other methods for 6.1.3:

Drop perpendicular from O onto AC bisecting AC and use the definitions in Rt angled triangle with side = 5.

Let OC and OA both be x and use cosine rule with 10^2 = etc and solve for x.

6.2 $C\hat{A}D = 180° - (x+y)$ ✓ (∠ sum in ΔACD; sum of <s in Δ; int <s of Δ)

$A\hat{C}B = \alpha$ (alt ∠s; horizon ∥ BC)

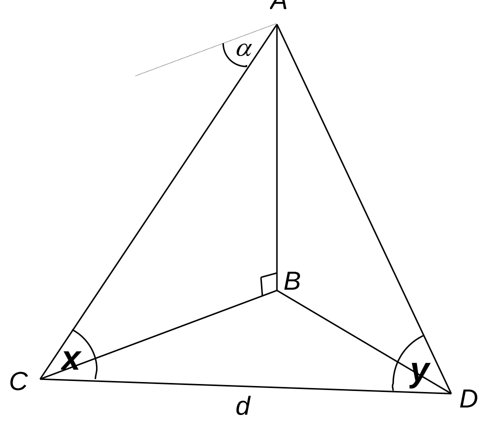
$\sin\alpha = \frac{AB}{AC}$ ✓

$\therefore AB = AC\sin\alpha$ ✓

$\frac{AC}{\sin y} = \frac{d}{\sin(180° - (x+y))}$ ✓

$\therefore AC = \frac{d \sin y}{\sin(x+y)}$ ✓

$\therefore AB = \frac{d \sin y . \sin\alpha}{\sin(x+y)}$ (−1if do not finish) (5)C



[16]

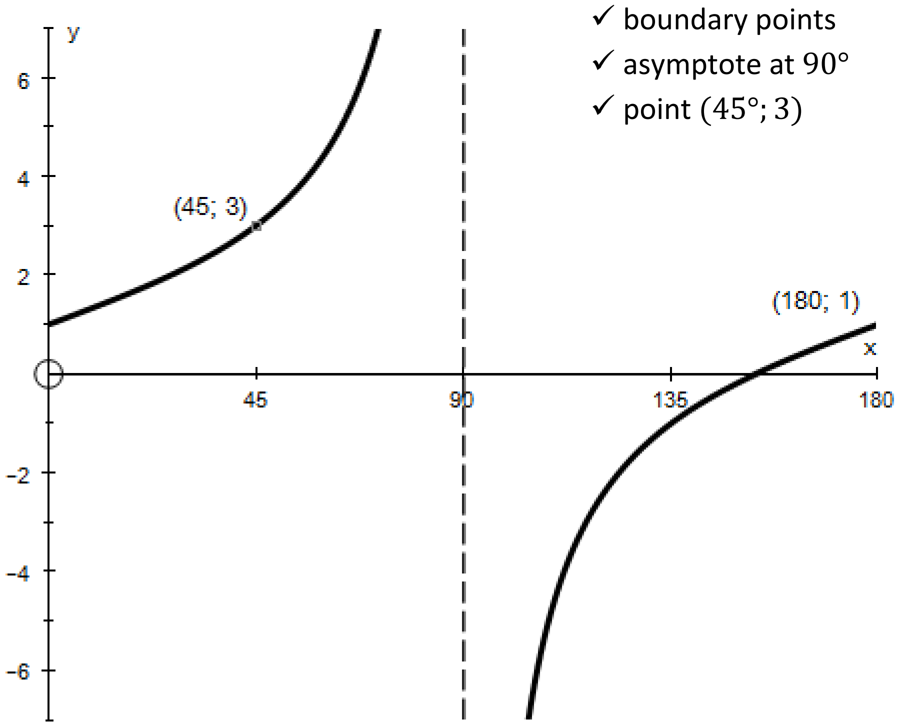
QUESTION 7

7.1.1 $p = 45°$ ✓ $a = \frac{1}{2}$ ✓ (2)R

7.1.2 Period: $\frac{360°}{\frac{1}{2}} = 720°$ ✓ (1)R

7.1.3 Range: $-4 \leq y \leq -2$ ✓✓ (2)C

7.2.1



✓ boundary points
✓ asymptote at 90°
✓ point (45°; 3)

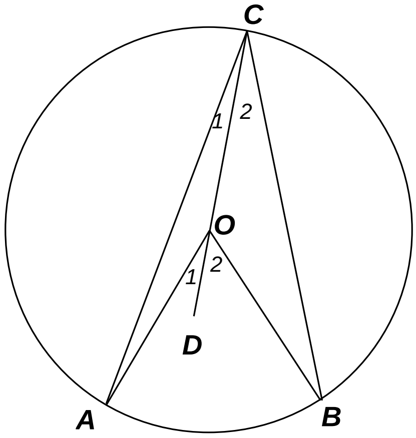
(3)R

7.2.2 $y = -2\tan x$ ✓ $- 3$ ✓ (2)C

[10]

QUESTION 8

8.1



Construct radius CO and extend to D.

