ST MARY'S DSG, KLOOF

MATHEMATICS

PAPER II

GRADE 11
TIME: 2½ HOURS
EXAMINER: Mrs van ROOYEN

NOVEMBER 2015
TOTAL: 124 MARKS
MODERATORS: Mrs DREW
MR NORTON

NAME:	MEMO (FINAL)
MATHS TEACHER:	

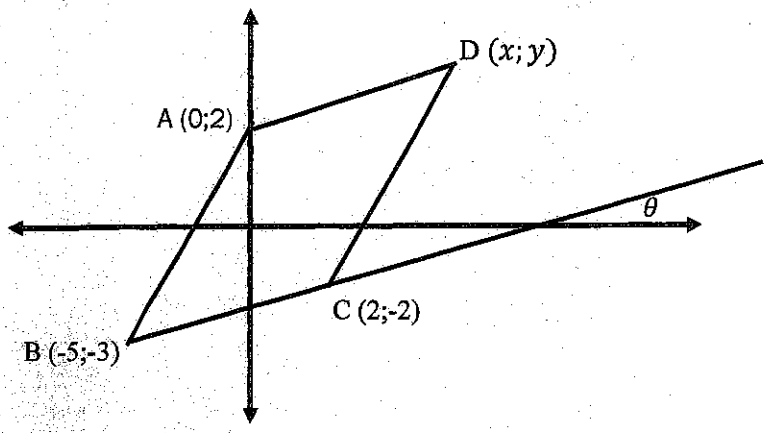
INSTRUCTIONS

1) Read all instructions carefully before you begin.
2) This paper consists of 18 numbered pages, including this cover page. Please check that your paper is complete.
3) Hand in each section separately.
4) Write down your name and your Maths teacher's name in the space provided at the start of each section.
5) All questions must be answered.
6) Calculators may be used unless otherwise stated.
7) Calculators must be in degree mode.
8) Show all your working details.
9) **Reasons must be given for all geometry calculations/ proofs.**
10) Round off all answers to two decimal places where necessary, unless otherwise stated.
11) Diagrams have not been drawn to scale.

SECTION A [34]

QUESTION 1

In the given figure the vertices of parallelogram ABCD are given as A (0;2) B(-5;-3) C (2;-2) and D $(x; y)$.



a) Calculate the size of θ (3)

$\tan\theta = \frac{-3-(-2)}{-5-2}$ $\tan\theta = m$

$= \frac{-1}{-7}$

$= \frac{1}{7}$ $\therefore \theta = 8,13°$

b) Give the co-ordinates of D (7; 3) (2)

c) Give the co-ordinates of E, the midpoint of AB. $E\left(\frac{-5}{2}; \frac{2-3}{2}\right) = \left(-\frac{5}{2}; -\frac{1}{2}\right)$ (2)

d) Show that ΔABC is isosceles (4)

$AB = \sqrt{5^2 + 5^2} = \sqrt{50}$

$BC = \sqrt{7^2 + 1^2} = \sqrt{50}$

$AC = \sqrt{2^2 + 4^2} = \sqrt{20}$

$\therefore \Delta ABC$ isos $(AB = BC)$

e) What type of parallelogram is $ABCD$? Rhombus (adj sides =) (1)

f) Will the circle with centre A and radius $\sqrt{50}$ pass through B, D and C? Give a reason for your answer. (2)

NO, not C. $AC \neq \sqrt{50}$.

$AC = \sqrt{20}$.

