ST STITHIANS GIRLS' COLLEGE

GRADE 11 – November 2016
CORE MATHEMATICS: PAPER 2

TIME:	3 hours	MARKS:	150
EXAMINER:	Mrs Wilde	MODERATOR:	Mrs de Lange

NAME: MEMO

TEACHER: **Mrs Kaur** ☐ **Mrs Marais** ☐ **Mr Ancillotti** ☐ **Mr Statham** ☐

Question	AIM3	AIM4	AIM5	Total	
1		/14			
2			/9		
3	/7				
4	/6				
5		/6			
6		/15			
7		/6			
8			/8		
9			/9		
10		/20			
11	/15				
12	/7				
13	/8				
14		/12			
15		/8			
Total	/43	/81	/26	/150	%

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 28 pages and 15 questions and a green formula sheet.
2. Answer all the questions on the question paper.
3. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
4. Round off your answers to two decimal digits where necessary, unless instructed to do otherwise.
5. All the necessary working detail must be clearly shown and reasons stated.
6. Diagrams are not drawn to scale.
7. All Euclidean Geometry requires reasoning to be shown.
8. It is in your own interests to write legibly and to present your work neatly.

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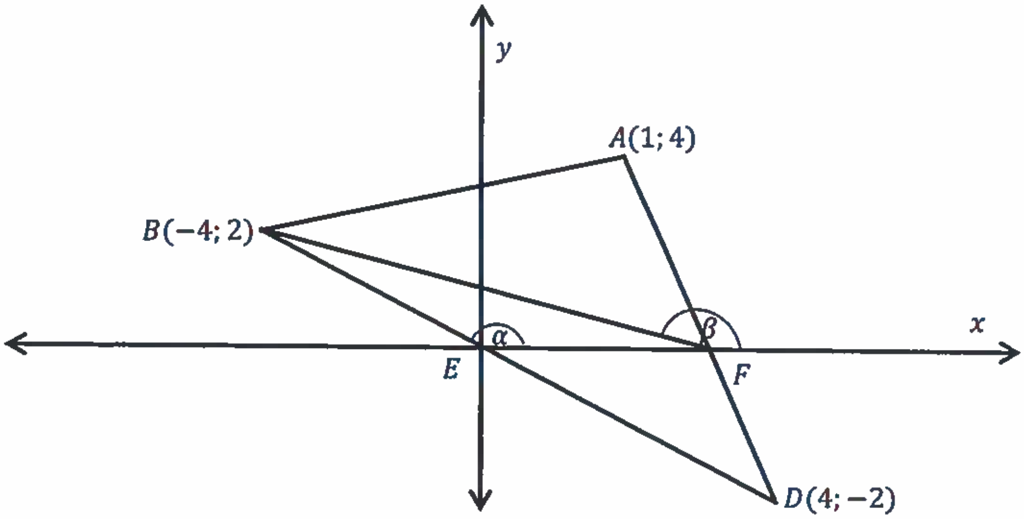
SECTION A:

Question 1: **14 Marks**

$A(1;4)$, $B(-4;2)$ and $D(4;-2)$ are the coordinates of the vertices of ΔABD . Line BF is drawn and the equation of AD is $y = -2x + 6$.



a. Determine the coordinates of point F. (2)

$0 = -2x + 6$

$2x = 6$ ✓ ✓

$x = 3$ $F(3;0)$

b. Calculate the gradient of BF. (2)

$m = \frac{0-2}{3+4}$ ✓

$= -\frac{2}{7}$ ✓

c. Calculate the size of β, the angle BF makes with the positive x-axis. (2)

$\tan\beta = -\frac{2}{7}$ ✓

$\beta = 164{,}05°$ ✓

d. Calculate the area of ΔBEF. (4)

$A = \frac{1}{2} b \perp h$ ✓

$= \frac{1}{2}(3)(2)$ ✓

$= 3 \text{ units}^2$ ✓

e. If B is the midpoint of GA, determine the coordinates of G. (4)

$-4 = \frac{x+1}{2}$ ✓ $\quad 2 = \frac{y+4}{2}$ ✓

$-8-1 = x \quad 4-4 = y$

$x = -9 \quad 0 = y$

$G(-9;0)$

✓ ✓

Question 2: **9 Marks**

a. 12 race times, in seconds, by an Olympic freestyle swimmer competing for a place in a relay team are given:

62; 56; 59; 63; 59; 59; 61; 60; 58; 63; 56; 52

Calculate the standard deviation of the swimmer's times. (2)

$\sigma = 3{,}08$ ✓✓

b. A second competitor for the same event records his times as follows:

58; 56; 63; 54; 57; 66; 65; 53; 51; 63; 63; 59

If you were a selector of the relay team, which competitor would you choose? Motivate your answer using suitable calculations. (3)

$\sigma = 4{,}76$ ✓ $\bar{x}_{above} = 59$

$\bar{x} = 59$

✓ ✓

Swimmer 1 as times more consistent around the average

c. The heights of several plants (in cm) was measured at a certain stage after planting, and the following data was recorded:

x = days after planting	14	20	8	15	18	11	14
h = height (cm)	6	11	3	8	10	4	

The record of the last height has been lost, but we do know that the regression line had equation $h = 0{,}72x - 3{,}31$.

