ST STITHIANS GIRLS COLLEGE

GRADE 11
CORE MATHEMATICS: PAPER 2

November 2015

TIME: 3 hours

MARKS: 150

EXAMINER: Mr Schaerer

MODERATOR: Ms du Plessis

NAME:	______________________________		
TEACHER:	**Mr Ancillotti/Mrs van Rooy** ☐	**Ms du Plessis** ☐	
	Mrs Kaur ☐	**Mr Schaerer** ☐	

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1 This question paper consists of 14 questions and 24 pages, including the cover pages.

2. Read all of the questions carefully.

3. You may use an approved non-programmable and non-graphical scientific calculator unless stated otherwise.

4. Round off all of your answers to **two** decimal places where necessary unless stated otherwise.

6. All of the necessary working and details must be clearly shown.

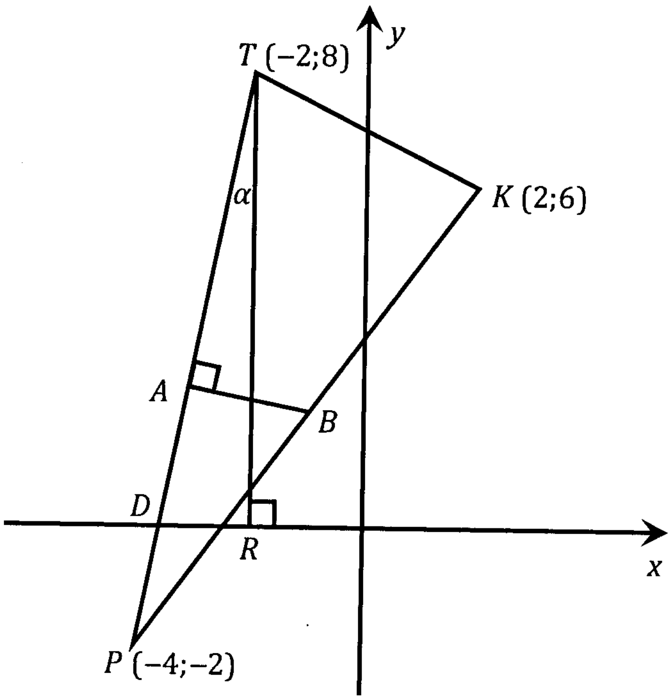
7. It is in your own interest to write legibly and to present your work neatly. Remember to check all of your answers.

ENSURE YOU CALCULATOR IS IN DEGREE MODE

PLEASE WRITE YOUR NAME ON THE BACK PAGE.

QUESTION: 1

$T\,(-2;8)$; $K\,(2;6)$ and $P\,(-4;-2)$ are the vertices of ΔTKP. $AB \perp TP$ and $TR \perp$ *x-axis.* $P\hat{T}R = \alpha$.



(a) Determine the co-ordinates of B, the midpoint of PK.

$B(x;y) = \left(\frac{2-4}{2} ; \frac{6-2}{2}\right)$

$= (-1;2)$

(2)

(b) If the equation of TP is $y-5x=18$, determine the equation of AB in the form of $y=mx+c$.

$y = 5x + 18$

$m_{AB} = -\frac{1}{5}$ through $(-1;2)$

$y-y_1 = m(x-x_1)$ OR $y = mx + c$

$y-2 = -\frac{1}{5}(x+1)$ $\quad 2 = (-\frac{1}{5})(-1) + c$

$y = -\frac{1}{5}x + \frac{9}{5}$ $\quad c = 9/5$

$\therefore y = -\frac{1}{5}x + 9/5$

(3)

(c) Determine the value of α.

$m_{TP} = 5$

$\therefore \tan T\hat{D}R = 5$

$\therefore T\hat{D}R = 78{,}69^\circ$

$\therefore \alpha = 90^\circ - 78{,}69^\circ$ ext $\angle \triangle$

$\alpha = 11{,}31^\circ$

(3)

(d) Given $N\,(n;5)$. Determine the value of n such that T, K and N are collinear.

$m_{TK} = \frac{8-6}{-2-2} = -\frac{1}{2}$

$\therefore m_{KN} = \frac{6-5}{2-n} = \frac{1}{2-n}$

$\therefore -\frac{1}{2} = \frac{1}{2-n}$

$-2+n = 2$

$n = 4$

(4)

(e) If $TKPQ$ is a paralellogram, determine the coordinates of point Q.

$Q(-8;0)$

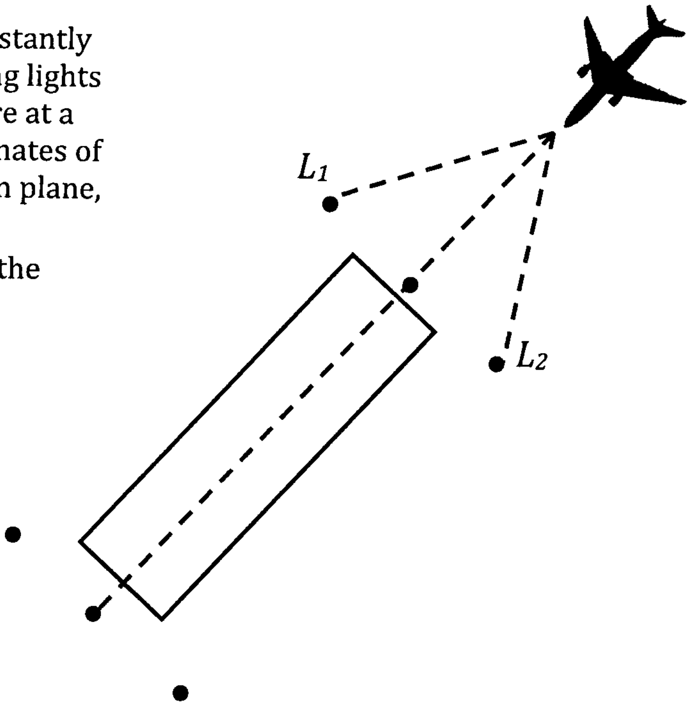
(2)

