

QUESTION 1

a) Solve for x :

i) $(2x - 1)(2^x - 1) = 0$ (3)

$$2x - 1 = 0$$

$$x = \frac{1}{2} \checkmark$$

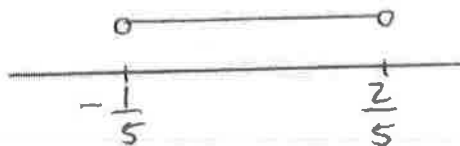
$$2^x - 1 = 0$$

$$\text{or } 2^x = 1 \checkmark$$

$$2^x = 2^0$$

$$x = 0 \checkmark$$

ii) $(5x + 1)(5x - 2) < 0; x \in \mathbb{Z}$ (3)



$$-\frac{1}{5} < x < \frac{2}{5} \checkmark$$

$$\therefore x \in \{0\} \checkmark$$

iii) $\sqrt{x+7} + 1 = 2x$ (5)

$$\sqrt{x+7} = 2x - 1 \quad \checkmark$$

$$x+7 = 4x^2 - 4x + 1 \quad \checkmark$$

$$4x^2 - 5x - 6 = 0 \quad \checkmark$$

$$(4x+3)(x-2) = 0$$

$$x \neq \frac{-3}{4} \quad \checkmark \quad \text{or} \quad x = 2 \quad \checkmark$$

b) Simplify

i) $(\sqrt{ab} + \sqrt{a})^2 - 2a\sqrt{b} - ab$ (3)

$$= ab + 2\sqrt{a}\sqrt{ab} + a - 2a\sqrt{b} - ab \quad \checkmark$$

$$= 2\sqrt{a^2b} + a - 2a\sqrt{b}$$

$$= 2a\sqrt{b} + a - 2a\sqrt{b}$$

$$= a \quad \checkmark$$

ii) $\frac{3^x - 1}{9^x - 1}$ (3)

$$= \frac{3^x - 1}{3^{2x} - 1} \quad \checkmark$$

$$= \frac{3^x - 1}{(3^x - 1)(3^x + 1)} \quad \checkmark$$

$$= \frac{1}{3^x + 1} \quad \checkmark$$



- c) Given $M = \frac{1}{\sqrt{2x-1}}$ determine the value(s) of x for which M is real. (2)

$$2x - 1 > 0 \checkmark$$

$$2x > 1$$

$$x > \frac{1}{2}$$

- d) If $2x^2 - 5xy - 12y^2 = 0$ and $xy > 0$

- i) Determine the value of $\frac{x}{y}$. (3)

$$(2x + 3y)(x - 4y) = 0 \checkmark$$

$$2x = -3y$$

$$\frac{x}{y} = -\frac{3}{2}$$

$$xy > 0$$

$$x - 4y = 0$$

$$\frac{x}{y} = 4 \checkmark$$

- ii) If $x + y = 4$ use this and the answer from c)i) to solve for x and y . (3)

$$\frac{x}{y} = 4 \quad \text{and} \quad x + y = 4$$

$$x = 4 - y$$

$$\frac{4-y}{y} \checkmark = 4$$

$$4-y = 4y$$

$$4 = 5y$$

$$y = \frac{4}{5} \checkmark$$

and

$$x = 4 - \frac{4}{5}$$

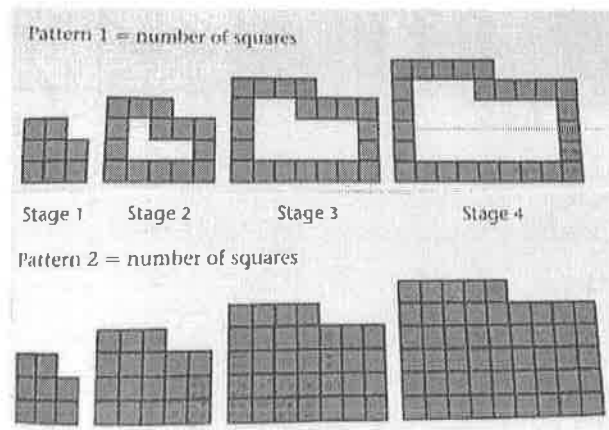
$$= \frac{16}{5} \checkmark$$

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QUESTION 2

a)