~~2.1~~ Estimate, to the nearest centimetre, what the last recorded height was. (1)

7cm ✓

~~2.2~~ Calculate the correlation coefficient for the data relating to the first 6 plants correct to four decimal places. (i.e. ignoring the last column). (1)

$r = 0{,}9839$ ✓

~~2.3~~ Sometime later another plant's height 25 days after planting was found to be 20 cm. Comment on how surprising (or not) this is in the light of your previous results. (2)

$h = 0{,}72(25) - 3{,}31$ ✓ ✓

$= 14{,}69$ cm expected, could an outlier $\therefore$ surprising

Not surprising - any valid reason (rain, fertilizer etc.)

Question 3: **7 Marks**

For each expression in Column 1 choose an equivalent expression from Column 2. Give the correct letter from column 2. (You may use a letter more than once).

Column 1	Column 2
i) $\tan x$	A. $\cos x$
ii) $\cos(-x)$	B. $-\sin x$
iii) 1	C. $-\cos x$
iv) $-\tan x . \cos x$	D. $\frac{\sin x}{\cos x}$
v) $\cos^2 x$	E. $-\tan x$
vi) $\sin(90° + x)$	F. $1 - \sin^2 x$
vii) $\tan(540° - x)$	G. $\sin^2 x + \cos^2 x$

i) D

ii) A

iii) G

iv) B

v) F

vi) A

vii) E

Question 4: **6 Marks**

Evaluate, **without** using a calculator:

$$\frac{\cos(90°+\theta) - \sin(\theta-360°) + 2\sin 90°}{\cos 360° + \sin(-\theta)}$$

$$= \frac{-\sin(\theta) - \sin(\theta) + 2}{1-\sin(\theta)}$$

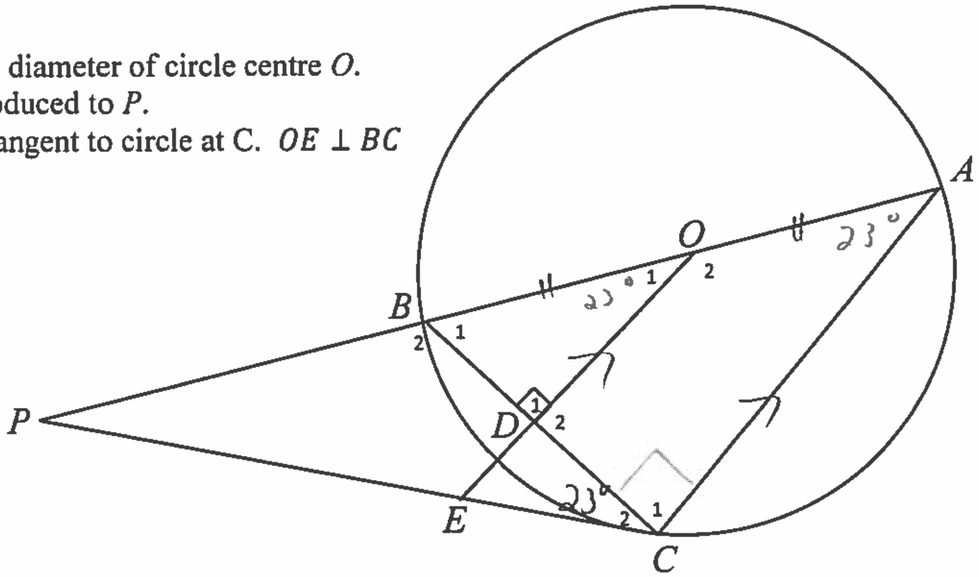
$$= \frac{2 - 2\sin(\theta)}{1-\sin(\theta)}$$

$$= \frac{2(1-\sin(\theta))}{1-\sin(\theta)}$$

$$= 2$$

Question 5: **6 Marks**

Given: AB is the diameter of circle centre O.
AB is produced to P.
PC is a tangent to circle at C. $OE \perp BC$



a. Prove that $OE \parallel CA$. (2)

$\hat{C}_1 = 90^\circ$ ∠ in semi circle ✓

$= \hat{D}_1$

$\therefore OE \parallel CA$ corresponding ∠'s = ✓

b. If $\hat{C}_2 = 23°$, name (with reasons) 2 other angles equal to $\hat{C}_2$. (2)