Let $\hat{C}_1 = x$ and $\hat{C}_2 = y$.

Then $\hat{A} = x$ and $\hat{B} = y$ ✓ radii; ∠s opp equal sides ✓

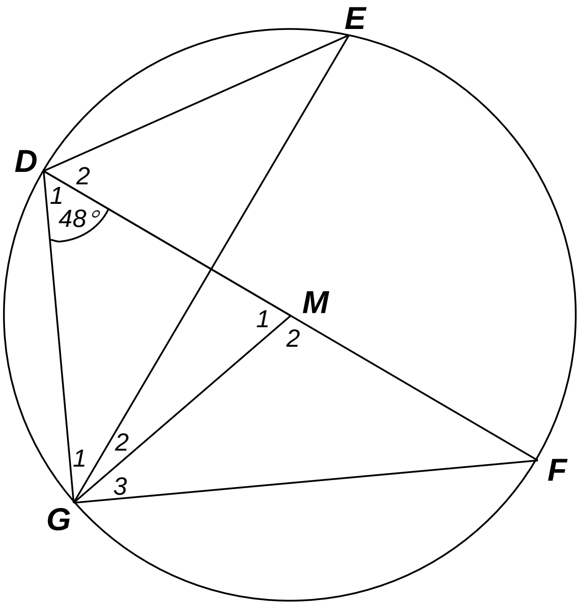
$\therefore \hat{O}_1 = 2x$ and $\hat{O}_2 = 2y$ ext ∠ of Δ ✓

$\therefore A\hat{O}B = 2x + 2y$ ✓

$= 2(x + y)$

$= 2(A\hat{C}B)$ ✓ (5)R

8.2



8.2.1 $\hat{M}_2 = 96°$ ✓ ∠ at centre = 2 × ∠ at circumference ✓ (2)R

8.2.2 $D\hat{G}F = 90°$ ✓ ∠ in semicircle ✓ (2)R

8.2.3 $\hat{G}_3 = \hat{F} = 42°$ ✓ ∠s opp equal sides; radii ✓
∠ sum in ΔMFG ✓ (3)R

8.2.4 $\hat{E} = \hat{F} = 42°$ ✓ ∠s in same seg ✓ (2)R

[14]

QUESTION 9

9.1 $AD = DB$ line from centre ⊥ to chord AB ✓

$= \frac{3}{4}r$ ✓

Construct $AO = r$ (radius)

$r^2 = \left(\frac{3}{4}r\right)^2 + OR^2$ ✓

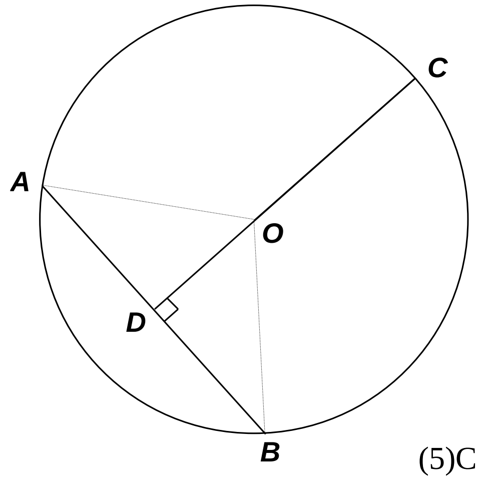
$OD^2 = r^2 - \frac{9}{16}r^2$ (Pythagoras)

$= \frac{7}{16}r^2$

$OD = \frac{\sqrt{7}}{4}r$ ✓

$CD = \frac{\sqrt{7}}{4}r + r$ ✓

$= \frac{(4+\sqrt{7})}{4}r$



(5)C

9.2.1 $\hat{R} = x$ ✓ tan chord theorem ✓

$\hat{P}_3 = x$ ✓ tan chord theorm ✓ **OR** tangent from common point

$\hat{Q}_1 = x$ ✓ corresp ∠s; $RP \parallel QT$ ✓ (6)R

9.2.2 $\hat{Q}_1 = \hat{P}_3$ ✓ ∴ $TNQP$ is cyclic (converse ∠s in same seg) ✓ (2)C