[14]

QUESTION 2

The diagram shows a circle centre O. A, B and C are points on the circumference. DCO is a straight line. DA is a tangent to the circle. $\hat{D} = 36^\circ$

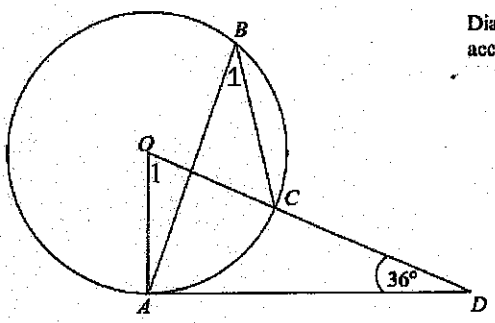


Diagram NOT accurately drawn

a) Find the size of $\hat{O}_1$ (3)

$\hat{A} = 90^\circ$ (rad ⊥ tangent)

$\hat{O}_1 = 180^\circ - (90^\circ + 36^\circ)$ (∠ Sum Δ)

$= 54^\circ$

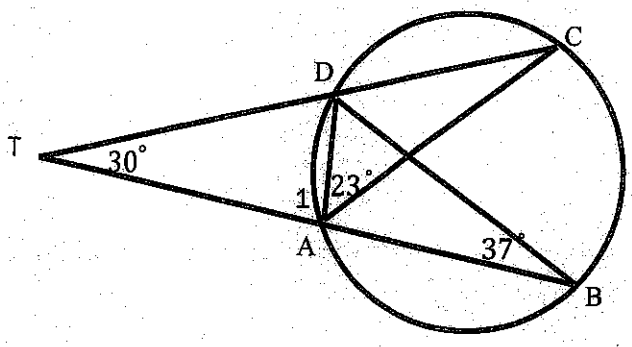
b) Find the size of $\hat{B}_1$ (2)

$\hat{B}_1 = 27^\circ$ (∠ at centre)

[5]

QUESTION 3

In the given diagram, TAB and TDC are two straight lines. A, B, C and D lie on the circumference of the circle.



a) Determine $\hat{A}_1$ (2)

$\hat{C} = \hat{B} = 37°$ (∠'s in same seg) ✓

$\hat{A}_1 = 180° - (30° + 37° + 23°)$ ✓ (∠ sum Δ)

$= 90°$

b) Is BD a diameter of the circle? Explain (2)

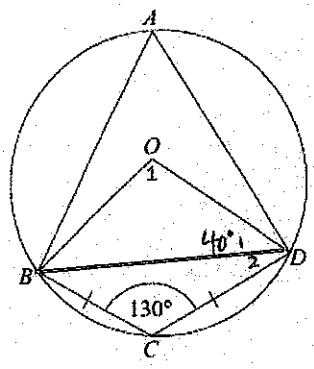
$B\hat{A}D = 90°$ (adj ∠'s st line) ✓

∴ BD diameter (conv. ∠ in semi-⊙) ✓

[4]

QUESTION 4

A, B, C and D are points on circle centre O. BC = CD. $\hat{C} = 130°$



a) Give the size of $\hat{O}_1$ (2)

$\hat{A} = 50°$ (opp ∠'s cyclic quad) ✓ OR $\hat{O}_2 = 260°$ (∠ at centre)

$\hat{O}_1 = 100°$ (∠ at centre) ✓ $\hat{O}_1 = 100°$ (∠'s at a pt)

b) Work out the size of $O\hat{D}C$ (3)

$\hat{D}_1 = \frac{180° - 100°}{2}$ ✓(∠ sum isos Δ, = radii)

$= 40°$

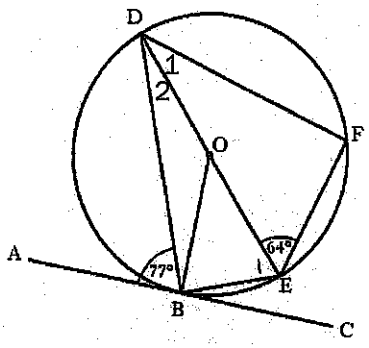
$\hat{D}_2 = \frac{180° - 130°}{2}$ ✓ (∠ sum isos Δ, BC = CD).

$= 25°$ [5]

$\therefore \quad O\hat{D}C = 65°$ ✓ ca.

QUESTION 5

AC is a tangent to the circle, centre O, with point of contact B. DE is the diameter of the circle and F is a point on the circumference. $A\hat{B}D = 77^\circ, D\hat{E}F = 64^\circ$.