$P\hat{A}C$ tan-chord ✓

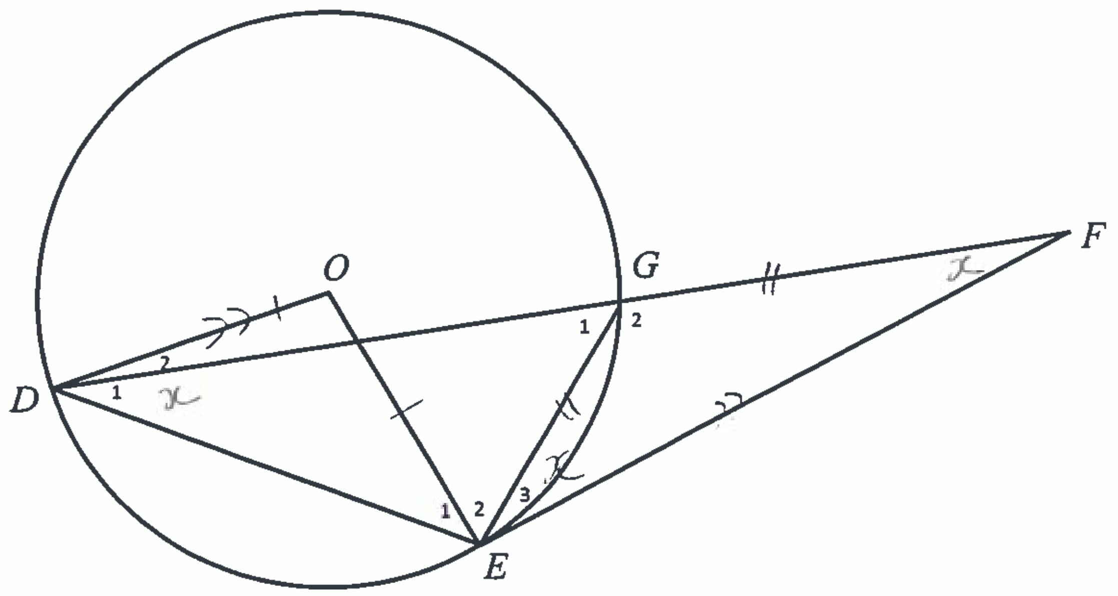
$\hat{O}_1$ corresponding ∠'s $OE \parallel CA$ ✓

c. Calculate the size of $\hat{P}$. (2)

$\hat{P} = 180^\circ - 90^\circ - 23^\circ - 23^\circ$

$= 44^\circ$ ✓ sum of △ PCA ✓

Question 6: **15 Marks**



Given: FE a tangent to circle centre O

D and F are joined so that $EG = GF$

a) If $\hat{E}_3 = x$ name, with reasons, 2 other angles equal to x. (4)

$\hat{D}_1 = x$ ✓ tan-chord ✓

$\hat{F} = x$ ✓ isos Δ ✓

b) Prove that $DE = EF$. (2)

$\hat{D}_1 = \hat{F} = x$ proven above ✓

$\therefore DE = EF$ Base $\angle$'s $=$ ✓

c) Express $D\hat{O}E$ in terms of x. (4)

$\hat{G}_1 = 2x$ ✓ ext ∠ of △ ✓

$D\hat{O}E = 4x$ ✓ ∠ at centre = 2× ∠ at circumf ✓

d) If it is given that $OD \parallel FE$, determine the value of x. (5)

$\hat{E}_{2+3} = 4x$ ✓ alt ∠'s $OD \parallel FE$ ✓

$\hat{E}_{2+3} = 90$ rad ⊥ tan ✓

$4x = 90$ ✓

$x = 22{,}5°$ ✓

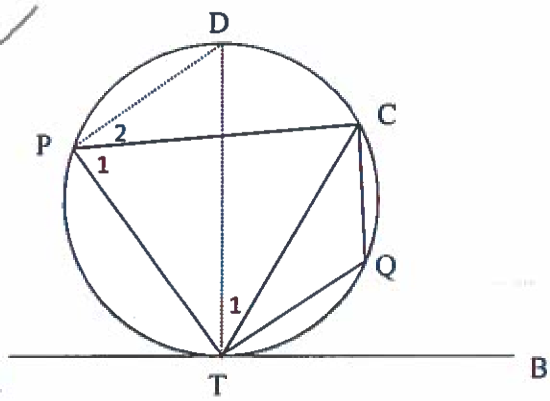
Question 7: **6 Marks**

Complete the following statement:

The angle between a tangent to a circle and a chord drawn from the point of contact is equal to the angle in the alternate segment ✓

Eugene is asked to prove the "*Tan-Chord Theorem*" based on the given information below:

Given: Circle O with TB a tangent at T. TC a chord and points P and Q on the major and minor arcs respectively.



Fill in the missing angles required for this proof: ✓

RTP: $C\hat{T}B = \hat{P}_1$ and $C\hat{T}A = \hat{Q}$

Constr: Draw diameter DT and join PD.

In this first part of the proof Eugene has made two errors only. Circle the errors.

Proof: $\hat{P}_1 + \hat{P}_2 = 90^\circ$ (∠ in a semi-circle)

$\hat{T}_1 + C\hat{T}B = 90^\circ$ (ext ∠ of Δ)

but $\hat{P}_2 = \hat{T}_1$ (∠ at centre = 2×∠ at circumference) ✓

∴ $C\hat{T}B = \hat{P}_1$

Eugene used correction fluid to 'cover' some incorrectly inserted angles.

Unfortunately he forgot to return to this question, once the correction fluid had dried, leaving some 'blanks'.

Fill in the FOUR missing angles in the spaces below.

$A\hat{T}C + C\hat{T}B = 180^\circ$ ✓ (∠'s on a str. Line)

$\hat{P}_1 + \hat{Q} = 180^\circ$ ✓ (Opp. ∠'s of a cyclic quad.)

