SACRED HEART COLLEGE

MATHEMATICS: PAPER 1

GRADE 11

NOVEMBER 2016

EXAMINER:
Mr M Phungula

TIME: 3 hours
MARKS: 150

MODERATORS:
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PLEASE READ THE INSTRUCTIONS CAREFULLY

1. This question paper consists of 10 page(s) and separate diagram sheet and formula sheet. Please check that your paper is complete.

2. Read the questions carefully.

3. Answer all the questions on the paper provided.

4. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.

4. All the necessary working details must be clearly shown, giving an answer only will not necessarily give you full marks.

6. It is in your own interest to write legibly and to present your work neatly.

7. Round all answers to **2 decimal place** unless told to do otherwise.

QUESTION 1

Solve for x in each of the following:

(a)
$$x(x-1)=30$$
$$x^2-x-30=0 \checkmark$$
$$(x-6)(x+5)=0 \checkmark$$
$$x=6 \text{ or } x=-5 \checkmark$$
(3)

(b)
$$3x^2-5x+1=0$$
$$a=3 \quad b=-5 \quad c=1$$
$$x=\frac{-(-5)\pm\sqrt{25-4(3)(1)}}{2(3)} \checkmark$$
$$=\frac{5\pm\sqrt{13}}{6} \checkmark$$
$$x=1,4 \text{ or } x=0,2 \checkmark$$
(3)

(c)
$$x+\sqrt{-4x-3}=0$$
$$(\sqrt{-4x-3})^2=(-x)^2$$
$$-4x-3=x^2 \checkmark$$
$$0=x^2+4x+3$$
$$0=(x+1)(x+3) \checkmark$$
$$x=-1 \text{ or } x=-3 \checkmark$$
(4)

(d)
$$15x-4<9x^2$$
$$9x^2-15x+4>0$$
$$(3x-4)(3x-1)>0$$
$$x<\frac{1}{3} \text{ or } x>\frac{4}{3} \checkmark$$

(4)

$$x^{\frac{4}{3}} = 13x^{\frac{2}{3}} - 36$$

$$x^{\frac{4}{3}} - 13x^{\frac{2}{3}} + 36 = 0$$

$$\left(x^{\frac{2}{3}} - 4\right)\left(x^{\frac{2}{3}} - 9\right) = 0 \quad \checkmark \quad \checkmark$$

(e) $x^{\frac{2}{3}} - 4 = 0$ or $x^{\frac{2}{3}} - 9 = 0$ (5)

$$x^{\frac{2}{3}} = 2^2 \qquad x^{\frac{2}{3}} = 3^2$$

$$(x^{\frac{2}{3}})^{\frac{3}{2}} = (2^2)^{\frac{3}{2}} \checkmark \qquad (x^{\frac{2}{3}})^{\frac{3}{2}} = (3^2)^{\frac{3}{2}}$$

$$\therefore x = \pm 8 \checkmark \quad \text{or} \quad x = \pm 27 \checkmark$$

(f)

$$\frac{x^2+7}{x^2-2x-3} + \frac{2}{x+1} = \frac{1}{3-x}$$

$$\therefore \frac{x^2+7}{(x-3)(x+1)} \checkmark + \frac{2}{x+1} = \frac{1}{3-x}$$

$$\therefore x^2 + 7 + 2(x-3) = -1(x+1) \checkmark$$

$$\therefore x^2 + 7 + 2x - 6 = -x - 1 \quad (5)$$

$$\therefore x^2 + 3x + 2 = 0 \checkmark$$

$$\therefore (x+2)(x+1) = 0$$

$$\therefore x \neq -1 \checkmark or \quad -2 \checkmark$$

[24]

QUESTION 2

Solve simultaneously for x and y in the following set of equations:

$y = x - 3$

sub ✓

$$x^2 - x(x-3) - 2(x-3)^2 - 7 = 0$$

$$x^2 - x^2 + 3x - 2(x^2 - 6x + 9) - 7 = 0 \quad ✓$$

$$0 = 2x^2 - 15x + 25 \quad ✓$$

$$0 = (2x-5)(x-5) ✓$$

$$x = \frac{5}{2} \quad \text{or} \quad x = 5 \quad ✓$$

$$y = -\frac{1}{2} \quad \text{or} \quad y = 2 \quad ✓$$

[6]

QUESTION 3

(a) If A= $\dfrac{3.2^{x+1} - 4.2^{x-1}}{2^{x-3}}$

(1) Determine A

$$A = \frac{3.2^{x+1} - 4.2^{x-1}}{2^{x-3}}$$

$$= \frac{2^x\left(3.2^1 - 4.2^{-1}\right)}{2^x.2^{-3}} \quad ✓ \quad ✓$$

$$= \frac{6-2}{\frac{1}{8}}$$

$$= 32 \quad ✓$$

(3)

(b) $\dfrac{x^2}{1+x}$ if $x=1+\sqrt{3}$ (no calculators) (3)

$$=\frac{\left(1+\sqrt{3}\right)\left(1+\sqrt{3}\right)}{1+\left(1+\sqrt{3}\right)} \checkmark$$

$$=\frac{4+2\sqrt{3}}{2+\sqrt{3}} \checkmark$$

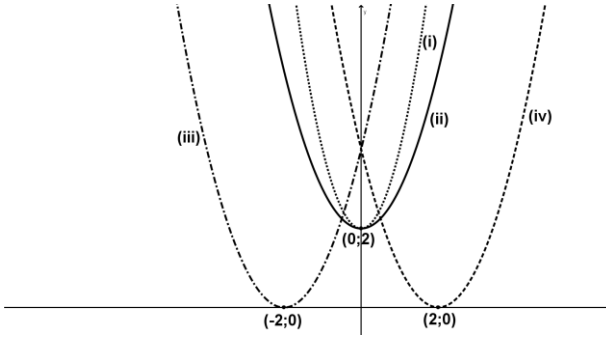
$$=\frac{2(2+\sqrt{3})}{(2+\sqrt{3})}$$

$$=2 \checkmark$$

[6]

QUESTION 4

(a) Match the following functions to the graphs (i) – (iv) in each case. Write down the equation and the number (i) – (iv) for each.



(a) (1) (ii)
(2) (i)
(3) (iii)
(4) (iv)

(4)

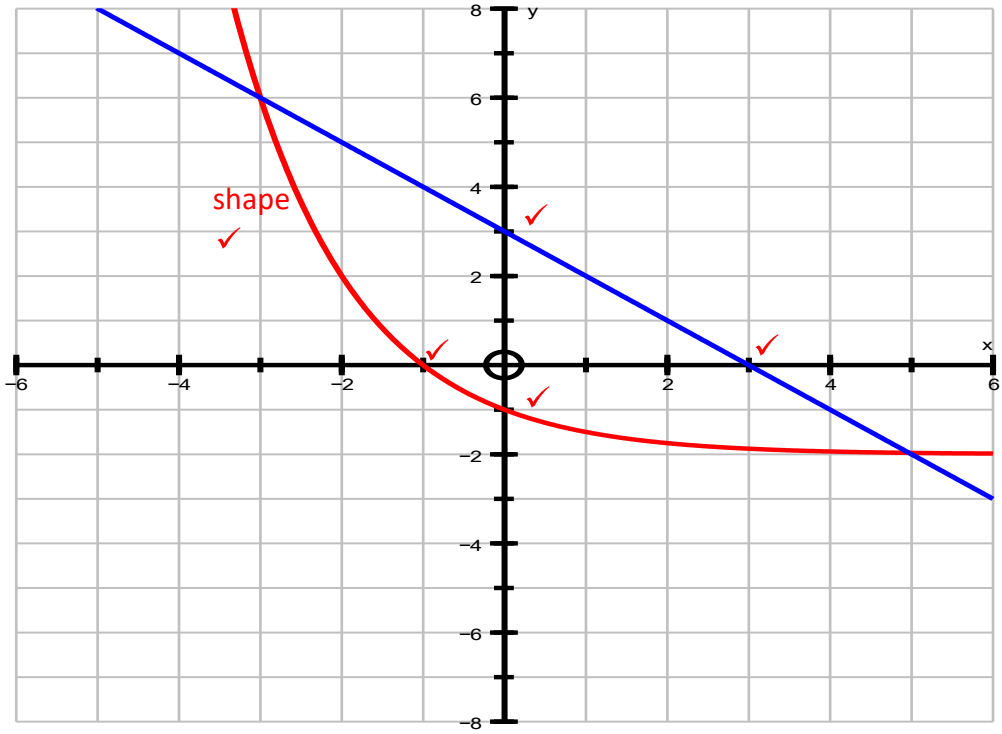
(b) Given: $f(x)=\left(\frac{1}{2}\right)^x-2$ and $g(x)=-x+3$