Find the sizes of the following, showing working and giving reasons:

a) $\hat{D}_1$ (3)

$\hat{F} = 90^\circ$ (∠ in semi-⊙) ; OR $\hat{E}_1 = 77^\circ$ (tan-chord

$\hat{D}_1 = 90^\circ - 64^\circ$ (∠ sum △)

$= 26^\circ$

b) $\hat{D}_2$ (3)

$\hat{E}_1 = 77^\circ$ (tan-chord th)

$\hat{D}_2 = 180^\circ - (77^\circ + 64^\circ + 26^\circ)$ (opp ∠s cyclic quad)

$= 13^\circ$

OR

$O\hat{B}A = 90^\circ$ (rad ⊥ tangent)

$\hat{B}_1 = \hat{D}_2 = 13^\circ$ (∠'s opp = radii) [6]

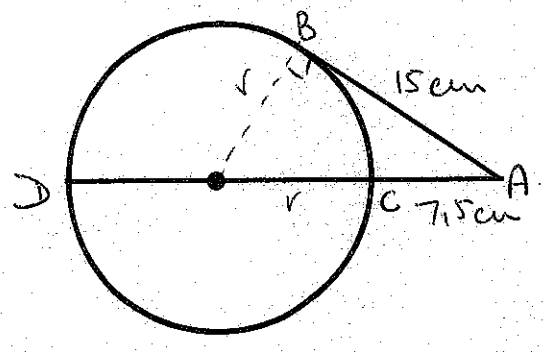
SECTION B [27]

NAME:_____

MATHS TEACHER:_____

QUESTION 6

In the given diagram, O is the centre of the circle and AB is a tangent at B. If AB = 15cm and AC = 7,5cm, calculate the radius of the circle.



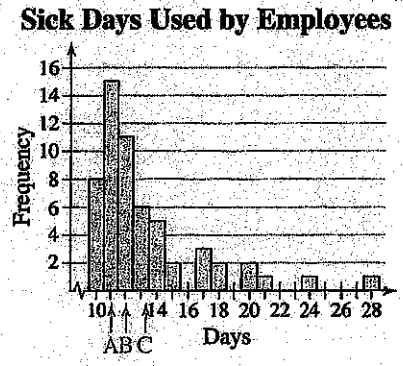
$\hat{B} = 90°$ (r ⊥ tangent)

$(r+7,5)^2 = r^2 + 15^2$ (Pythag)

$r^2 + 15r + 56,25 = r^2 + 15^2$

$15r = \frac{675}{4}$ $r = \frac{45}{4} = 11,25$ cm [3]

QUESTION 7



In the given diagram the letters A, B and C are marked on the horizontal axis

a) Specify which is the mean, which is the median and which is the mode. Give a reason for your choice each time. (3)

A mode most frequent

B median +ve skew

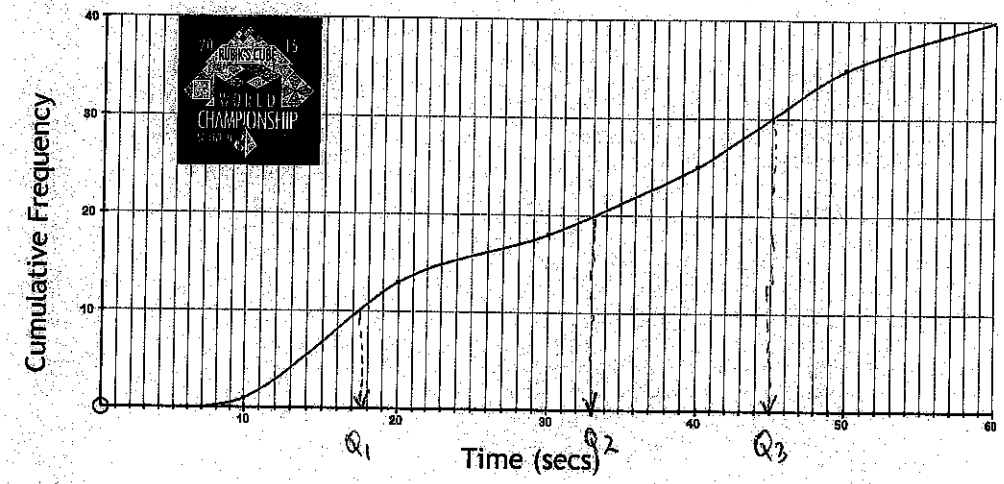
C mean +ve skew

b) How many workers are represented in the histogram? 57 (2)