- i) Write down the first four terms of pattern 1 and pattern 2. (2)

8 ; 14 ; 20 ; 26 ✓

8 ; 18 ; 32 ; 50 ✓

- ii) Which pattern is linear and which is quadratic? (2)

pattern 1 is linear ✓
 pattern 2 is quadratic ✓

- iii) Write down a formula for T_n , the n^{th} term of pattern 1. (2)

$$T_n = 6n + 2 \checkmark$$

- iv) Write down a formula for T_n , the n^{th} term of pattern 2. (5)

8 ; 18 ; 32 ; 50

10 14 18 ✓

4 4 ✓

$$3a + b = 10$$

$$3(2) + b = 10$$

$$b = 4$$

$$2a = 4$$

$$a = 2$$

$$T_n = 2n^2 + 4n + 2 \checkmark$$

$$a + b + c = 8$$

$$2 + 4 + c = 8$$

$$c = 2.$$

- b) A quadratic pattern has a third term equal to 2, a fourth term equal to -2 and a sixth term equal to -16 . Calculate the second difference of this quadratic pattern. (5)

$$T_3 = 2 \quad T_4 = -2 \quad T_6 = -16$$

$$\begin{array}{cccc} 2 & -2 & x & -16 \\ -4 & & x+2 \checkmark & -16-x \checkmark \end{array}$$

$$x+2+4 = \checkmark -x-16-x-2$$

$$x+6 = -2x-18$$

$$3x = -24$$

$$x = -8 \checkmark$$

$$\therefore \begin{array}{cccc} 2 & -2 & -8 & -16 \\ -4 & -6 & -8 & \\ -2 & -2 & \checkmark \text{second diff.} & \end{array}$$

- c) It is given that 27 and $\frac{64}{27}$ are the 1st and 7th terms of a geometric sequence respectively. Determine the common ratio(s). (5)

$$a = 27 \checkmark$$

$$T_7 = ar^6 = \frac{64}{27} \checkmark$$

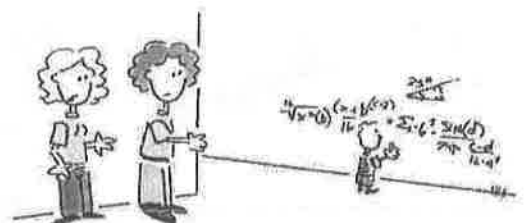
$$27 \sqrt[6]{r^6} = \frac{64}{27}$$

$$r^6 = \frac{64}{729}$$

$$r^6 = \frac{2^6}{3^6}$$

$$r = \pm \sqrt[6]{\frac{2^6}{3^6}} \checkmark$$

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"Of course I'll punish him, but not until I can figure out if he's on to something."

QUESTION 3

a) Show that $x^2 + 2x + 3 = 0$ has no real roots.

(3)

$$\begin{aligned}\Delta &= b^2 - 4ac \\ &= (2)^2 - 4(1)(3) \checkmark \\ &= 4 - 12 \\ &= -8 \checkmark < \checkmark 0 \\ \therefore &\text{ no real roots.}\end{aligned}$$

b) Given $f(x) = -(x+5)^2 + 8$ determine possible value(s) of m if $g(x) = f(x) + m$ has non-real roots. (2)

$$m < \checkmark -8 \checkmark$$

$$-(x^2 + 10x + 25) + 8 + m$$

$$= -x^2 - 10x - 25 + 8 + m$$

$$= -x^2 - 10x - 17 + m.$$

$$\Delta = (-10)^2 - 4(-1)(-17+m)$$

$$= 100 + -68 + 4m$$

$$= 32 + 4m \checkmark$$

$$\Delta < 0$$

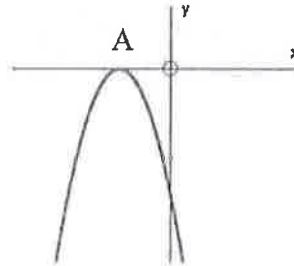
$$32 + 4m < 0$$

$$4m < -32$$

$$m < -8 \checkmark$$

c) The graph of $y = -x^2 + bx - 4$ is sketched below. The parabola touches the x -axis at A.