9.2.3 $\hat{Q}_2 = \hat{N}_2 = x$ ∠s in same seg ✓

∴ $\hat{Q}_1 = \hat{Q}_2$ ∴ TQ bisects $N\hat{Q}P$ ✓ (2)C

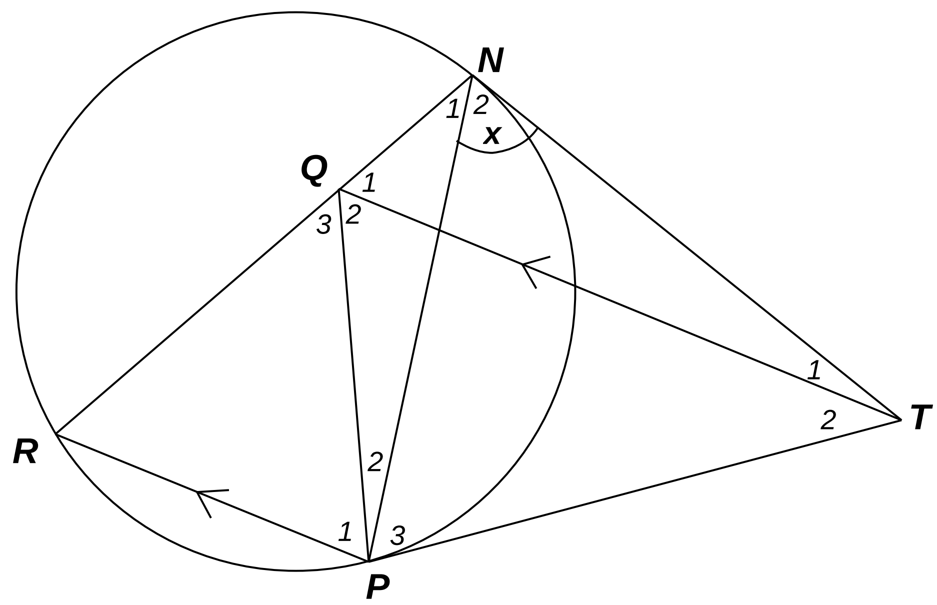
9.2.4 $\hat{P}_1 = x$ alt ∠s; $QT \parallel RP$ ✓

∴ $\hat{P}_1 = \hat{R}$ ✓

∴ $QR = QP$ sides opp equal ∠s ✓

∴ ΔPRQ is isosceles (3)C

[18]



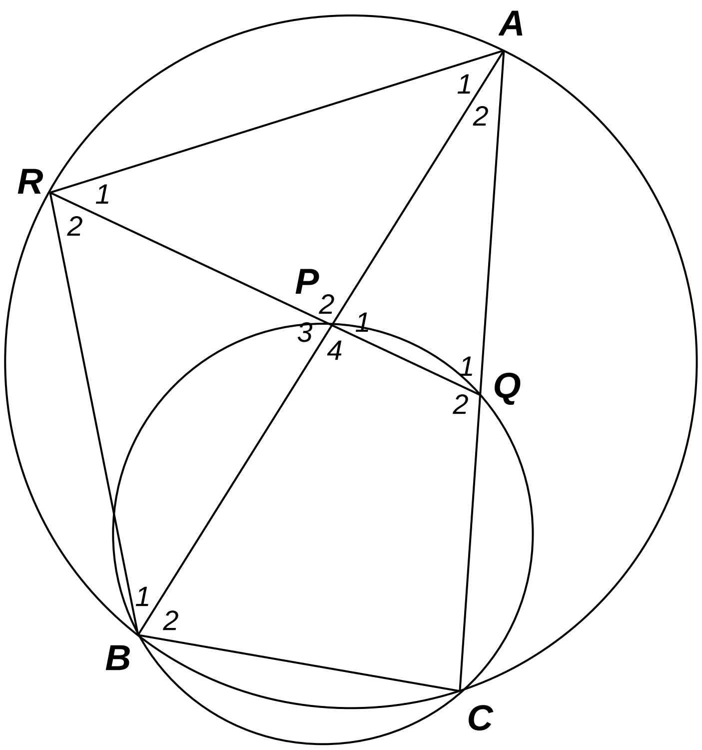
QUESTION 10

10.1 The exterior angle of a cyclic quadrilateral is:
equal to the interior ✓ *opposite* ✓ *angle.* (2)R

10.2 Let $\hat{A}_1 = x$ and let $\hat{B}_1 = y$.

$\hat{R}_{1+2} = 180° - x - y$		∠ sum in ΔRAB	✓S/R
$\hat{C} = x + y$	✓S	opp ∠s cyclic quad $RACB$	✓R
$\hat{P}_1 = x + y$	✓S	ext ∠ of cyclic quad $PQCB$	✓R

$\therefore \hat{P}_1 = \hat{A}_1 + \hat{B}_1$ (5)P



[7]

[[TOTAL 150]]