(1) Determine the intercepts of f with the x- and y-axes. (2)

$y=-1$

$x=-1$

(2) Draw neat sketch graphs of f and g on the same system of axes on the DIAGRAM SHEET. Clearly indicate asymptotes, turning points and intercepts with the axes. (5)



(3) Find the points of intersection by using graphs drawn above. (2)

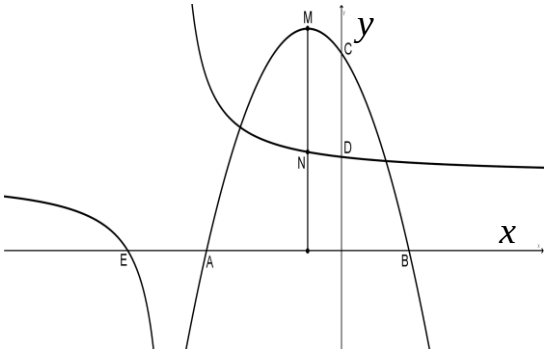
$(-3;6)$ *and* $(5;-2)$

(4) What transformation must f undergo to have no real roots? (1)

Move up two units

The figure below represents the graphs of the following functions:

$f(x) = -2x^2 - 4x + 16$ and $g(x) = \frac{8}{x+5} + 6$



(a) Determine the coordinates of A, B, C, D, E (6)

$-2x^2 - 4x + 16 = 0$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x = -4 \text{ or } 2$ ✓

$\therefore A(-4;0),\ B(2;0),\ C(0;16)$ ✓ ✓ ✓

$y = \frac{8}{0+5} + 6 = \frac{38}{5}$

$D\left(0;\frac{38}{5}\right)$ ✓

$0 = \frac{8}{x+5} + 6$ ✓

$-6(x+5) = 8$

$x = -\frac{19}{3}$

$E\left(-\frac{19}{3};0\right)$ ✓

(b) If M is the turning point of $f(x)$, determine the length of MN (4)

$x = \frac{-b}{2a} = \frac{4}{-4} = -1$ ✓

$M: y = f(-1) = -2 + 4 + 16 = 18$ ✓

$N: y = g(-1) = \frac{8}{4} + 6 = 8$ ✓

$\therefore MN = 10 units$ ✓

(c) How many points of intersection do f and g share? (1)

3

(d) State the equations of the vertical and horizontal asymptotes of $g(x)$ (2)

$vert: x = -5$

$horiz: y = 6$

(e) Give the range of f (2)

$(-\infty;18]$ *or* $y \leq 18$ ✓✓

(f) Determine the equation of $f(x)$ if it is translated 3 units to the right and 2 units down. (2)

$f(x) = -2(x+1)^2 + 18$

Translated: $-2(x-2)^2 + 16$ *or* $-2(x-3)^2 - 4(x-3) + 14$ ✓✓

QUESTION 5

(a) An old school tractor is worth R30 000. Compare the length of time it will take to reach a value of R1000 if the depreciation rate is 11%p.a. calculated:

(1) on a straight line basis (3)

$1000 = 30000(1 - 0{,}11 \times n)$ ✓

$\frac{1}{30} = 1 - 0{,}11n$

$0{,}11n = \frac{29}{30}$ ✓

$n \approx 9$ years ✓

(2) on a reducing balance basis (3)

$1000 = 30000(1 - 0{,}11)^n$ ✓

$\frac{1}{30} = (1 - 0{,}11)^n$

$\frac{1}{30} = (0{,}89)^n$ ✓

$n \approx 30$ years ✓

(3) Explain the difference between your answers to (1) and (2) (2)

The amount reduces more slowly with reducing balance because the amount subtracted gets less and less with time, whereas with straight line reducing the amount subtracted remains the same.

(b) A school decides that they are going to invest R60 000 at 12% p.a. compounded monthly in order to put up floodlights on the Hockey AstroTurf. Three years later the interest rate drops by 1% and it was compounded quarterly. Four years after the first investment, R10 000 is withdrawn to buy rugby poles.
What is the total investment worth 13 years after the first investment was made? (5)

$A = 60000\left(1 + \frac{12\%}{12}\right)^{36}\left(1 + \frac{11\%}{4}\right)^{40} - 10000\left(1 + \frac{11\%}{4}\right)^{36}$ ✓✓✓

$= R254093{,}72 - R26554{,}98$

$= R227538{,}74$ ✓

QUESTION 6

(a) The distance of an object (in metres) from a starting point at a particular time (in seconds) is recorded in the table below:

Time in seconds	**0**	**1**	**2**	**3**	**4**	**5**	**6**
Distance from starting point	**3**	**4**	**9**	**18**	**31** ✓	**48** ✓	**69** ✓

1) Complete the table for the next three seconds. (3)

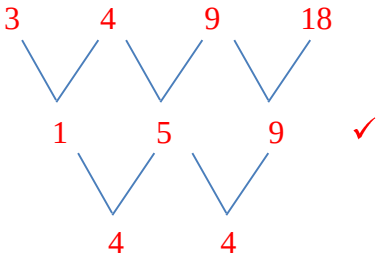
2) The reationship between the distance travelled (d) in a particular time (t) is modelled by the equation:

$d = at^2 + bt + c$

i) Determine the value of c when $t = 0$. (1)

$c = 3$✓

ii) Determine the value of a and b. (3)



$2a = 4$ $\quad\quad$ $3a + b = 5$ ✓

$\therefore a = 2$✓ $\quad\quad$ $\therefore b = -1$ ✓

OR

add 1 *to t since you have the extra term at* $t = 0$

$d = 2(t+1)^2 - 5(t+1) + 6$ ✓✓

$= 2(t^2 + 2t + 1) - 5t - 5 + 6$

$= 2t^2 - t + 3$✓

iii) Determine how far from the starting point the object is after 10 seconds. (2)

At $t = 10$

$d = 2(10)^2 - 10 + 3$✓

$d = 193\ m$✓

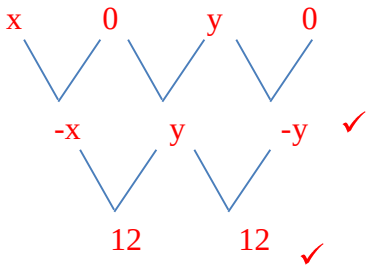
iv) How long does it the object to travel a distance of *1038 m* from the starting point? (4)

$1038 = 2t^2 - t + 3$ ✓

$2t^2 - t - 1035 = 0$ ✓

$t = 23 \text{ or } t \neq -22,5$ ✓

(b) A given quadratic pattern $T_n = an^2 + bn + c$ has $T_2 = T_4 = 0$ and a second difference of 12. Determine the value of the 1st and 3rd terms of the pattern. (6)