[14]

QUESTION: 2

The pilot of a plane coming in to land has to make sure that his plane is constantly equidistant from the two outer landing lights L_1 and L_2. The line of landing lights are at a right angle to the runway. The coordinates of L_1 and L_2, when plotted on a Cartesian plane, are (16;30) and (20;25) respectively.

Find the equation of his flight path in the form $ax+by+c=0$



$m = \frac{30-25}{16-20} = -\frac{5}{4}$ → m of landing lights

$m_\perp = \frac{4}{5}$

Midpoint $= \left(\frac{16+20}{2} ; \frac{30+25}{2}\right)$

$= (18 ; 27\frac{1}{2})$

$y - y_1 = m(x - x_1)$ OR $y = mx + c$

$y - 27\frac{1}{2} = \frac{4}{5}(x - 18)$ $27\frac{1}{2} = (\frac{4}{5})(18) + c$

$y = \frac{4}{5}x + \frac{131}{10}$ $\therefore c = {}^{131}/_{10}$

$\therefore 10y - 8x - 131 = 0$ $y = {}^{4}/_{5}x + {}^{131}/_{10}$

$\therefore 10y - 8x - 131 = 0$

[5]

QUESTION: 3

(a) Prove that $\frac{-\tan x}{\cos x} + \frac{1}{\sin x \cos^2 x} = \frac{1}{\sin x}$

LHS: $\frac{1}{\sin x \cos^2 x} - \frac{\tan x}{\cos x}$

$= \frac{1}{\sin x \cos^2 x} - \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$

$= \frac{1 - \sin^2 x}{\sin x \cos^2 x}$

$= \frac{\cos^2 x}{\sin x \cos^2 x}$

$= \frac{1}{\sin x}$

$=$ RHS

(5)

(b) Simplify:

$\frac{\sin(180^\circ + x).\sin(x - 90^\circ)}{\tan(180^\circ + x).\cos^2(180^\circ - x)} = \frac{(-\sin x)(-\cos x)}{(\tan x)(-\cos x)^2}$

$= \frac{(-\sin x)(-\cos x)}{(\tan x)(\cos^2 x)}$

$= \frac{\sin x}{\frac{\sin x}{\cos x} \cdot \frac{\cos x}{1}}$

$= \frac{\sin x}{\sin x}$

$= 1$

(5)

(c) Determine without the use of a calculator: $\dfrac{\cos 150^\circ . \sin 20^\circ . \tan 225^\circ}{\cos 110^\circ . \sin 300^\circ}$

$= \dfrac{(-\cos 30^\circ)(\sin 20^\circ)(\tan 45^\circ)}{(-\sin 20^\circ)(-\sin 60^\circ)}$

OR $-\cos 70^\circ$

$= -\tan 45^\circ$

$= -1$

(4)

(d) Solve for x: $\sin^2 x + 5\sin x - 2\cos^2 x = 0 \quad x \in [0^\circ; 90^\circ]$

$\sin^2 x + 5\sin x - 2(1 - \sin^2 x) = 0$

$3\sin^2 x + 5\sin x - 2 = 0$

$(3\sin x - 1)(\sin x + 2) = 0$

$\sin x = 1/3$ or $\sin x = -2$

$\therefore x = 19,47^\circ + k.360^\circ$ No solution.

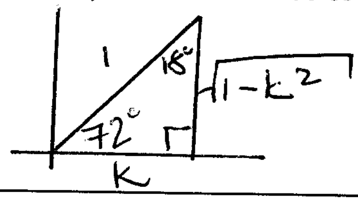
OR $x = 160,53^\circ + k.360^\circ$

$k \in \mathbb{Z}$

$x = \{19,47^\circ\}$

(6)

(e) Given $\cos 72^{\circ} = k$. Determine the following in terms of k, without the use of a calculator:



(1) $\sin 18^{\circ}$

$= k$

(2)

(2) $\tan 162^{\circ}$

$= \tan(180^{\circ} - 18^{\circ})$

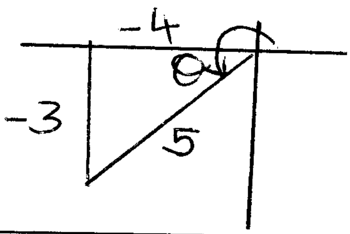
$= -\tan 18^{\circ}$

$= \dfrac{-k}{\sqrt{1-k^2}}$

(3)

(f) Determine, without the use of a calculator, the value of:

$4\cos\theta + 2\sin\theta$, if $\dfrac{3}{\tan\theta} = 4$ and $\theta \in [180^{\circ};360^{\circ}]$



$3 = 4\tan\theta$

$\therefore \tan\theta = 3/4$

$\therefore 4\cos\theta + 2\sin\theta$

$= 4(-4/5) + 2(-3/5)$

$= -16/5 - 6/5$

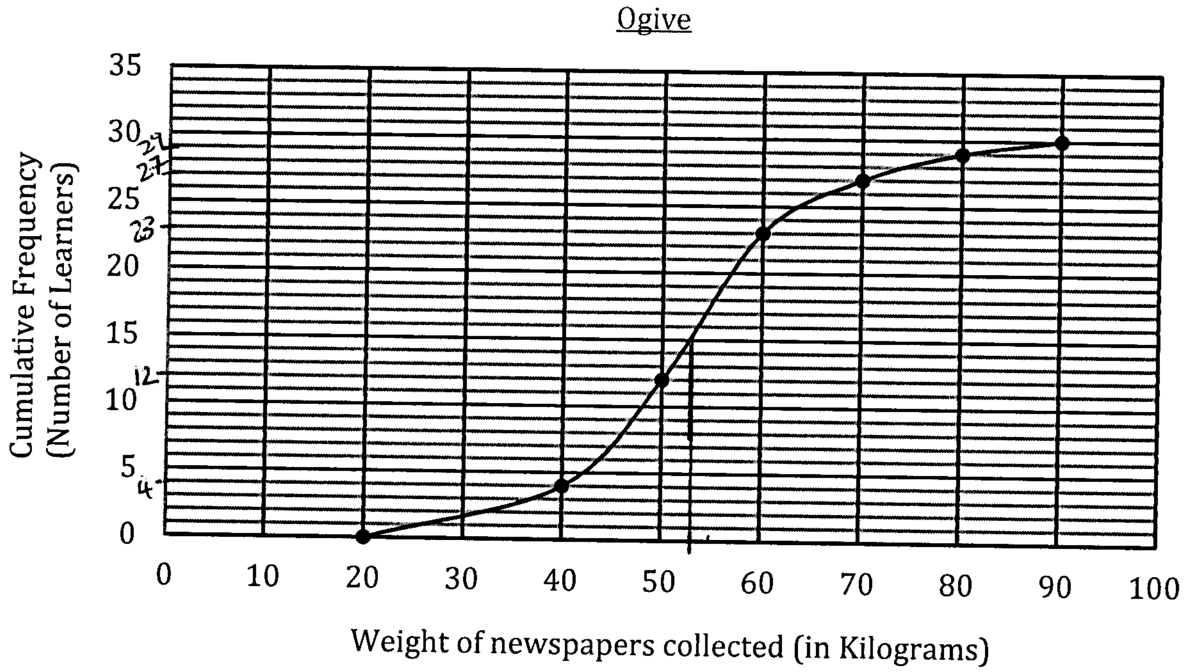
$= -22/5$

(6)

[31]

QUESTION: 4

As part of an environmental awareness initiative, learners were requested to collect newspapers for recycling. The Ogive below shows the total weight of the newspapers (in kilograms) collected over a period of 6 months by learners.



(a) Determine the modal class of the weight of the newspapers collected.

50 – 60 kgs

(1)

(b) Determine the median weight of newspapers collected by the learners.

53 kg

(1)

(c) How many learners collected 60*kgs* or more of newspapers?

7 learners

(2)

[4]

QUESTION: 5

The resting pulse rate of 10 runners are listed below:
45; 50; 55; 63; 72; 60; 78; 57; 61; 53

(a) Determine the mean pulse rate.

$\bar{x} = 59,4$

(2)

(b) Determine the standard deviation.

$\sigma = 9,39$

(2)

(c) How many runner's pulse rates lie within one standard deviation of the mean?

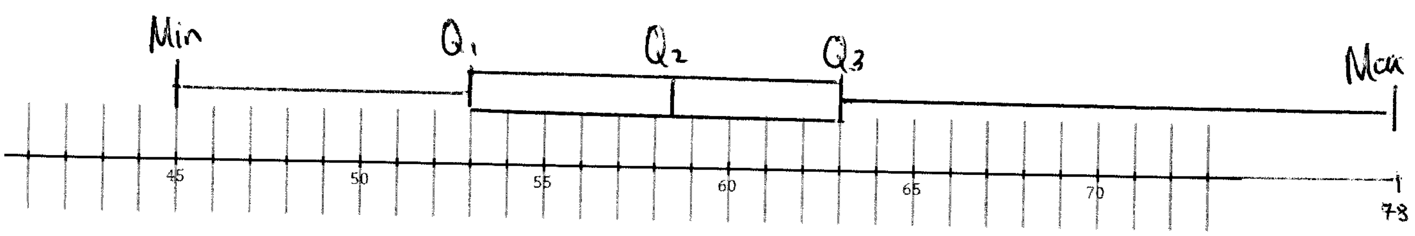
50,01 – 68,39
6 learners

(1)

(d) Draw the box and whisker plot of the data above, showing all calculations.