[5]

QUESTION 8

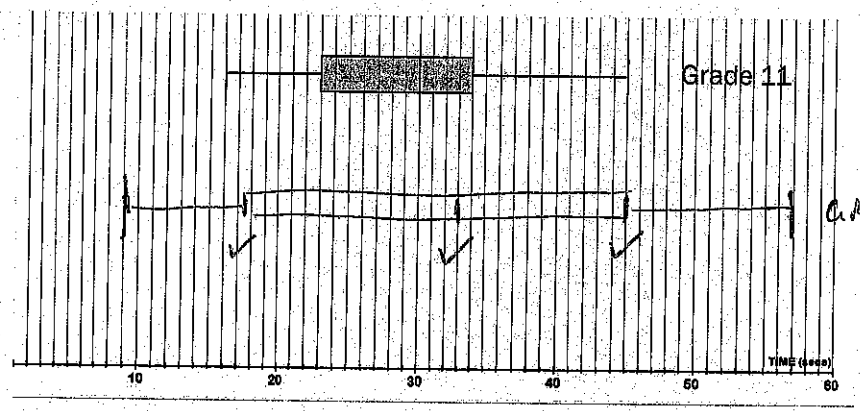
Following the 2015 Rubiks Cube championships in Brazil, 40 grade 10 Kearsney boys challenged 40 grade 11 Kearsney boys to a Rubiks Cube competition. The cumulative frequency graph given shows the times taken for the grade 10's to complete the challenge of the cube.



a) Use the graph to estimate the median time taken by the grade 10's to complete the challenge. (1)

33 seconds.

b) A box and whisker plot showing information about the grade 11's times to complete the challenge is given.
The least time taken to complete the cube by a grade 10 was 9 secs
The longest time take to complete the cube by a grade 10 was 57 secs
Draw the box and whisker for the grade 10's between the grade 11's and the x axis. (3)



c) Based on the box and whisker diagrams : ANY REASONABLE ANSWER! (1)
Give a way the grade 11's could argue that they were the better team.

i) Smaller range of times. Longest time 45 secs (matches Q3 of gr 10).

ii) Give a way the grade 10's could argue that they were the better team. (1)

almost 25% of the grade 10's took less time than the minimum grade 11 time.

[6]

QUESTION 9

A group of students wrote a stats test and the following data was obtained:

Std deviation	σ
Mean	72
Median	64

a) The teacher decides to add 5 marks to each person's mark. For the new set of marks, write down the:

i) Mean (1)

77

ii) Standard deviation (1)

σ

iii) Median (1)

69

b) If the standard deviation of a given data set is equal to zero, what can we say about the data values included in the given data set? (1)

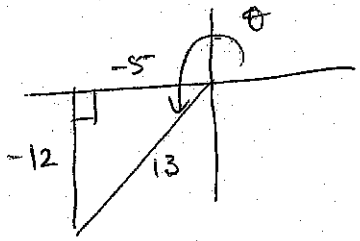
all scores the same.