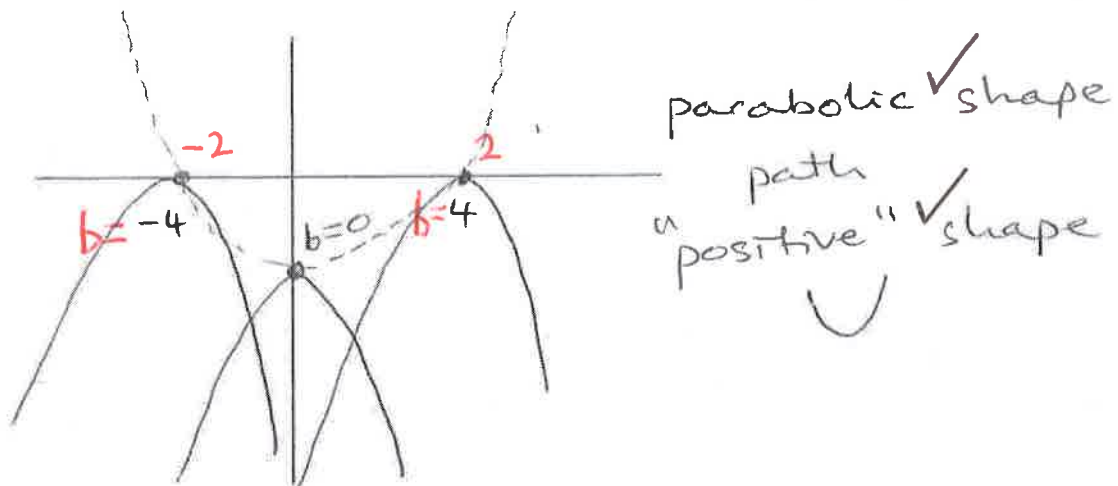
i) Determine the value of b . (2)



$$b = -4 \checkmark \checkmark$$

$$x^2 + 4x + 4 = (x+2)^2 \therefore x = -2.$$

ii) Describe what happens to the graph of $y = -x^2 + bx - 4$ when b varies from -4 to 4 ? Draw a sketch to illustrate the path of A, the turning point of the graph. (2)



Not sufficient to say
moves horizontally.

QUESTION 4

- a) i) Given the points $(3; 1)$ and $(2; \frac{1}{3})$ on the graph of $f(x) = a \cdot b^{x-3}$, determine the values of a and b . (4)