Min = 45
Q_1 = 53
Q_2 = 58,5
Q_3 63
Max = 78

45; 50; 53; 55; 57; 60; 61; 63; 72; 78

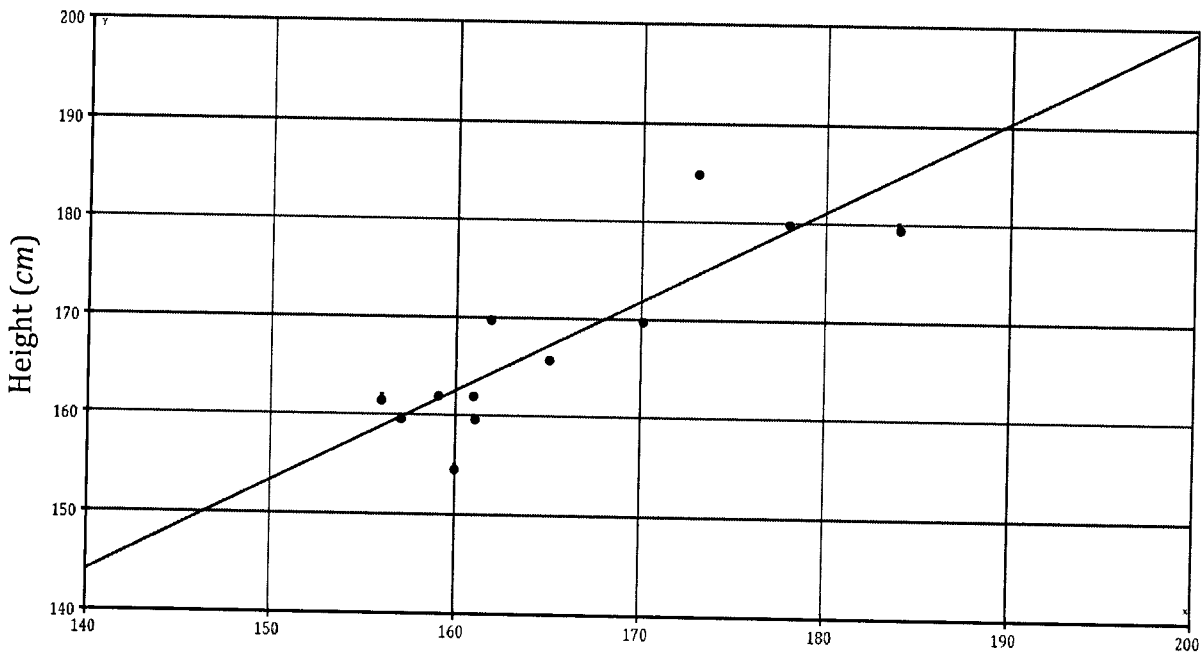


(3)
[8]

QUESTION: 6

Measurements were collected from 12 people to determine whether people with long arms are taller than people with short arms.

Candidate	Arm Span (cm)	Height (cm)
A	156	162
B	157	160
C	159	162
D	160	155
E	161	160
F	161	162
G	162	170
H	165	166
I	170	170
J	173	185
K	178	180
L	184	180



(a) Use your calculator to determine:

(1) The equation of the least squares regression line, in the form $y = A + Bx$. Give your answers correct to 4 decimal digits.

$y = 14,0566 + 0,9282x$ (3)

(2) The correlation coefficient, and describe what this tells us.

$r = 0,87$

Strong, positive correlation

(3)

(b) Marie's arm span is measured to be 138 cm.

(1) What would her estimated height be?

$y = 14,0566 + 0,9282(138)$

$y = 142,15$

(2)

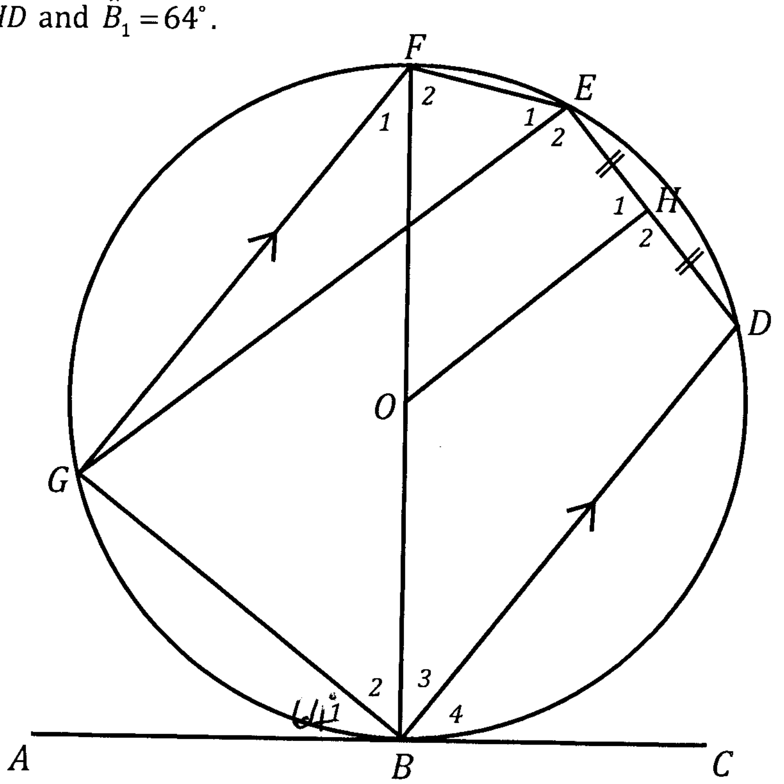
(2) Is this an accurate estimation of her height? Explain.