[4]

QUESTION 10

If $\cos\theta = \frac{-5}{13}$ and $\theta \epsilon [180^\circ; 360^\circ]$, without solving for θ but using a sketch, determine

1st quad
$\checkmark y = -12$



a) $\tan\theta = \frac{12}{5}$ (3)

b) $\sin(360^\circ - \theta)$ (2)

$= -\sin\theta$

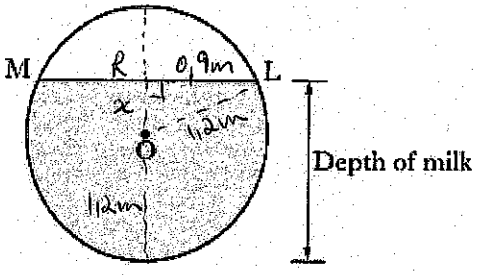
$= -\left(\frac{-12}{13}\right)$

$= \frac{12}{13}$

[5]

QUESTION 11

The diagram shows the circular cross-section of a milk tank. The radius of the circle, centre O, is 1,2 m. The width of the surface of the milk in the tank (ML in the diagram) is 1,8 m. Calculate the depth of milk in the tank.



Draw in radii

$\hat{R}_1 = 90°$ (rad to midpoint chord).

$x^2 = \sqrt{1,2^2 - 0,9^2}$ (Pythag)

$= 0,79\text{m}$

∴ milk depth $= 1,2 + 0,79$ m

$= 1,99\text{m}$.

[4]

SECTION C [33]

NAME:_____

MATHS TEACHER:_____

QUESTION 12

Simplify without the use of a calculator:

a) $\frac{\sin(-10^\circ)}{\cos(-80^\circ)}$ (3)

$\frac{-\sin 10^\circ}{\cos 80^\circ}$

$= -1$ →

$\frac{\sin(350^\circ)}{\cos(280^\circ)} = \frac{-\sin 10^\circ}{\cos 80^\circ} = -1$

b) $\frac{\cos(180^\circ - p)\sin(p - 90^\circ) - 1}{\tan^2(540^\circ + p)\sin(90^\circ + p)\cos(-p)}$ (7)

$\frac{-\cos p(-\cos p) - 1}{\tan^2 p(\cos p)(\cos p)}$

$= \frac{\cos^2 p - 1}{\frac{\sin^2 p}{\cos^2 p} \times \cos^2 p}$

$= \frac{\cos^2 p - 1}{\sin^2 p}$ / Factorise numerator as $(\cos p + 1)(\cos p - 1)$

$= -\frac{(1 - \cos^2 p)}{\sin^2 p}$

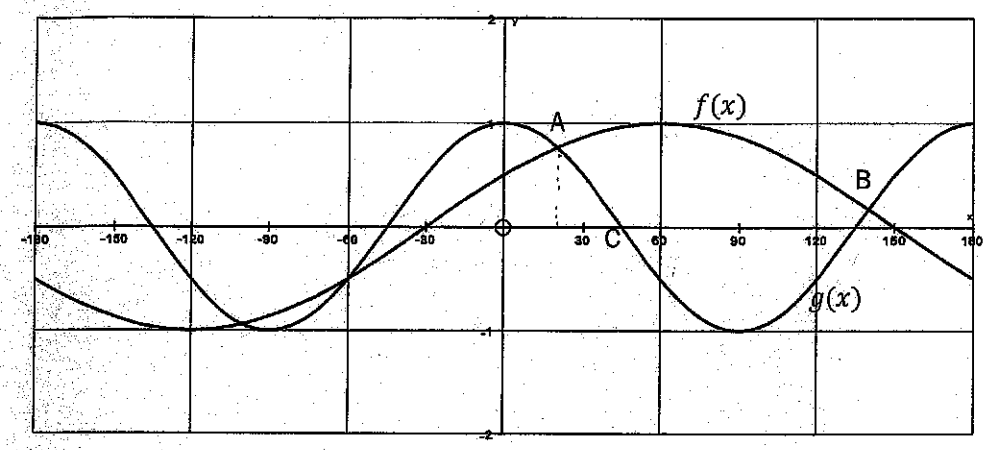
$= -\frac{\sin^2 p}{\sin^2 p}$

$= -1$ → [10]

Question 13

The graphs below represent the functions $g(x) = cos2x$ and $f(x) = \sin(x + 30^\circ)$.