$$1 = a \cdot b^{3-3} \checkmark$$

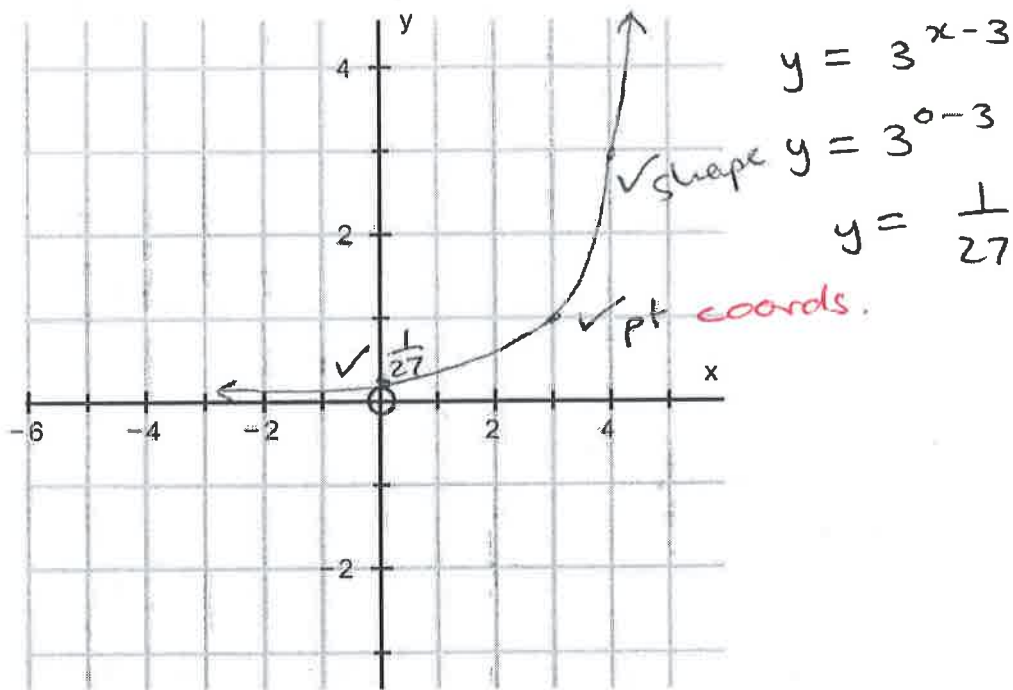
$$1 = a \cdot b^0$$

$$a = 1 \checkmark$$

$$\frac{1}{3} = b^{2-3} \checkmark$$

$$\frac{1}{3} = b^{-1} \therefore b = 3 \checkmark$$

- ii) Sketch the graph of $f(x)$ on the axes given below. Clearly indicate any intercepts with the axes and any asymptotes. (3)

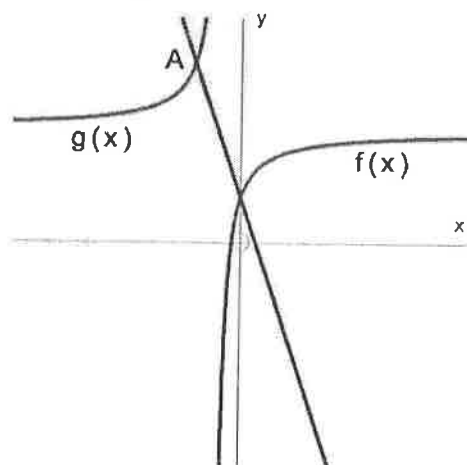


- iii) State the range of the graph of $h(x)$, where $h(x)$ is the graph of $f(x)$ reflected over the x -axis and shifted one unit down. (2)

$$y \in (-\infty; -1)$$

$$y < -1$$

b) The graphs of $f(x) = \frac{-3}{x+1} + 5$ and $g(x) = -3x + 2$ are sketched:



i) Write down the axes of symmetry of $f(x)$. (2)

$$y = x + 1 + 5 \therefore y = x + 6 \checkmark$$

$$y = -(x + 1) + 5 \therefore y = -x + 4 \checkmark$$

ii) Determine the coordinates of A, one of the points of intersection of $f(x)$ and $g(x)$. (5)

$$\frac{-3}{x+1} + 5 = -3x + 2$$

$$-3 + 5(x+1) = (-3x+2)(x+1)$$

$$-3 + 5x + 5 = -3x^2 - x + 2$$

$$3x^2 + 6x = 0$$

$$3x(x+2) = 0 \checkmark$$

$$x = 0 \text{ or } x = -2 \checkmark$$

$$g(0) = 2 \quad g(-2) = 8$$

$$A(-2; 8) \checkmark$$

iii) For which values of x is $\frac{f(x)}{g(x)} \leq 0$?