No. Extrapolation.
The data value falls outside the given data set

(2)
[10]

QUESTION: 7

In the diagram below, ABC is a tangent to the circle with centre O. FOB is a diameter. $GF \parallel BD$, $EH = HD$ and $\hat{B}_1 = 64^\circ$.



Calculate (with reasons) the sizes of the following angles:

(a) $\hat{H}_1$ 90° line from centre (2)

(b) $\hat{F}_1$ 64° tan chord (2)

(c) $\hat{B}_2$ 26° diam ⊥ tan (2)

(d) $\hat{E}_1$ 26° ∠s in same segment (2)

(e) $\hat{B}_3$ 64° alt ∠s GF // BD (2)

(f) $\hat{E}_2$ 90° opp ∠s cyclic quad (2)

(g) Show that $GE \parallel OH$

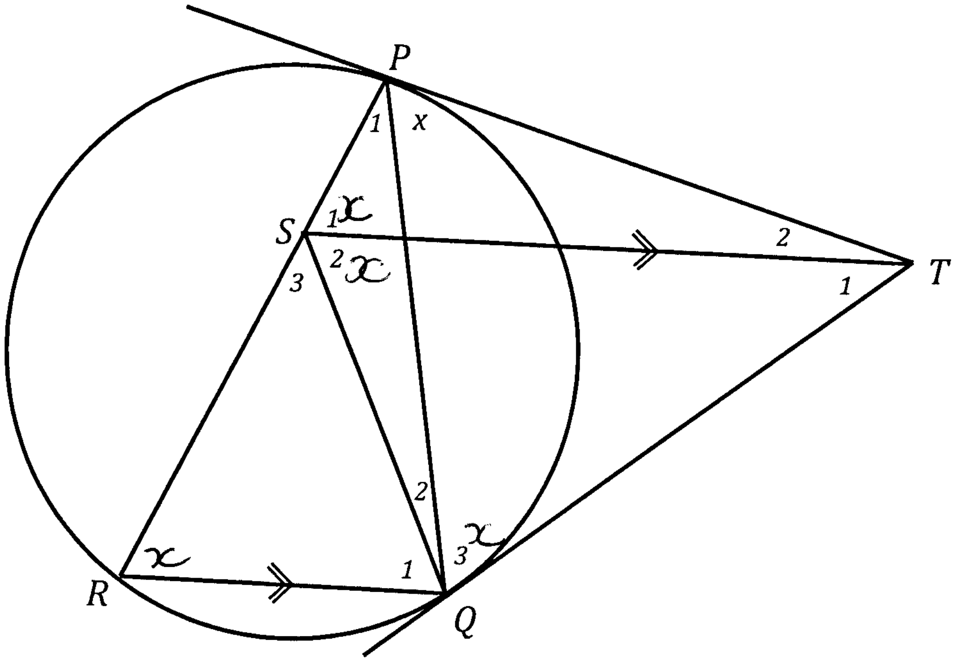
$\hat{E}_2 + \hat{H}_1 = 180°$

∴ GE // OH converse co-int ∠s 180° (2)

[14]

QUESTION: 8

In the figure TP and TQ are tangents to the circle PQR, $T\hat{P}Q = x$ and $RQ // ST$



(a) Name, with reasons, 3 other angles equal to x.

$\hat{Q}_3 = T\hat{P}Q = x$ isos Δ

$x = \hat{R}$ tan chord

$x = \hat{S}_1$ corr $\angle^s$ $ST // RQ$

(6)

(b) Prove that $TPSQ$ is a cyclic quadrilateral.

$\hat{Q}_3 = x = \hat{S}_1$ proven above

$\therefore$ TPSQ a cyclic quad converse $\angle^s$ in same segment

(2)

(c) Hence or otherwise prove $P\hat{S}Q$ is bisected by TS.

$\hat{S}_2 = x$ $\angle^s$ in same segment

$\therefore \hat{S}_1 = \hat{S}_2 = x$

(2)

(d) Hence or otherwise prove ΔRQS is an isosceles triangle.