B has co-ordinates $(140^\circ ; 0{,}17)$.



a) Give the period of $f(x)$ (1)

360° ✓aa

b) Give the co-ordinates of C. (1)

$(45^\circ ; 0)$ ✓aa

c) For which values of x are $f(x)$ and $g(x)$ both increasing? (2)

$x \in (-90^\circ ; 0)$ ✓values ✓notation.

d) i) Using your calculator, find $g(20^\circ)$ $0{,}766 \approx 0{,}77$ ✓aa (1)

ii) Using your calculator, find $f(20^\circ)$ $0{,}766 \approx 0{,}77$. ✓aa. (1)

e) For which values of x is $g(x) < f(x)$ for $x \in [0^\circ; 180^\circ]$ (2)

✓values ✓notation.

use A, B and their values. $x \in (20^\circ ; 140^\circ)$

then zero if don't get $g(20^\circ) = f(20^\circ)$ accept 15° to 25° or lower bound

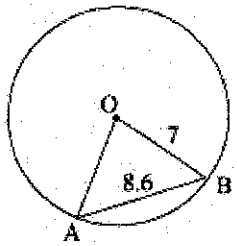
f) If $f(x)$ is translated 30° to the left to make $h(x)$, give the equation of $h(x)$ (2)

$h(x) = \sin(x + 60^\circ)$ ✓✓aa.

[10]

QUESTION 14

a) AB is a chord of a circle, centre O and radius 7cm. If AB = 8,6 cm, calculate $A\hat{O}B$ (3)



$8,6^2 = 7^2 + 7^2 - 2(7)(7)\cos A\hat{O}B$ ✓m Cos Rule. ± sgt.

$0,245306\ldots = \cos A\hat{O}B$ ✓m

$\hat{O} = 75,8°$ ✓ans.

b) Determine the general solution of $sin^2\theta - 3sin\theta = -2$ (5)

$\sin^2\theta - 3\sin\theta + 2 = 0$ ✓

$(\sin\theta - 2)(\sin\theta - 1) = 0$ ✓m

$\sin\theta = 2 \qquad \sin\theta = 1$ ✓

X $\qquad \theta = 90° + k360°, \quad k \in Z$ ✓

✓m for gen solution excluding one 2 option.

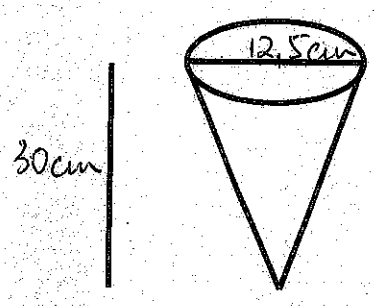
must be via legitimate means

[8]

QUESTION 15

a) The circular opening at the top of a rain-gauge has an internal diameter of 12,5 cm. The rain-gauge has a perpendicular depth of 30cm. Show that the volume of water it can hold before it overflows is 1227 cm^3 to the nearest whole number. (2)

Possible formulae $V = \pi r^2 H$ $V = \frac{1}{3}\pi r^2 H$ $V = \frac{4}{3}\pi r^3$

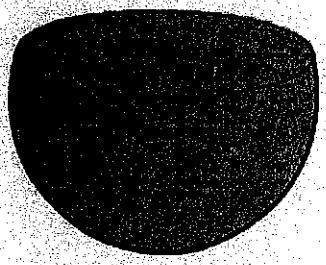


$V = \frac{1}{3}\pi(6{,}25)^2(30)$ ✓

$= 1227{,}184\ldots$

$\approx 1227\ cm^3$.

b) A hemispheric container also collects water during the rain. What is the minimum radius that the hemisphere must have to collect the same amount of water? (3)



$1227 = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$ ✓m $\frac{1}{2}$ of sphere formula.

✓a correct form.

$\frac{3}{2} \times \frac{1227}{\pi} = r^3$

$\sqrt[3]{\quad} = r$ [5]

$8{,}367\ldots = r$

$8{,}37\ cm = r$ ✓aa

accept $8{,}38$.

if used $1227 = \frac{1}{2}\left(\frac{3}{4}\pi r^3\right)$ to get $r = 10{,}14$ get 2/3.