✓

$x + y = 12$ ✓ $\quad -2y = 12$ ✓

$y = -6$ ✓ *and* $x = 18$ ✓

QUESTION 7

(a) The events A and B are independent. P(A) = 0,4 and P(B) = 0,5. Determine:

(1) P(A and B) (1)

$P(A \text{ and/en } B) = P(A) \times P(B)$
$= 0,4 \times 0,5$
$= 0,2$ ✓

(2) P(A or B) (2)

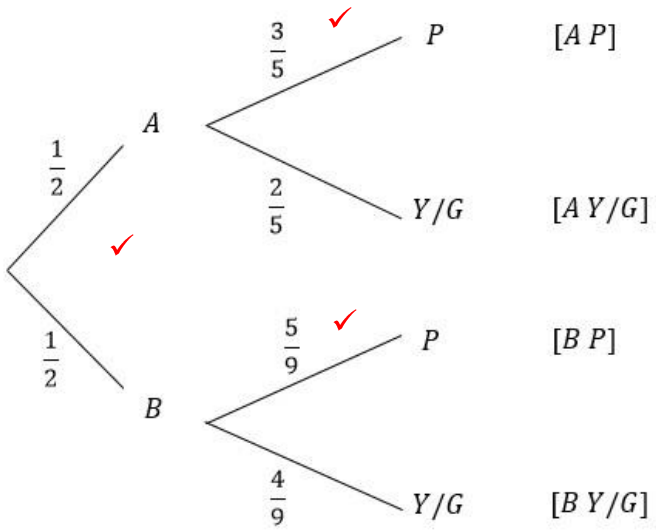
$P(A \text{ or } B) = 0,4 + 0,5 - 0,2$ ✓
$= 0,7$ ✓

(3) P(not A and not B) (2)

$P(\text{not } A \text{ and en not } B) = 1 - P(A \text{ or/of } B)$
$= 1 - 0,7$ ✓
$= 0,3$ ✓

(b) Two identical bags are filled with balls. Bag A contains 3 pink and 2 yellow balls. Bag B contains 5 pink and 4 yellow balls. It is equally likely that Bag A or Bag B is chosen. Each ball has an equal chance of being chosen from the bag. A bag is chosen at random and a ball is then chosen at random from the bag.

(1) Represent the information by means of a tree diagram. Clearly indicate the probability associated with each branch of the tree diagram and write down all the outcomes. (3)



(2) What is the probability that a yellow ball will be chosen from **Bag A**? (1)

$$P(\text{yellow ball from bag } A) = \frac{2}{5}$$

(3) What is the probability that a pink ball is chosen? (2)

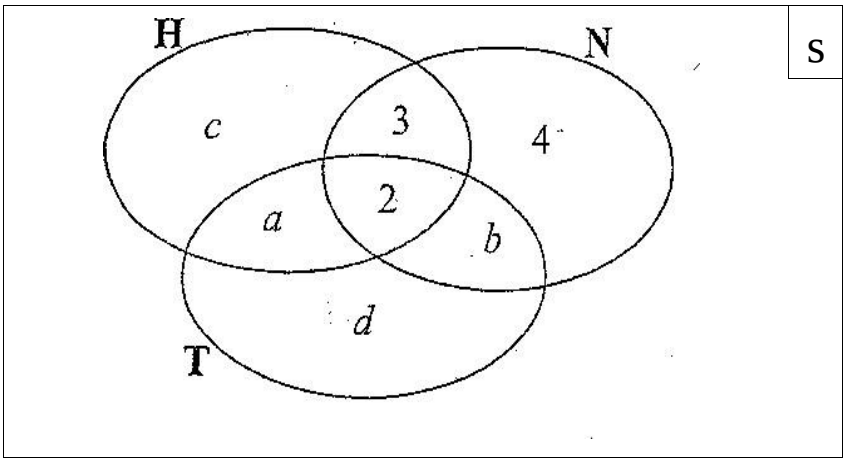
$$P(Pink \backslash Pienk) = \frac{3}{10} + \frac{5}{18}$$

$$= \frac{26}{45}\ (0{,}58)$$

c) A group of 33 learners were surveyed at a school. The following information from the survey is given:

- 2 learners play tennis, hockey and netball
- 5 learners play hockey and netball
- 7 learners play hockey and tennis
- 6 learners play tennis and netball
- A total of 18 learners play hockey
- A total of 12 learners play tennis
- 4 learners play netball only

A Venn diagram representing the survey results is given below:



(1) Use the information provided to determine the values of a, b, c, and d. (4)

$a = 5 \quad b = 4 \quad c = 8 \quad d = 1$

(2) How many of the learners do not play any of the sports on the survey? (1)

6

d) A survey was conducted among 100 boys and 60 girls to determine how many of them watched TV in the period during which examinations were written. Their responses are shown in the partially completed table below.

	WATCHED TV DURING EXAMINATIONS	**DID NOT WATCH TV DURING EXAMINATIONS**	**TOTALS**
Male	80	A = 20	100
Female	48	12	60
Totals	B =128	32	160

(1) Calculate the values of a and b. (2)

$a = 20 \quad and \quad b = 128$

(2) Are the events 'being a male' and 'did not watch TV during examinations' mutually exclusive? Give a reason for your answer. (2)

No, since there are there are 20 people who satisfy both

(3) If a learner who participated in this survey is chosen at random, what is the probability that the learner:

(i) Watched TV in the period during which the examinations were written? (1)

$\frac{128}{160} = \frac{4}{5}$

(ii) Is not a male and did not watch TV in the period during which examinations were written? (2)

$\frac{12}{160} = \frac{3}{40}$

QUESTION 8

(a) In what way must the axes be moved so that the graph defined by $y = x^2 + 1$ becomes $y = x^2 + 2x$? (3)

$y = x^2 + 1$ has a TP: $(0;1)$

$y = x^2 + 2x$

$= x^2 + 2x + 1 - 1$

$= (x+1)^2 - 1$ has a TP: $(-1;-1)$ ✓

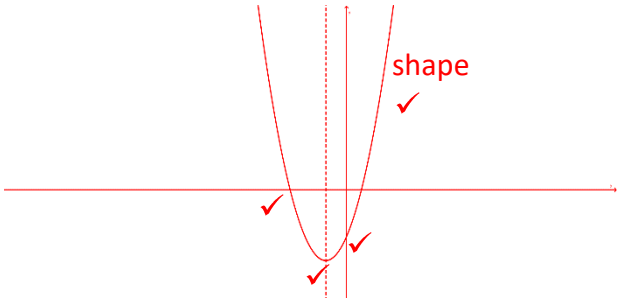
$\therefore y-$ axis moves down 2 units ✓

$x-$ axis moves 1 unit to the left ✓

(b) Draw a rough sketch of the parabola if $y = ax^2 + bx + c$ if $a > 0$, $b > 0$ and the roots are real but have opposite signs. (4)

$y = ax^2 + bx + c$ with $a > o$; $b > 0$, roots real, opposite signs

$x = \frac{-b}{2a} = -\frac{+}{2(+)} = -$



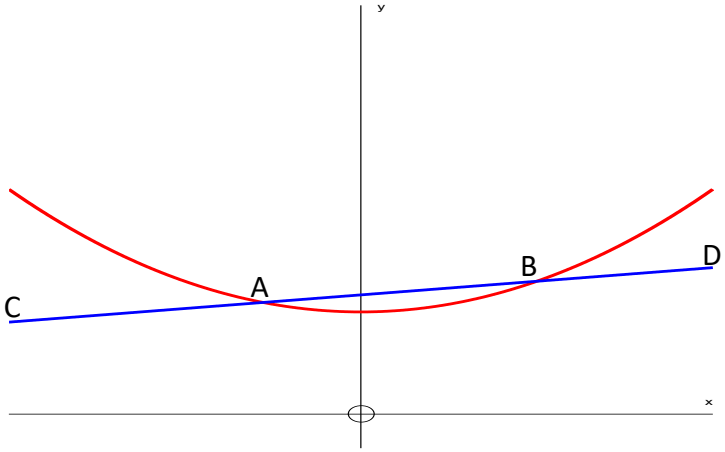
(c) A Metro council installed Christmas lights in John Ross Street. A garland of lights was suspended in the shape of parabola with equation:

$$y = \frac{x^2}{10} + 3$$

where y is the height of the garland (in metres) above the road and x is the horizontal distance (in metres) from the centre of the road.

A rope CD holding up other decorations is connected to the garland at A and B and is described by the function:

$$y = \frac{2x}{15} + \frac{7}{2}$$



Centre of Road

Determine the difference in height above the road of the two points A and B.
Give your answer to the nearest centimetre. (5)

$$\frac{x^2}{10} + 3 = \frac{2x}{15} + \frac{7}{2} \quad \checkmark$$

$$3x^2 + 90 = 4x + 105$$

$$3x^2 - 4x - 15 = 0 \quad \checkmark$$

$$(x-3)(3x+5) = 0 \quad \checkmark$$

$$x = 3 \qquad or \qquad x = \frac{-5}{3}$$

$$y = 390 \quad or \quad y = 327,7 \quad \checkmark$$

$$\therefore difference = 62\ cm \quad \checkmark$$

QUESTION 9

(a) Calculate the exact value of:

$$\frac{\sqrt{10^{2009}}}{\sqrt{10^{2011}}-\sqrt{10^{2007}}}$$

$$=\frac{10^{\frac{2009}{2}}}{10^{\frac{2011}{2}}-10^{\frac{2007}{2}}}$$

$$=\frac{10^{\frac{2009}{2}}}{\checkmark 10^{\frac{2007}{2}}\left(10^2-1\right)}\ \checkmark$$

$$=\frac{10}{99}\ \checkmark$$

(3)

(b) Calculate the value of:

$$1234567893\times 1234567894-1234567895\times 1234567892$$ (3)

$$(x+1)(x+2)-(x+3)\times x\ \checkmark$$

$$x^2+3x+2-x^2-3x\ \checkmark$$

$$2\ \checkmark$$

c) A man buys a new car today for R230 000. He knows that in five years it will depreciate to R102 052 and that, over the same period of time the price of a new car will increase to R353 884 due to inflation.

He wishes to make 3 equal payments, one 20 months from now, one 40 months from now and one 60 months from now in order to be able to replace the car 60 months from now, immediately after making the last payment. If he will earn 12% per annum compounded monthly then how much must each payment be? You can assume that he trades in his old car at the depreciated value. (5)

$$x\left(1+\frac{0.12}{12}\right)^{40\checkmark}+x\left(1+\frac{0.12}{12}\right)^{20\ \checkmark\checkmark}+x=353\,884-102052\ \checkmark$$

$$\therefore x\left(1.01^{40}+1.01^{20}+1\right)=251\,832$$

$$\therefore x=R67\,896.56\ \checkmark$$

(d) In still waters, a man can row at a speed of 10 km/h. He needs 40 minutes more to row 16 km upstream than to row 16 km downstream. Determine the speed of the current (x). (6)

Let speed of the current be x

$$\therefore \frac{16}{10-x} - \frac{16}{10+x} = \frac{40}{60}$$

$$\text{LCD} = 60(10-x)(10+x)$$

$$960(10+x) - 960(10-x) = 40(10-x)(10+x)$$

$$1920x = 4000 - 40x^2$$

$$x^2 + 48x - 100 = 0$$

$$(x-2)(x+50) = 0$$

$$\therefore x = 2 \text{ or } -50$$

$$\therefore \text{Speed of current is } 2 km/h$$