$\hat{Q}_1 = \hat{S}_2 = x$ alt $\angle^s$ $ST \parallel RQ$

$\therefore \hat{Q}_1 = x = \hat{R}$

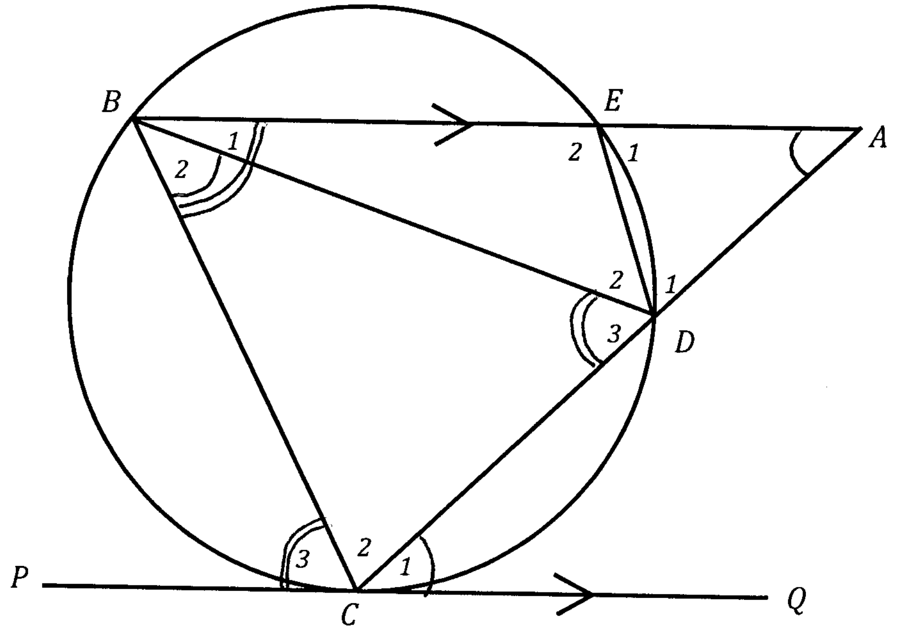
$\therefore \Delta RQS$ isos Δ base $\angle^s$ equal

(2)

[12]

QUESTION: 9

PQ is a tangent to the circle at C. AEB and ADC are straight lines. $PQ // AB$.



Prove, with reasons:

(a) $\hat{A} = \hat{B}_2$

$\hat{C}_1 = \hat{B}_2$ tan chord

$\hat{C}_1 = \hat{A}$ alt $\angle^s$ AB//PQ

$\therefore \hat{A} = \hat{B}_2$

(4)

(b) $\hat{B}_1 + \hat{B}_2 = \hat{D}_3$

$\hat{C}_3 = \hat{D}_3$ tan chord

but $\hat{C}_3 = \hat{B}_1 + \hat{B}_2$ alt $\angle^s$ PQ//AB

$\therefore \hat{B}_1 + \hat{B}_2 = \hat{D}_3$

(4)

(c) $\hat{D}_3 = \hat{D}_1$

$\hat{D}_1 = \hat{B}_1 + \hat{B}_2$ ext ∠ cyclic quad

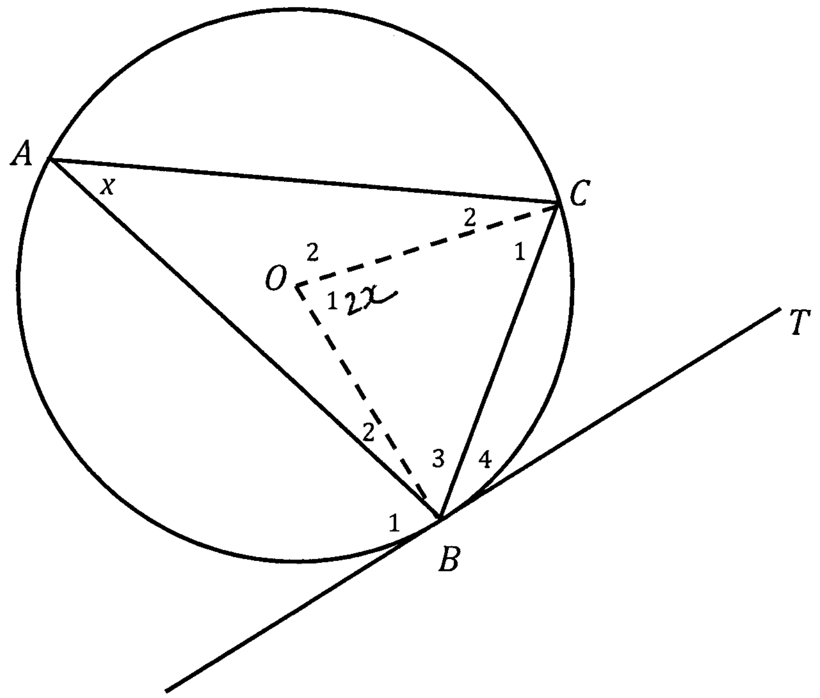
$\therefore \hat{D}_1 = \hat{D}_3$

(2)

[10]

QUESTION: 10

In the diagram shown, BT is a tangent to the circle at B. O is the centre, and A and C are points on the circle. Let $\hat{A} = x$



Doris is required to prove $\hat{A} = \hat{B}_4$. However, she has never been taught the tan-chord theorem. She does know all the other theorems. Show how Doris can complete the proof, without making use of the tan-chord theorem, using the given diagram.

$\hat{O}_1 = 2x$ ∠ @ cntr 2x ∠ @ circ

$\hat{B}_3 = \hat{C}_1 = \dfrac{180° - 2x}{2}$ isos △

$\therefore \hat{B}_3 = \hat{C}_1 = 90° - x$

$\hat{B}_3 + \hat{B}_4 = 90°$ rad ⊥ tan

$\therefore \hat{B}_4 = 90° - \hat{B}_3$

$= 90° - (90° - x)$

$\therefore \hat{B}_4 = x$

$\therefore \hat{A} = \hat{B}_4 = x$

$\therefore$ BT a tangent to circle at B.

[6]

QUESTION: 11

$SA_{sphere} = 4\pi r^2$ $Vol_{sphere} = \frac{4}{3}\pi r^3$

Figure 1 *Figure 2*

(a) A hemisphere has a total volume of $1000cm^3$, as shown in *figure 1*. What is the radius of the hemisphere?