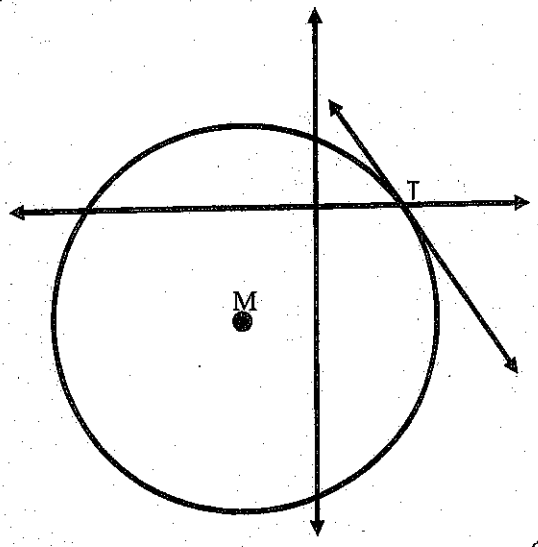
SECTION D [30]

NAME:_____

MATHS TEACHER:_____

QUESTION 16

In the diagram below, the circle centre M is drawn with a tangent at T $(x; 0)$.
The equation of the circle is $x^2 + 8x + y^2 + 16y = 20$



a) Show that M has co-ordinates $(-4; -8)$ (5)

$x^2 + 8x + 16 + y^2 + 16y + 64 = 20 + 16 + 64$ ✓m balancing ✓ca

$(x+4)^2 + (y+8)^2 = 100$ ✓a ✓a

$\therefore$ centre M $(-4; -8)$

b) Give the radius of the circle (1)

10 ✓ca

c) Find the equation of the tangent to the circle at T $(x; 0)$. (6)

T $(x; 0)$ ✓m sub $y=0$ $x^2 + 8x = 20$

$x^2 + 8x - 20 = 0$

$(x+10)(x-2) = 0$

$x = -10$ / $x = 2$. ✓

$M_{Tm} = \frac{-8-0}{-4-2} = \frac{-8}{-6} = \frac{4}{3}$ ✓ca $\therefore$ tangent $m = -\frac{3}{4}$ ✓m

$y = -\frac{3}{4}x + c$

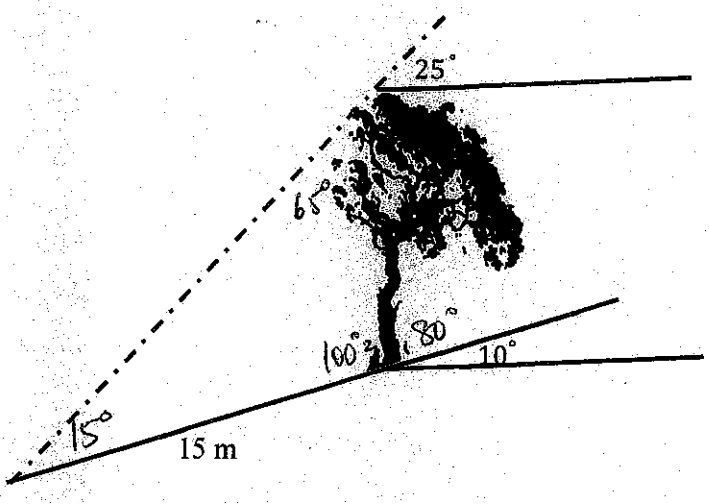
(2; 0) $0 = -\frac{3}{2} + c$ ✓ m sub into eq.

$\frac{3}{2} = c$

$y = -\frac{3}{4}x + \frac{3}{2}$ ✓ca. [12]

QUESTION 17

A tree grows vertically on the side of a hill. The hill has a constant upward slope of 10°. When the angle of elevation of the sun is 25°, the shadow of the tree falls 15m down the hill. How tall is the tree?