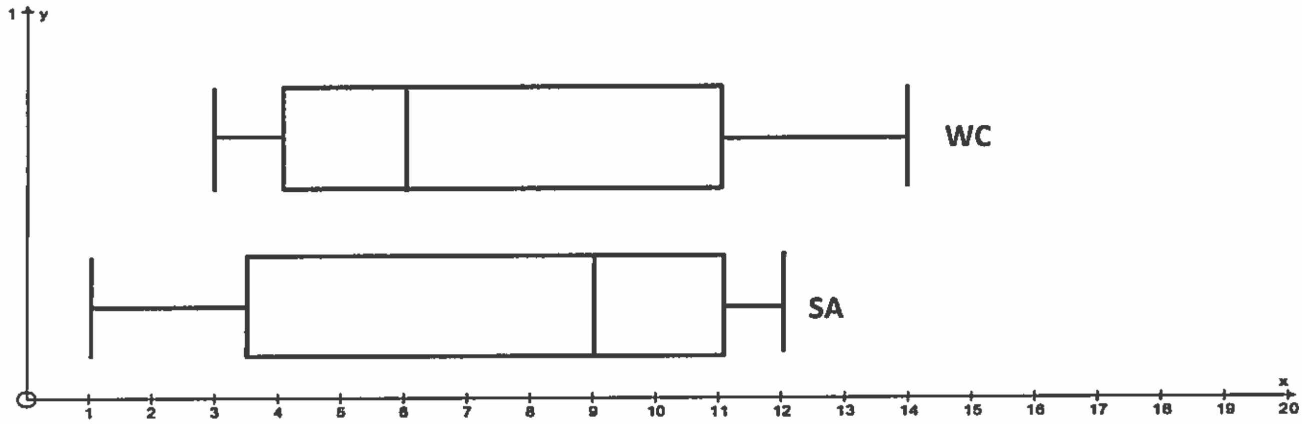
but $C\hat{T}B = \hat{P}_1$ (Proved above)

∴ $A\hat{T}C = \hat{Q}$ ✓

SECTION B:

Question 8: **8 Marks**

Admission fees of a picnic area in the Western Cape (WC) and in the rest of South Africa (SA) were investigated. The results are shown in 2 box and whisker diagrams below.



State, with calculations/reasons, if the following statements are true or false:

a. The range of fees for WC and SA are equal. (2)

WC = 14 - 3 SA = 12 - 1

= 11 = 11 ✓

TRUE ✓

b. Half of WC paid an admission fee of R6. (2)

False ✓ 50% paid more then R6 ✓

or the median fee is R6

c. Most fees in WC lie between R8 and R14. (2)

False ✓ less than 50% lie between

R8 - R14 ✓

d. SA fees are positively skewed. (2)

False ✓, negatively skewed ✓

Question 9: **9 Marks**

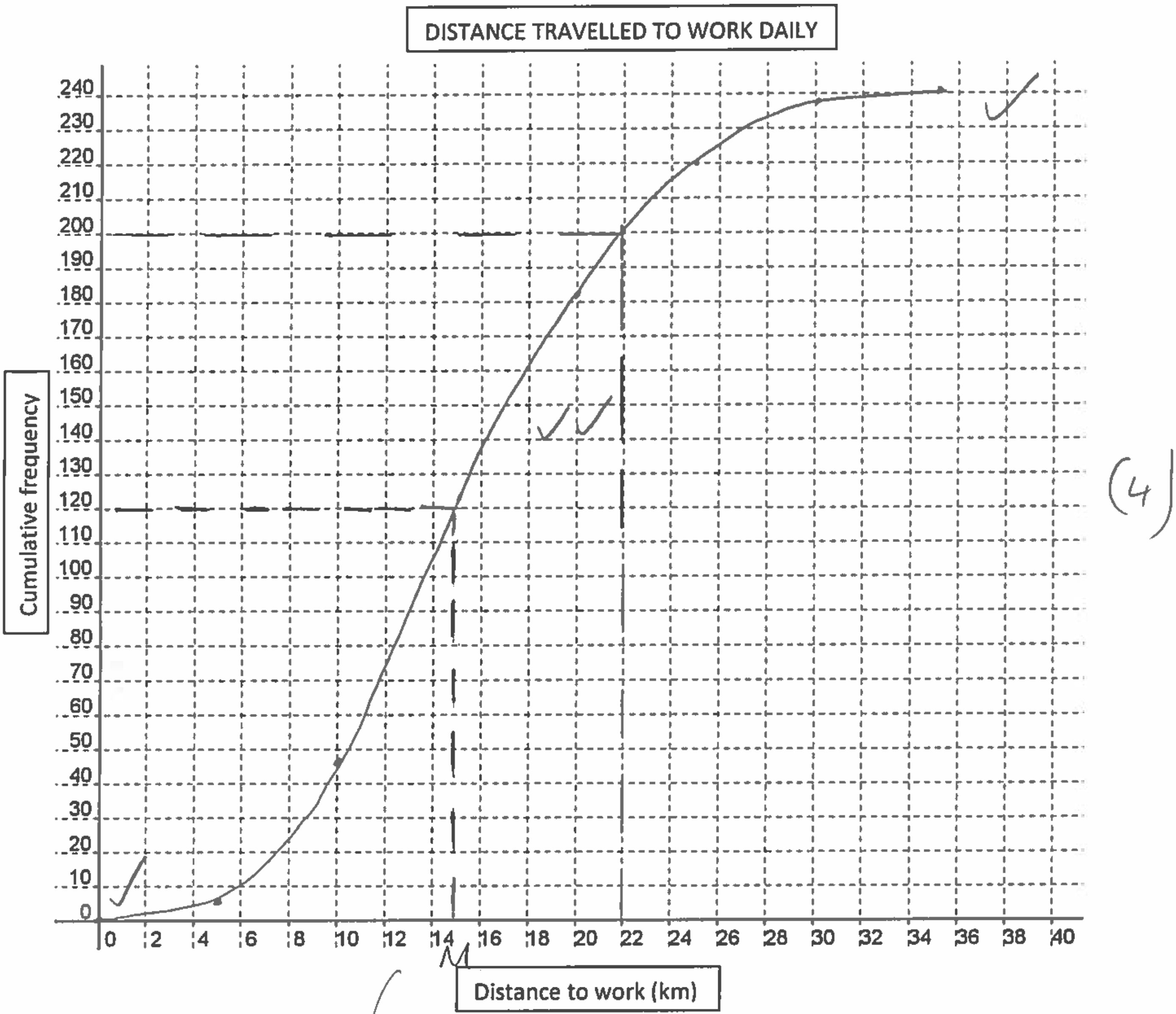
A survey was done on 240 people to determine distances travelled to work daily. The results are shown in the table below:

Distance (km)	Frequency	Cumulative Frequency
$0 < d \leq 5$	5	5
$5 < d \leq 10$	41	46
$10 < d \leq 15$	77	123
$15 < d \leq 20$	58	181
$20 < d \leq 25$	39	220
$25 < d \leq 30$	17	237
$30 < d \leq 35$	3	240

✓✓

a. Complete the cumulative frequency column in the table above. (2)

b. Draw an ogive, representing this information on the axes given on the next page. (4)



c. Use the ogive to determine the median distance travelled. Indicate where you read it off with M. (2)

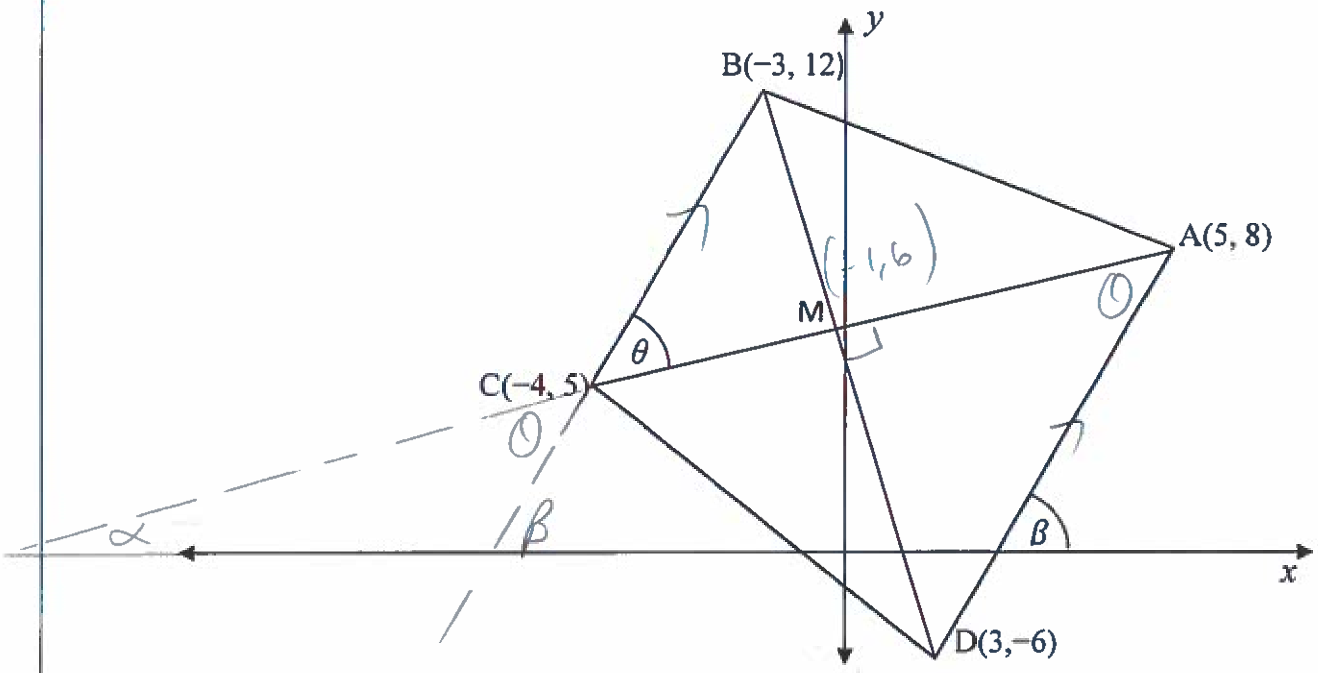
14,7 km (± 0,2 km)

d. How many people travel less than or equal to 22km to work daily? (1)