$1000 = \frac{1}{2}(\frac{4}{3}\pi r^3)$

$\therefore r^3 = 477,46...$

$\therefore r = 7,82$ cm

(3)

(b) A smaller hemisphere is scooped out from the larger hemisphere to create a container, shown in *figure 2*. The diameter of the larger hemisphere is twice that of the smaller hemisphere.

(1) Find the area of the flat surface of the top of the container.

$\therefore$ radius smaller ⊙ = 3,91 cm

Area flat surface = $\pi(7,82)^2 - \pi(3,91)^2$

$= 144,09$ cm^2

(3)

(2) What volume can the container hold?

$V_{figure\ 2} = \frac{1}{2}(\frac{4}{3} \cdot \pi \cdot 3,91^3)$

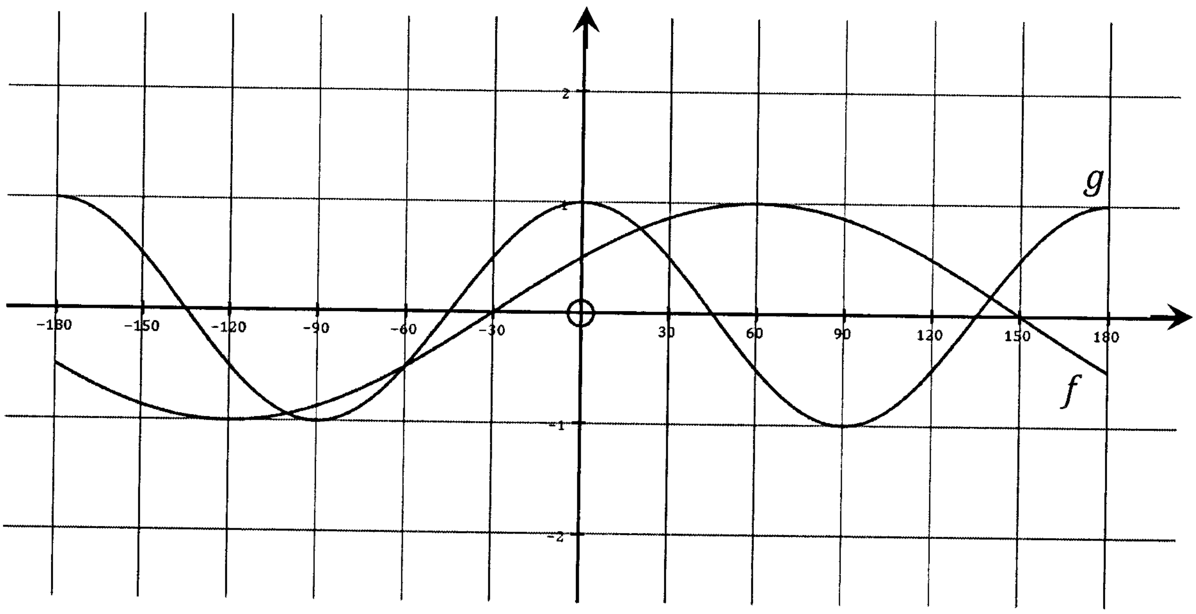
$= 125,20$

(2)

[8]

QUESTION: 12

In the diagram of $f(x) = \sin(x - a)$ and $g(x) = \cos bx$ for $x \in [-180^\circ; 180^\circ]$ are drawn.



(a) Determine the values of a and b.

$a = -30^\circ$

$b = 2$

(2)

(b) Give the period of g.

180°

(1)

(c) Give the amplitude of $3f(x)$.

3

(1)

(d) Solve for x, if $\sin(x+30^\circ)=\cos 2x \quad x\in[-180^\circ;0^\circ]$

$\sin(x+30^\circ) = \cos 2x$

$\cos[90^\circ-(x+30^\circ)] = \cos 2x$ OR $\sin(x+30^\circ) = \sin(90^\circ-2x)$

$\cos(60^\circ-x) = \cos 2x$ $\quad x+30^\circ = 90^\circ-2x+k.360^\circ$

$60^\circ-x = \pm 2x + k.360^\circ$ $\quad 3x = 60^\circ + k.360^\circ$

(+) $-3x = -60^\circ + k.360^\circ$ $\quad x = 20^\circ + k.120^\circ$

$\therefore x = 20^\circ + k.120^\circ$ OR $x+30^\circ = 90^\circ+2x+k.360^\circ$

OR (−) $x = -60^\circ + k.360^\circ$ $\quad \therefore x = -60^\circ + k.360^\circ$

$k\in\mathbb{Z}$

$\therefore x = \{-100^\circ; -60^\circ\}$

(7)

(e) For which values of x will:

$f(x) > g(x)$ for $x\in[-180^\circ;0^\circ]$

$-100 < x < -60$

OR $x\in(-100^\circ;-60^\circ)$

2 (~~4~~)

(f) Give the new equation if f is shifted 20° to the left.