$25^\circ - 10^\circ = 15^\circ$ (alt ∠s betw. ∠'s). ✓a

$T_1 = 80^\circ$ (

$T_2 = 100^\circ$ (adj ∠s str line) ✓a

Top ∠ of $\Delta = 65^\circ$ ✓a

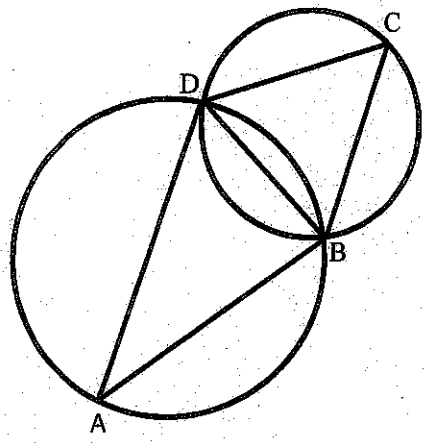
$\frac{15m}{\sin 65^\circ} = \frac{Tree}{\sin 15^\circ}$ ✓use Sine Rule.

$4,28m = Tree$ ✓ca.

[5]

QUESTION 18

In the two circles given, BD is a common chord, the circles can be any size. What condition must be met for $\hat{A} = \hat{C}$?

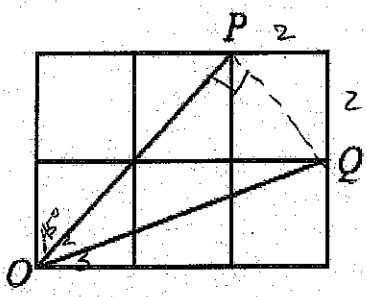


circles must be same size. ✓✓

[2]

QUESTION 19

In the diagram, the six small squares all have side length 2. Lines are drawn from O to P and O to Q. Find $P\hat{O}Q$.



45° ✓

$\hat{Q}_3$ √a. 18,43° ✓Tan m.

$\hat{O}_2$ √a.

$PQ = \sqrt{8}$ (Pythag) ✓

$PO = 2\sqrt{8}$ ✓

$P_1 + P_3 = 90°$ (two diags, two squares meet). ✓

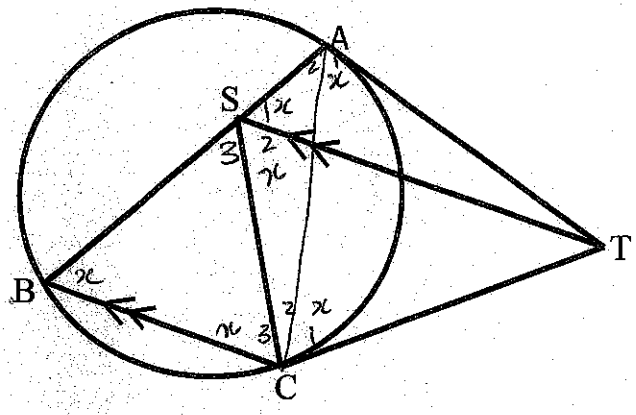
$\text{Tan } P\hat{O}Q = \frac{\sqrt{8}}{2\sqrt{8}} < \frac{1}{2}$ ✓m. $\therefore P\hat{O}Q = 26,57°$ ✓a

Cos Rule.

[4]

QUESTION 20

TA and TC are tangents. ST // BC.



Prove that :

a) ASCT is a cyclic quad (4)

Draw in AC. ✓

$\hat{S}_1 = \hat{B} = x$ (corresp ∠'s, ST // BC) ✓

$\hat{C}_1 = \hat{B} = x$ (tan-chord thm). ✓

$\therefore \hat{S}_1 = \hat{C}_1 = x$

$\therefore$ ASCT cyclic quad (conv. ∠'s in same seg) ✓

b) ΔSBC is isosceles (3)

$\hat{B} = \hat{A} = x$ ✓ (tan chord OR $\hat{C}_1 = \hat{A} = x$ tang-from ext pt)

$\hat{A} = \hat{S}_2 = x$ ✓ (∠'s in same seg)

$\hat{S}_2 = \hat{C}_3 = x$ ✓ (alt ∠'s ST // BC)

$\therefore$ Δ SBC isos (= angles). [7]