



NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2010

MATHEMATICS: PAPER I

Time: 3 hours

150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 9 pages, and a green insert (pages i – iv) consisting of an Answer Sheet for Question 5 and an Information Sheet. Please make sure that your paper is complete.
 2. Read the questions carefully.
 3. Answer all the questions. Question 5 should be answered on the Answer Sheet provided, and hand this in with your Answer Book.
 4. Number your answers exactly as the questions are numbered.
 5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
 6. Round off your answers to one decimal digit where necessary.
 7. All the necessary working details must be clearly shown.
 8. It is in your own interest to write legibly and to present your work neatly.
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SECTION A**QUESTION 1**(a) Solve for x :

(1) $2x^2 - 3x + 1 = 0.$ (2)

(2) $x(x - 3) \leq 4.$ (4)

(3) $\log_x 8 = 2$, leaving your answer in surd form. (2)

(b) Simplify fully:

(1) $(x - 2)^{-1} - (x - 1)^{-1}$ (3)

(2) $\left(\frac{\sqrt{y} + \sqrt{y^3}}{\sqrt{y}} - 1 \right)^2, y > 0$ (4)

[15]**QUESTION 2**(a) Given the arithmetic sequence: $21 ; 20\frac{1}{4} ; 19\frac{1}{2}$

(1) Determine the n^{th} term T_n in terms of n . Simplify your answer. (3)

(2) Show that $T_{29-n} + T_{29+n} = 0$ for all values of n , $n \leq 28$, $n \in \mathbb{N}_0$. (3)

(b) Evaluate:

$$\sum_{k=3}^5 (-1)^k \frac{2}{k}$$
 (3)

(c) Find $\frac{dy}{dx}$ if $y = (x^3 - p^2)(x^2 + 1)$ where p is a constant. (4)

(d) Given $f(x) = x^3 - \frac{3}{x^3}$, determine the slope of the tangent to the graph of f at the point $\left(3; \frac{242}{9}\right)$. (5)

[18]

QUESTION 3

- (a) A survey conducted in December 2007 determined that 5,7 million South Africans were living with HIV.



The researchers used a model of exponential growth

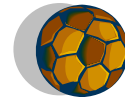
$$A = P(1 + i)^n$$

to predict that there will be 6 million people living with HIV in December 2014.

Calculate as a percentage, the annual rate of increase that the researchers used for the 7 years.

(4)

- (b) The Schmidt family from Germany visited South Africa in June 2009 and enjoyed their stay so much that they decided to return to attend the World Cup Soccer in 2010.



In June 2009 (when the exchange rate was 10,825 Rands to the Euro) they were charged R400 per person per night at a B&B in Durban.

This same place put up their prices to R2 100 per person per night during the World Cup. At this stage the exchange rate was 10,516 Rands to the Euro.

Calculate the percentage increase for accommodation that the Schmidts experienced.

(5)

- (c) Diana decided when she started work that she would invest R500 each month into a retirement fund earning interest at 8,5% p.a. compounded monthly. The first payment is made after 1 month and the final payment earns no interest.

(1) Calculate how much she could expect to be in her account at the end of 30 years.

(4)

(2) Alex also started work at the same time as Diana and invested in the same scheme but decided to deposit R600 per month. After n years, Alex's investment accumulated to A , where A is the answer obtained in (1) above. Calculate the value of n , to the nearest whole number.

(5)

[18]

QUESTION 4

A series of diagrams, using three types of triangles, is shown. The triangles are grey, white or black.

Diagram 1



Diagram 2

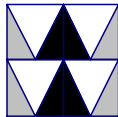


Diagram 3

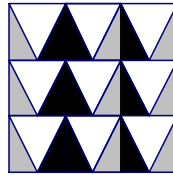
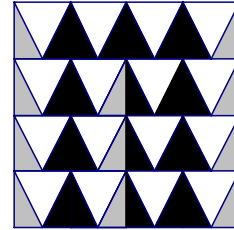


Diagram 4



The table below shows the numbers of each type of triangle used in the diagrams.

Diagram	1	2	3	4		n
Grey triangles	2	4	6	A		X
White triangles	1	4	9	B		Y
Black triangles	0	2	6	C		Z

- (a) Determine the values of A, B and C, the number of the different types of triangle in Diagram 4. (1)
 - (b) Determine simplified expressions for X, Y and Z, the number of triangles in the n^{th} diagram. (5)
 - (c) Hence determine which diagram has 1 260 black triangles. (4)
- [10]**