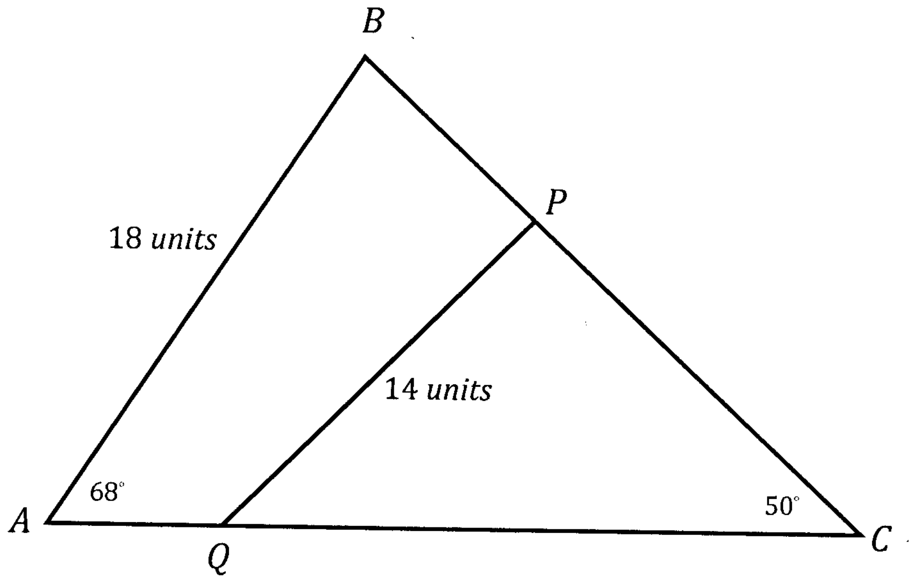
$f(x) = \sin(x+50^\circ)$

(1)

[~~16~~]
14

QUESTION: 13

In the diagram below, an acute-angled triangle ABC is drawn. A line PQ is drawn, where P lies on the line BC and Q lies on the line AC. The length of PQ is 14 *units* and the length of AB is 18 *units*. $\hat{A} = 68°$ and $\hat{C} = 50°$.



(a) Show that BC is 21,79 *units*.

$$\frac{BC}{\sin 68°} = \frac{18}{\sin 50°}$$

$$\therefore BC = \frac{18 \sin 68°}{\sin 50°}$$

$\therefore BC = 21,79$ units

3 (~~2~~)

(b) If the ratio of $BP:PC$ is 2:3, determine the size of $P\hat{Q}C$.

$21,79 \div 5 \times 3 = 13,07 \rightarrow PC$

$$\therefore \frac{\sin \hat{Q}}{13,07} = \frac{\sin 50°}{14}$$

$$\therefore \sin \hat{Q} = \frac{13,07 \sin 50°}{14}$$

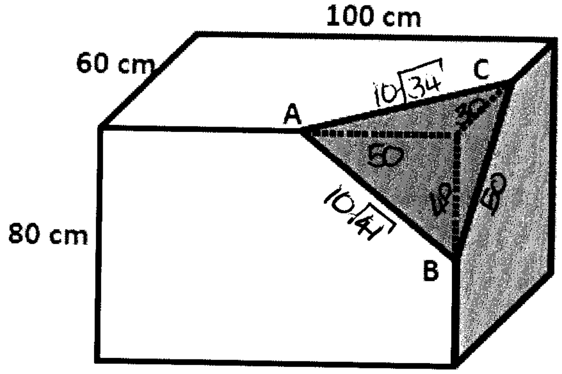
$\therefore \hat{Q} = 45,66°$

4 (~~2~~)

[~~5~~] 7

QUESTION: 14

A rectangular block of wood measures 80 cm × 100 cm × 60cm.
One corner is cut away from the block, in such a way that three of the edges are cut through their midpoints A, B and C.



Determine the area of the triangular face ABC created by the cut. Give your answer correct to one decimal digit.

$BC = 50\,cm$ OR $AC = \sqrt{3400} = 10\sqrt{34} = 58,31\,cm$ OR $AB = \sqrt{4100} = 10\sqrt{41} = 64,03\,cm$

$BC^2 = AC^2 + AB^2 - 2(AC)(AB)\cos A$ $AC^2 = BC^2 + AB^2 - 2(BC)(AB)\cos B$ $AB^2 = AC^2 + BC^2 - 2(AC)(BC)\cos C$

$\cos A = \dfrac{50^2 - (10\sqrt{34})^2 - (10\sqrt{41})^2}{-2(10\sqrt{34})(10\sqrt{41})}$ $\cos B = \dfrac{(10\sqrt{34})^2 - 50^2 - (10\sqrt{41})^2}{-2(50)(10\sqrt{41})}$ $\cos C = \dfrac{(10\sqrt{41})^2 - (10\sqrt{34})^2 - 50^2}{-2(10\sqrt{34})(50)}$

$\hat{A} = 47,9646 \ldots \to A$ $\therefore \hat{B} = 60,016 \ldots \to B$ $\hat{C} = 72,0192 \ldots \to C$

$\text{Area}\triangle ABC = \frac{1}{2}(10\sqrt{34})(10\sqrt{41})\sin(A)$
$= 1386,54\,cm^2$

OR

$\text{Area}\triangle ABC = \frac{1}{2}(10\sqrt{41})(50)\sin(B)$
$= 1386,54\,cm^2$

OR

$\text{Area}\triangle ABC = \frac{1}{2}(10\sqrt{34})(50)\sin(C)$ **[7]**
$= 1386,54\,cm^2$