(5)

$$-3x + 2 = 0$$

$$x = \frac{2}{3}$$

$$\frac{-3}{x+1} + 5 = 0$$

$$\frac{-3}{x+1} = -5$$

$$-3 = -5(x+1)$$

$$-3 = -5x - 5$$

$$5x = -2$$

$$x = -\frac{2}{5}$$

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$$\frac{f(x)}{g(x)} \leq 0$$

$$\begin{array}{c} + \\ - \end{array} \quad \vee \quad \begin{array}{c} - \\ + \end{array}$$

$$-1 < x \leq -\frac{2}{5} \quad \vee \quad x > \frac{2}{3}$$

$$-0,4$$

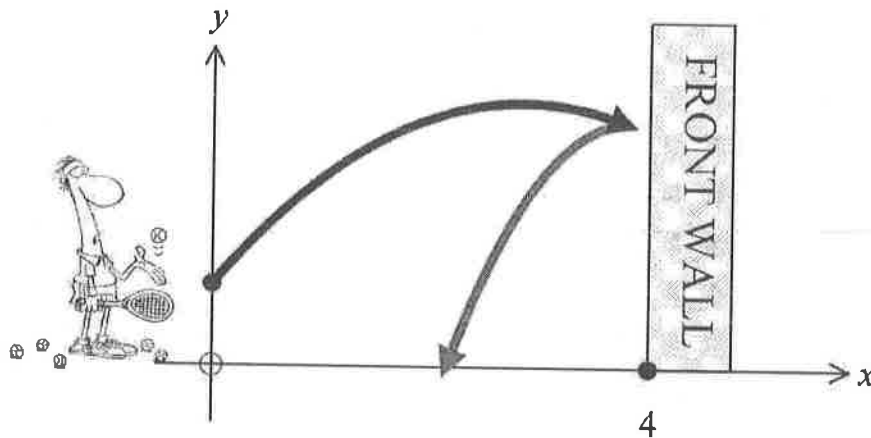
$$0,67$$

QUESTION 5

A tennis player hits a ball against a practice wall 4m away and it rebounds as shown in the diagram below. The initial path of the ball as it is struck by the player is given by the equation:

$$y = -\frac{1}{4}x^2 + px + 1,$$

where y is the height, in metres that the ball is above the ground and x is the distance, in metres, that the ball is away from the player.



- a) From what height is the tennis ball struck? (1)

$$y = 1 \text{ metre } \checkmark$$

- b) Given that the ball strikes the wall at $x = 4$, determine the height it strikes the wall in terms of p . (4)

$$\begin{aligned} y &= -\frac{1}{4}(4)^2 + p(4) + 1 \\ &= -4 + 4p + 1 \\ &= 4p - 3 \end{aligned} \checkmark$$

- c) Show, by completing the square, that the turning point of the initial path of the ball as it is struck by the player is given by $(2p; p^2 + 1)$ (5)

$$\begin{aligned}
 y &= -\frac{1}{4}x^2 + px + 1 \\
 &= -\frac{1}{4}(x^2 - 4px + 4p^2 - 4p^2) + 1 \\
 &= -\frac{1}{4}(x^2 - 4px + 4p^2) + p^2 + 1 \\
 &= -\frac{1}{4}(x - 2p)^2 + p^2 + 1 \checkmark \\
 \therefore \text{TP } (2p; p^2 + 1) \\
 &\quad \text{given}
 \end{aligned}$$

- d) The ball rebounds off the wall along the curve defined by:

$$y = -3x^2 + 24x - 45$$

Determine the value of x at which the ball hits the ground. (3)

$$\begin{aligned}
 0 &= -3x^2 + 24x - 45 \\
 3x^2 - 24x + 45 &= 0 \\
 0 &= x^2 - 8x + 15 \checkmark \\
 0 &= (x - 5)(x - 3) \\
 x &\neq 5 \quad \text{or } x = 3 \text{ metres.} \\
 \checkmark &
 \end{aligned}$$

QUESTION 6

- a) Mrs Kennedy bought new projectors for the Maths department worth R18 000. The depreciation is calculated at a rate of 12% p.a. on a straight line basis. Calculate the value of the projectors at the end of 5 years. (2)

$$\begin{aligned}
 A &= 18000 (1 - 0,12 \times 5) \\
 &= 18000 \left(\frac{2}{5}\right) \\
 &= 7200 \quad \checkmark
 \end{aligned}$$

- b) Nicholas inherits R50 000. He invests this money at 7% p.a. compounded monthly for the first 3 years and then at 11% p.a. compounded quarterly for the next 4 years. How much will he have at the end of the 7 years if he only withdraws R25 000 at the end of the 5th year? (8)