QUESTION 5 ANSWER ON THE ANSWER SHEET PROVIDED

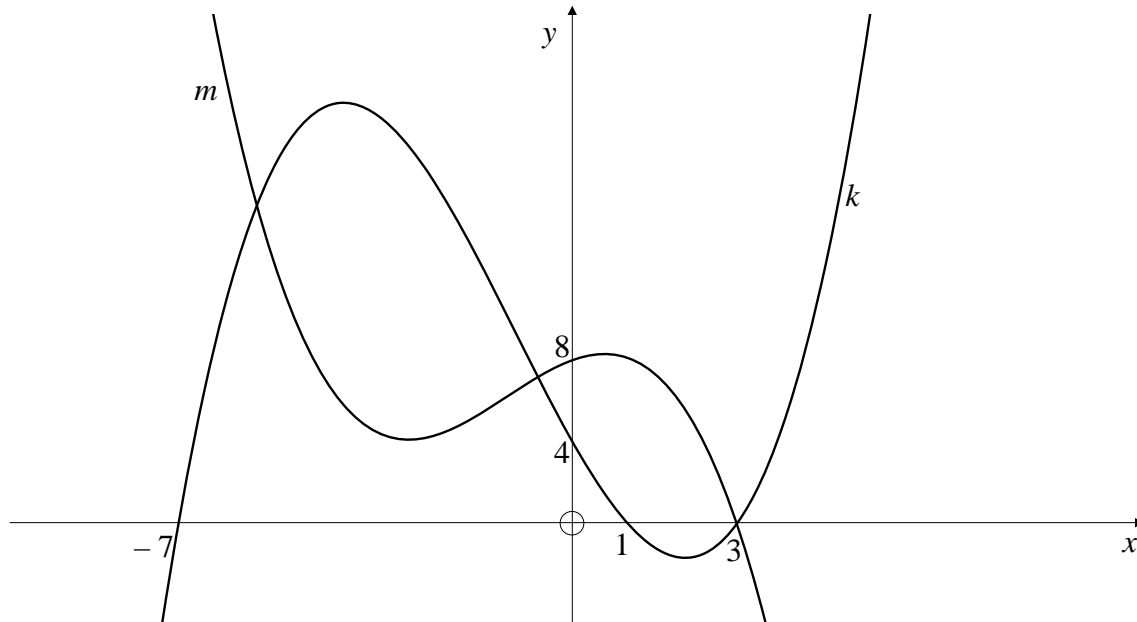
- (a) Given: $p(x) = -3x^2$
Determine the equation of the inverse of p stating its domain and range. (5)
 - (b) Given: $f(x) = 2^x$, $g(x) = f(x-2)$ and $h(x) = f^{-1}(x)$
 - (1) Write down the equations of g and h in the form $y = \dots$ (2)
 - (2) On the set of axes where f is already drawn for you, add the graphs for g and h . (4)
 - (3) Solve for x if $g(x) = h(x)$. (1)
- [12]**

73 marks

SECTION B

QUESTION 6

Refer to the figure showing cubic graphs $y = k(x)$ and $y = m(x)$ with intercepts indicated.



Use the graphs to answer the following questions:

- (a) Determine the values of x for which:
 - (1) $k(x) < 0$ (2)
 - (2) $k(x) \cdot m(x) > 0$ (2)

 - (b) Evaluate:
 - (1) $k(1) \times m(1)$ (1)
 - (2) $k(m(3))$ (2)

 - (c) Determine:
 - (1) the average gradient of $k(x)$ in the interval $[0; 1]$ (2)
 - (2) a common factor of the two functions (1)
- [10]**