± 200 people (195 - 205)

Question 10: **20 Marks**

ABCD is a quadrilateral with vertices A(5 ; 8), B(– 3 ; 12), C(– 4 ; 5) and D(3 ; – 6).
BD ⊥ AC at M and the equation of BD is $y = -3x + 3$



a. Prove that ABCD is a trapezium. (M) (5)

$m_{BC} = \frac{12-5}{-3+4}$

$= 7$

$m_{AD} = \frac{8+6}{5-3}$

$= \frac{14}{2}$

$= 7$

∴ BC ∥ AD (= gradients)

∴ ABCD a trapezium (one pr opp sides ∥)

b. Show that the coordinates of M are (−1; 6). (5)

✓ m (gradient)

$M_{AC} = \frac{1}{3}$

$y - 8 = \frac{1}{3}(x - 5)$ ✓ subst

$y = -3x + 3$

$y = \frac{1}{3}x - \frac{5}{3} + 8$

$3y = x - 5 + 24$

$3y = x + 19$ ✓

$3(-3x + 3) = x + 19$ ✓m

$-9x + 9 = x + 19$

$\therefore y = -3(-1) + 3$ ✓m

$-10x = 10$

$= 3 + 3$

$x = -1$

$= 6$

$M(-1, 6)$

c. Show that $CM = \frac{1}{3}CA$ (4)

$CM = \sqrt{(6-5)^2 + (-1+4)^2}$

$CA = \sqrt{(8-5)^2 + (5+4)^2}$

$= \sqrt{1+9}$

$= \sqrt{9+81}$

$= \sqrt{10}$ ✓

✓m

$= \sqrt{90}$

$= 3\sqrt{10}$ ✓

$\frac{3\sqrt{10}}{\sqrt{10}} = 3$ ✓

$\therefore CA = 3CM$

d. Calculate θ, the angle $A\hat{C}B$. (6)

$\tan\beta = 7$ $\tan\alpha = \frac{1}{3}$

$\beta = 81{,}869°\ldots$ $\alpha = 18{,}43°\ldots$

$81{,}869° = 18{,}43° + \theta$ (ext < of △)

$\theta = 63{,}43°$

OR $\tan\theta = \frac{BM}{CM}$

$= \frac{2\sqrt{10}}{\sqrt{10}}$

$= 63{,}43°$

Question 11: **15 Marks**

a. i. Show that $\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=\frac{2}{\sin\theta}$ (5)

$$LHS: \frac{\sin^2\theta+(1+\cos\theta)^2}{\sin\theta(1+\cos\theta)}$$

$$= \frac{\sin^2\theta+1+2\cos\theta+\cos^2\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{2+2\cos\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{2\cancel{(1+\cos\theta)}}{\sin\theta\cancel{(1+\cos\theta)}}$$

$$= \frac{2}{\sin\theta} = RHS$$

ii. Hence, solve for $\theta\in[-360^\circ;360^\circ]$ if $\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=4$ (5)

$$\frac{2}{\sin\theta} = 4$$

$$4\sin\theta = 2$$

$$\sin\theta = \frac{1}{2}$$

$$\theta = 30^\circ + k360^\circ \qquad \theta = 150^\circ + k360^\circ \qquad k\in\mathbb{Z}$$

$$\theta\in\{-330^\circ, -210^\circ, 30^\circ, 150^\circ\}$$

b. Give the general solution of:

$\cos 2x = \sin(70° + x)$ (5)

$\cos 2x = \cos(90° - (70° + x))$

$2x = \pm(90° - (70° + x)) + k360°$

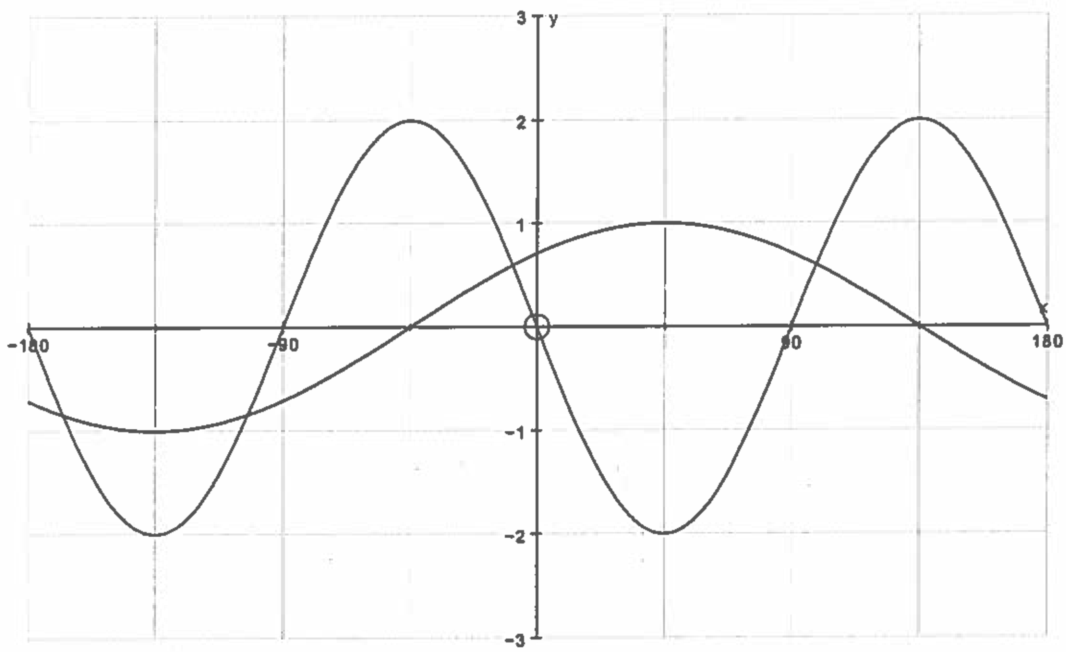
$2x = \pm(20° - x) + k360°$ $k \in \mathbb{Z}$