T_0 T_3 T_5 T_7

50 000
- 25 000

$\frac{7\%}{12}$
 $\frac{11\%}{4}$

$$\begin{aligned}
 A &= 50000 \left(1 + \frac{0,07}{12}\right)^{12 \times 3} \left(1 + \frac{0,11}{4}\right)^{4 \times 4} \\
 &- 25000 \left(1 + \frac{0,11}{4}\right)^{4 \times 2} \\
 &= 95151,61 - 31059,51 \\
 &= 64092,10 \quad \checkmark \checkmark \\
 &\quad \text{rounding}
 \end{aligned}$$



c) Vladimir needed R500 urgently. A 'loan shark' agreed to give it to him for one month but he would have to repay R600.

- i) Determine the monthly interest rate that he (the loan shark) is charging for this one-month loan. (2)

$$600 = 500(1+i) \checkmark$$

$$\frac{6}{5} - 1 = i$$

$$i = 0,2$$

$$= 20\% \checkmark \text{ per month.}$$

- ii) If this monthly rate is compounded for 12 months, then determine the equivalent effective interest rate per annum. (3)

$$(1 + i_{\text{eff}})^1 = \left(1 + \frac{2,4}{12}\right)^{12} \checkmark$$

$$20\% \times 12 \checkmark$$

$$= 240\%$$

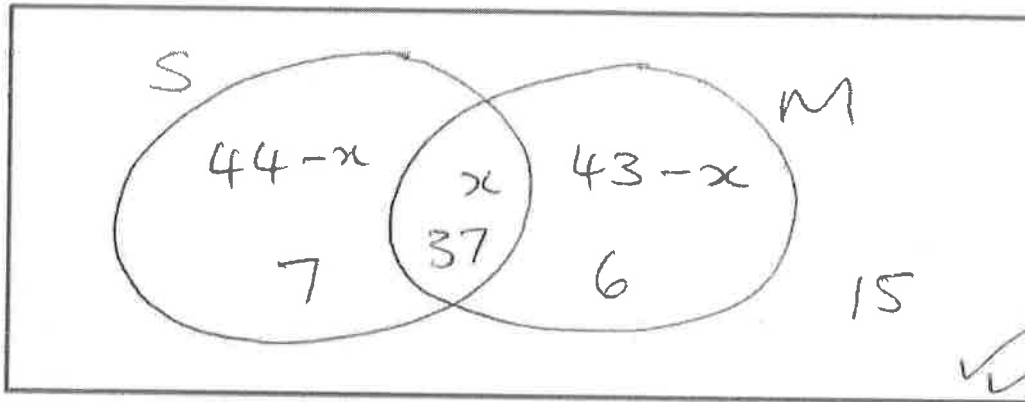
$$i_{\text{eff}} = \left(\frac{6}{5}\right)^{12} - 1$$

$$= 791,61\% \checkmark \text{ p.a.}$$

compounded annually.

QUESTION 7

- a) From a class of 65 boys, 44 played sport on Saturday, 43 went to movies and x did both. 15 did not do either.



- i) Determine making use of a Venn diagram the number of boys who went to movies **and** played sport. (4)



$$44 - x + x + 43 - x + 15 = 65$$

$$x = 37 \checkmark$$

- ii) Determine making use of a Venn diagram the probability that a boy chosen at random went to movies only. (2)

$$\frac{6}{65} \checkmark \checkmark = 0,09$$

- b) For two events, A and B, it is given that:

$$P(A) = 0,2$$

$$P(B) = 0,63$$

$$P(A \text{ and } B) = 0,126$$

Are the events, A and B independent? Justify your answer with appropriate calculations.