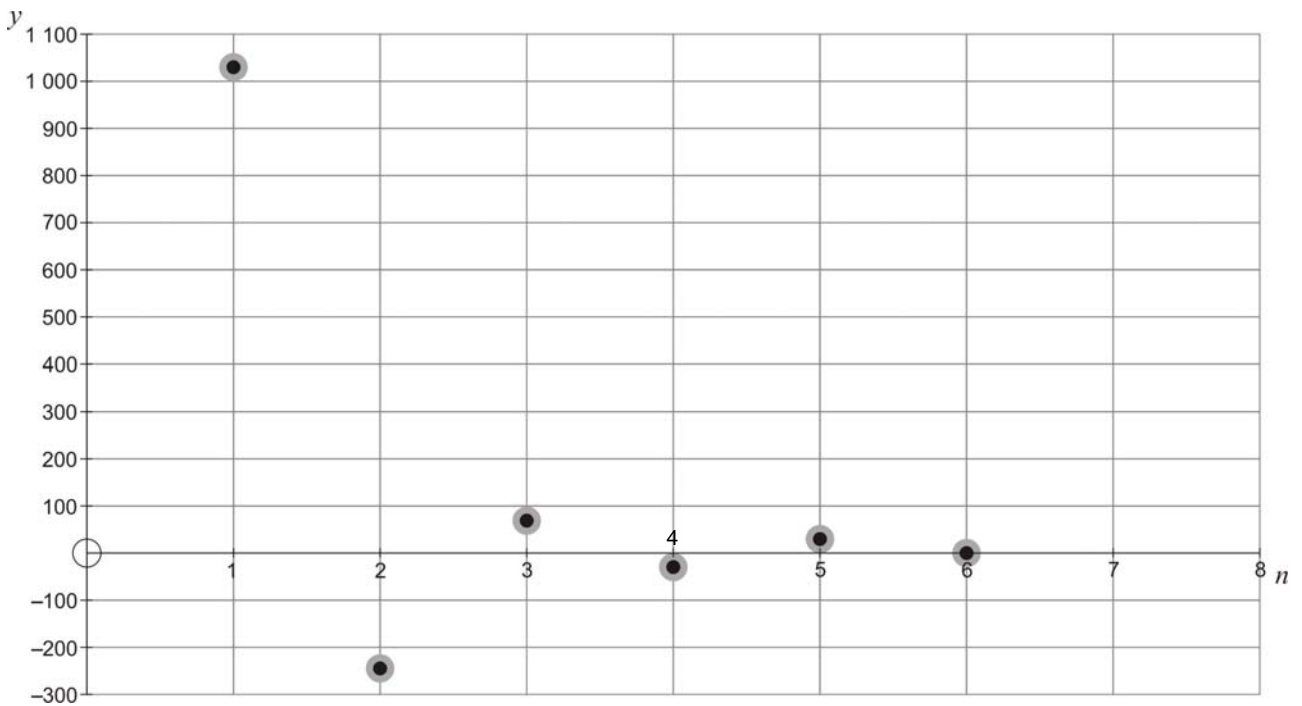
QUESTION 7

(a) Given the series: $8 + 16 + 24 + 32 + \dots$ (n terms)

(1) Determine a simplified expression for the sum of the first n multiples of 8 ($n \in N$). (3)

(2) Hence verify that this sum is one less than the square of an odd number. (Hint: An even number can be represented by $2n$, $n \in N$) (3)

(b) Refer to the figure showing discrete points $(1 ; 1\ 024)$, $(2 ; -256)$, $(3 ; 64)$, etc.



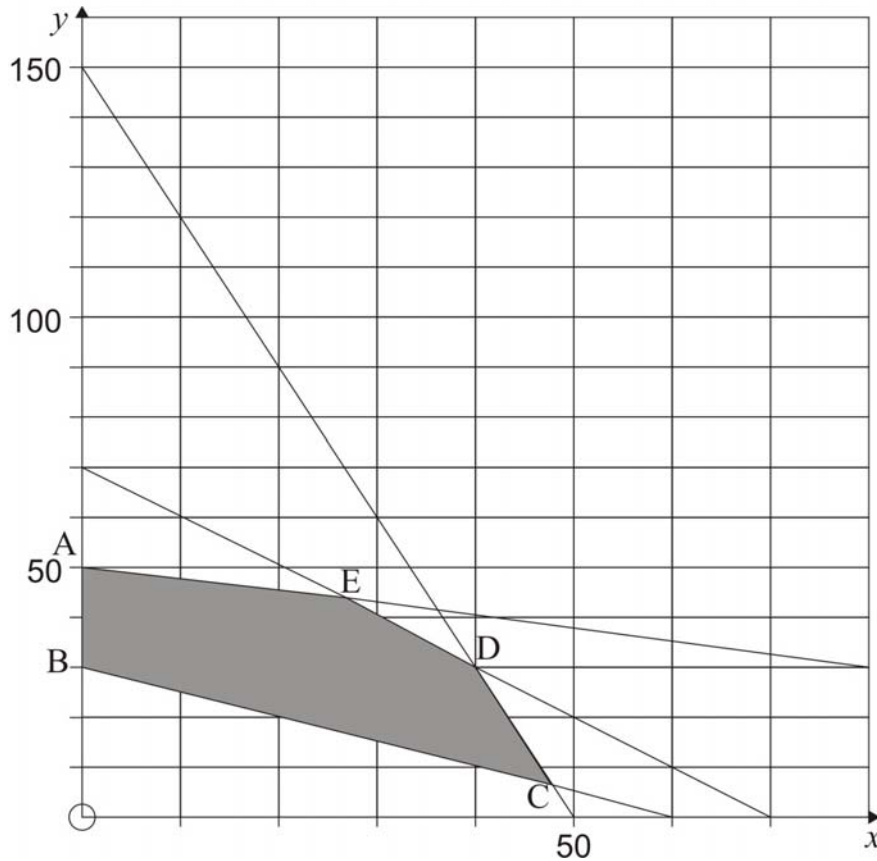
(1) Calculate the y -coordinate of the 20th point, leaving your answer in exponential form. (4)

(2) If the y -coordinates of these points are added together, the sum converges to a number P . Determine the value of P . (2)

[12]

QUESTION 8

Boitsheko set up a business selling South African flags. She dealt with small plastic flags as well as large cloth flags. There were a number of restrictions on how many flags she was able to have on her stand each day. These are represented in the following figure where x represents the number of small flags and y represents the number of large flags and a feasible region ABCDE.