$2x = 20° - x + k360°$ or $2x = -20° + x + k360°$

$3x = 20 + k360°$ $x = -20° + k360°$

$x = 6,67° + k120°$

Question 12: **7 Marks**



Given graphs f and g

$f(x) = -a\sin bx$ and $g(x) = \cos(x - c^\circ)$ for $x \in [-180^\circ; 180^\circ]$

a. Find the values of a, b, and c. (3)

$a = 2$ ✓

$b = 2$ ✓

$c = 45^\circ$ ✓

b. Find the value(s) of x for which $f(x).g(x) \geq 0$ (4)

$x \in [-180^\circ, -90^\circ] \cup [45^\circ; 0^\circ] \cup [90^\circ, 135^\circ] \cup [180^\circ]$

✓ ✓ ✓ ✓

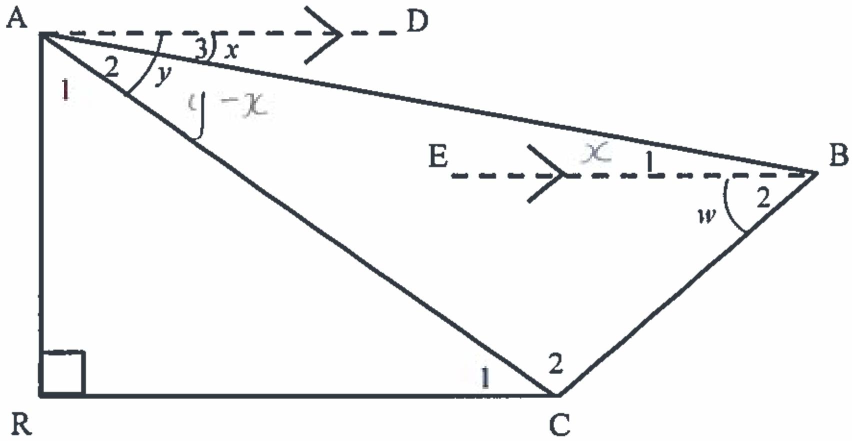
Question 13: **8 Marks**

Given: $C\hat{A}D = y$

$B\hat{A}D = x$

$E\hat{B}C = w$

$AD \parallel EB$



a. Determine the size of $\hat{C}_2$ in terms of w and y (2)

$\hat{B}_1 = x$ alt ∠'s $AD \parallel EB$ ✓(m)

$\hat{C}_2 = 180^\circ - (y - x) - x - w$ sum of △

$= 180^\circ - y + x - x - w$

$= 180^\circ - (y + w)$ ✓

b. Prove that $AC = \dfrac{AB\sin(w+x)}{\sin(y+w)}$ (3)

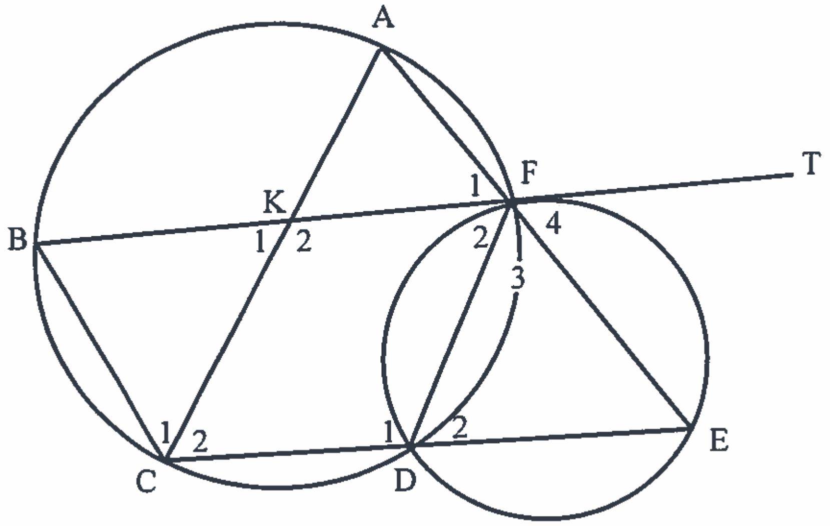
$\dfrac{AC}{\sin(x+w)}$ ✓ $= \dfrac{AB}{\sin(180^\circ - (y+w))}$ ✓

$AC = \dfrac{AB\sin(x+w)}{\sin(y+w)}$ ✓

c. Hence, if $x = 23^\circ; y = 54^\circ$; AB = 800m and AC = 788m, calculate the area of ΔABC (3)

$$\text{Area} = \frac{1}{2}(AB)(AC)\sin \hat{A}$$
$$= \frac{1}{2}(800)(788)\sin(54^\circ - 23^\circ)$$
$$= 162\,340,00\text{m}^2$$

Question 14: **12 Marks**



Given: 2 circles intersect at F and D

BFT is a tangent to the small circle at F. AFE and CDE are straight lines.