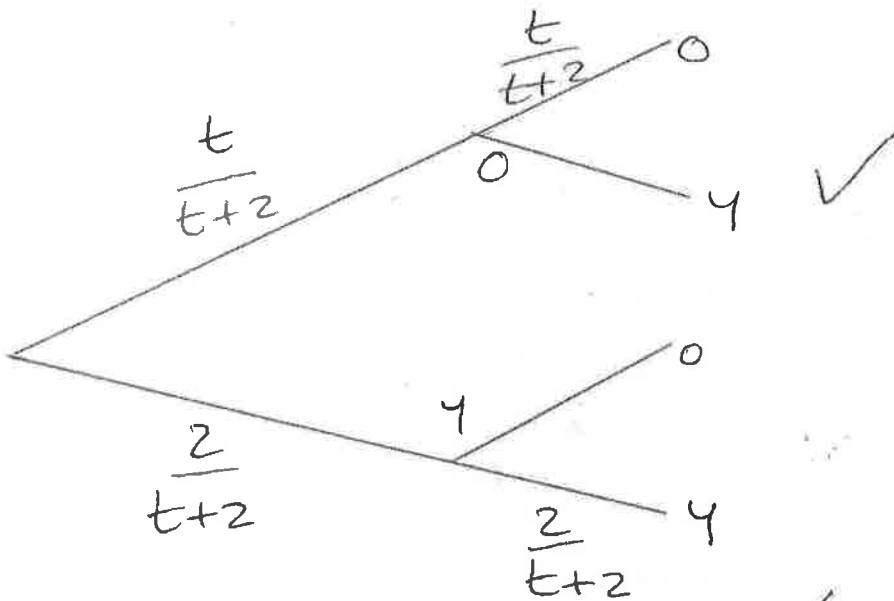
If independent then: $P(A \text{ and } B) = P(A) \times P(B)$ (3)

$$0,126 \quad | \quad 0,2 \times \checkmark 0,63$$

$$= 0,126 \checkmark$$

\therefore A and B are \checkmark independent

- c) There are t orange balls and 2 yellow balls in a bag. Matthew randomly selects one ball from the bag, records his choice and returns the ball to the bag. He then randomly selects a second ball from the bag, records his choice and returns it to the bag. It is known that the probability that Matthew will select two balls of the same colour from the bag is 0,52. Calculate how many orange balls in the bag. (6)



$$\frac{t}{t+2} \times \frac{t}{t+2} \checkmark + \frac{2}{t+2} \times \frac{2}{t+2} \checkmark = 0,52$$

$$t^2 + 4 = 0,52 (t+2)^2$$

$$t^2 + 4 = 0,52 (t^2 + 4t + 4)$$

$$t^2 + 4 = 0,52t^2 + 2,08t + 2,08$$

$$0,48t^2 - 2,08t + 1,92 \checkmark = 0$$

$$t = 3 \checkmark \quad \text{or} \quad t = \frac{4}{3} \checkmark$$



QUESTION 8

In still waters a man rows at a speed of 10 km/hr. He needs 40 minutes more to row 16 km upstream than to row 16 km downstream. Let the speed of the current be x km/hr.

- a) Write down the speed of the rower, in terms of x , as he travels upstream. (1)

$$10 - x \quad \checkmark$$

- b) Hence determine x , the speed of the current. (5)

$$\text{down } 10 + x$$

$$d = 16$$

$$t_{\text{up}} = \frac{16}{10 - x}$$

$$t_{\text{down}} = \frac{16}{10 + x} \quad \checkmark$$

$$\frac{16}{10 - x} - \frac{16}{10 + x} = \frac{40}{60} = \frac{2}{3} \quad \checkmark$$

$$16(3)(10 + x) - 16(3)(10 - x) = 2(10 - x)(10 + x) \quad \checkmark$$

$$48(10 + x) - 48(10 - x) = 2(100 - x^2)$$

$$480 + 48x - 480 + 48x = 200 - 2x^2$$

$$2x^2 + 96x - 200 = 0$$

$$x^2 + 48x - 100 = 0 \quad \checkmark$$

$$(x + 50)(x - 2) = 0$$

$$x = -50 \quad \text{or} \quad x = 2 \quad \checkmark \quad [6]$$

\therefore speed

$$= 2 \text{ km/hour}$$

[Total: 125 marks]