- (a) Two of Boitsheko's restrictions generate the constraints

$$x \geq 0 \quad \text{and} \quad y \leq -\frac{x}{4} + 50$$

Determine the other three constraints involving boundary lines of the feasible region.

(6)

- (b) Write down the largest number of large flags that Boitsheko could have on her stand complying with all the constraints.

(1)

- (c) Boitsheko made R8 profit on each small flag that she sold and R4 on each large flag. Determine her possible maximum profit.

(5)

- (d) After the World Cup, Boitsheko reduced the profit of her small flags to R4. The profit on large flags was not changed.

Determine 3 pairs $(x; y)$ that will ensure a maximum profit.

(6)

[18]

QUESTION 9

- (a) The strength of the reaction of a person's body to x units of a drug is given by:

$$R(x) = x^2 \left(\frac{C}{2} - \frac{x}{3} \right)$$

where C is the maximum number of units that can be given.

(1) Show that $R(C) = 2R\left(\frac{C}{2}\right)$. (4)

(2) $R'(x)$ is used as a measure of the sensitivity of the body to the drug. Determine x such that the sensitivity is maximised. (5)

- (b) Show that the curve $y = 4x^2 + \frac{1}{x}$ has only one turning point and determine whether it is a maximum or minimum. (6)
- [15]**

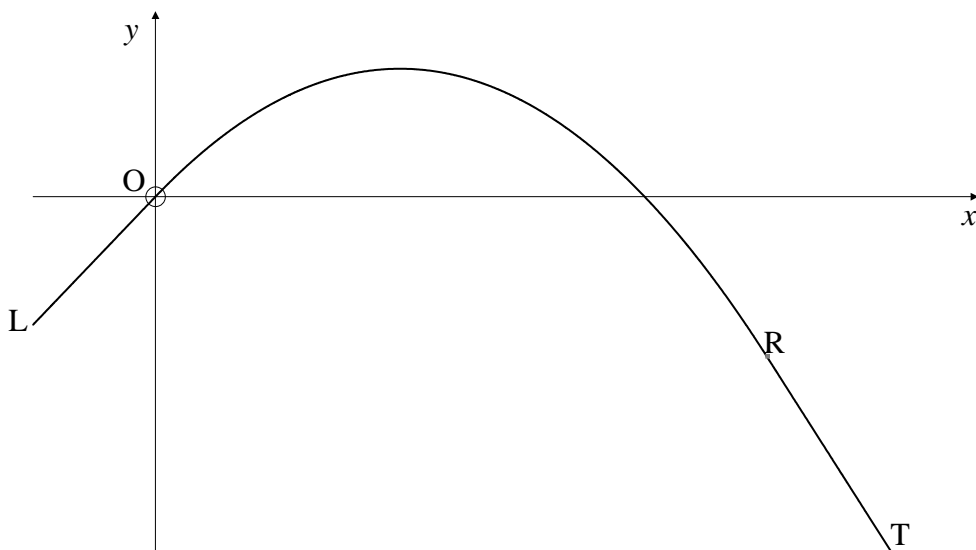
QUESTION 10

- (a) The height of a roller-coaster track at a fun-park is given by:

$$f(x) = \frac{1}{2450}(x - 50)(x - 100)^2 \quad \text{for } 40 \leq x \leq 90$$

Determine the coordinates of the highest point of this section of track. (7)

- (b) Refer to the diagram that shows another part of the roller-coaster track.



The section LO is a straight line with gradient of 2.
 The section RT is also a straight line with gradient -3 .
 The section OR is a parabola $y = ax^2 + bx$.

The point R is 20 metres from the y -axis.
 The designers of the track had to ensure that there were no bumps along the track, so the straight line sections are tangents to the parabola at O and R.

Determine the values of a and b .

(6)
 [13]

QUESTION 11

- (a) Tshepang rushed from home to the soccer stadium at 165 km.h^{-1} to see her favourite team.
 They lost the game so, feeling sad, she drove home taking the same route, at 110 km.h^{-1} .
 Calculate Tshepang's average speed for the whole trip.
 It is not $137,5 \text{ km.h}^{-1}$.

(5)

- (b) Without using a calculator and showing all working, evaluate:

$$\sqrt{5967564928^2 - 5967564926 \times 5967564930}$$

(4)
 [9]

77 marks

Total: 150 marks