FD = FE. AC and BF cut at K

Prove the following with reasons:

a. BT // CE (3)

Let $\hat{F}_4 = x$

$\hat{D}_2 = x$ tan-chord ✓

$= \hat{E}$ isos △ ✓

$\therefore BT \parallel CE$ alt ∠'s = ✓

b. BCEF is a parallelogram (5)

✓ $\hat{D}_2 = \hat{B} = x$ (ext ∠ of cyclic quad) ✓

proven $= \hat{F}_4$ ✓

$\therefore BC \parallel FE$ (corresp ∠'s =) ✓

$\therefore$ BCEF a pgm (both pairs opp sides ∥) ✓

c. $\hat{A} = \hat{E}$ (2)

$\hat{B} = \hat{A} = x$ ∠'s in same seg ✓

$\hat{E} = x$ pioren ✓

$\hat{A} = \hat{E} = x$

d. AC = BF (2)

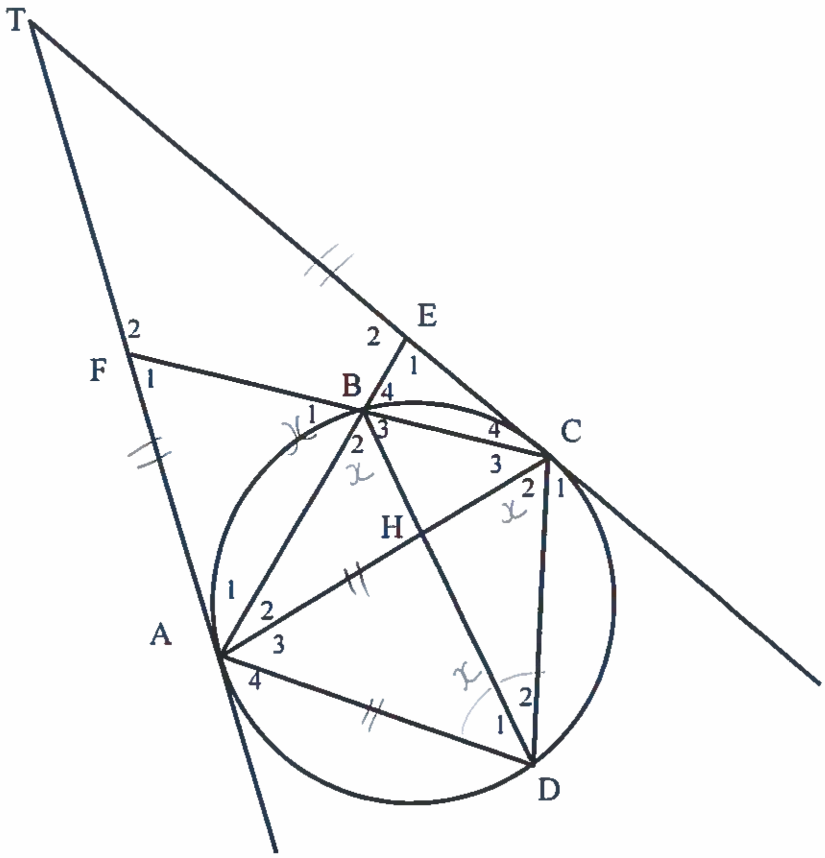
$\hat{A} = \hat{E}$ pioren

$\therefore AC = CE$ isos △ ✓

$CE = BF$ opp sides of parm ✓

$\therefore AC = BF$

Question 15: **8 Marks**



Given: **ABCD is cyclic**

AD = AC

Tangents TFA and TEC meet the circle at A and C respectively.

FBC, ABE and AHC are straight lines.

Prove the following:

a. $\hat{B}_1 = \hat{B}_2$ (5)

Let $\hat{C}_2 = x$

$\hat{C}_2 = \hat{D}_{1+2} = x$ ✓ isos Δ ✓

$\hat{B}_1 = x$ ✓ ext < of c-quad ✓

$\hat{B}_2 = x$ <'s in same segmt ✓

$\hat{B}_1 = \hat{B}_2$

b. BECH is cyclic (3)

$\hat{C}_{3+4} = \hat{O}_{1+2} = x$ ✓ tan chord.

$\hat{B}_2 = x = \hat{D}_{1+2}$ ✓

$\therefore$ BECH is cyclic (ext ∠ = int opp ∠) ✓

THE END

SPACE FOR ROUGH WORK: