NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2010

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours | 150 marks

These marking guidelines were used as the basis for the official IEB marking session. They were prepared for use by examiners and sub-examiners, all of whom were required to attend a rigorous standardisation meeting to ensure that the guidelines were consistently and fairly interpreted and applied in the marking of candidates' scripts.

At standardisation meetings, decisions are taken regarding the allocation of marks in the interests of fairness to all candidates in the context of an entirely summative assessment.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines, and different interpretations of the application thereof. Hence, the specific mark allocations have been omitted.

SECTION A

QUESTION 1

No working
1 mark if 1 correct
0 marks for incorrect answer

(a) (1)

$$2x^2 - 3x + 1 = 0$$

$$(2x-1)(x-1) = 0 \quad \textbf{A}$$

$$x = \tfrac{1}{2} \text{ or } x = 1 \quad \textbf{CA}$$

$$x = \frac{3 \pm \sqrt{9-8}}{4}$$

(2)

(2)

$$x(x-3) \leq 4$$

$$x^2 - 3x - 4 \leq 0 \quad \textbf{M}$$

$$(x-4)(x+1) \leq 0 \quad \textbf{A}$$

−1   4
\+ 0   −   0 +

$$-1 \leq x \leq 4 \quad \textbf{A A}$$

RHS = 0 $x^2 - 3x + 4 \leq 0$ No solution $\frac{3}{4}$

(4)

(3)

$$\log_x 8 = 2 \log_x$$

$$x^2 = 8 \quad \textbf{M}$$

Converting to Exponential

$$x = \pm\sqrt{8}$$

$$x = \sqrt{8} \quad \textbf{A}$$

(2)

(b) (1)

$$(x-2)^{-1} - (x-1)^{-1}$$

$$= \frac{1}{x-2} - \frac{1}{x-1} \quad \textbf{A}$$

$\frac{-3}{(x)}$ $\frac{2}{3}$

$$= \frac{(x-1)-(x-2)}{(x-2)(x-1)} \quad \textbf{M}$$

LCD

$$= \frac{x-1-x+2}{(x-2)(x-1)}$$

$$= \frac{1}{(x-2)(x-1)} \quad \textbf{A}$$

(3)

(2)

$$\left(\frac{\sqrt{y}+\sqrt{y^3}}{\sqrt{y}} - 1\right)^2$$

$$= \left(\frac{\sqrt{y}+y\sqrt{y}}{\sqrt{y}} - 1\right)^2 \quad \textbf{M}$$

Simplifying surd

$$\left(\frac{\sqrt{y}+\sqrt{y^3}-\sqrt{y}}{\sqrt{y}}\right)^2$$

$$= \left(\frac{\sqrt{y}(1+y)}{\sqrt{y}} - 1\right)^2 \quad \textbf{A}$$

$$= \left(\frac{\sqrt{y^3}}{\sqrt{y}}\right)^2$$

$$= (1+y-1)^2 \quad \textbf{A}$$

$$= \frac{y^3}{y}$$

$$= y^2 \quad \textbf{A}$$

$$= y^2$$

(4)

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QUESTION 2

(a) (1) $21\ ;\ 20\tfrac{1}{4}\ ;\ 19\tfrac{1}{2}\ \dots$

A.P. $a = 21;\quad d = -\dfrac{3}{4}$ **A**

$T_n = 21 + (n-1)\left(-\dfrac{3}{4}\right)$ **M**

$= 21 - \dfrac{3n}{4} + \dfrac{3}{4}$ **A**

$= 21{,}75 - \dfrac{3}{4}n \;/\; \dfrac{87-3n}{4}$

$/\; \dfrac{3(29-n)}{4}$

Sub. in T_n for AP $\quad bn + c \quad \dfrac{-3n}{4} +$

$21\dfrac{3}{4} - \dfrac{3}{4}(29-n)$ (3)

(2) $T_{29-n} + T_{29+n}$

$= \dfrac{87-3(29-n)}{4} + \dfrac{87-3(29+n)}{4}$ **M** **A**

$= \dfrac{3n}{4} + \dfrac{-3n}{4}$ **A**

$= 0$

Sub. into T_n found in (a)

Using 1 value for n calculating $\dfrac{2}{3}$ & marking zero (3)

1 more example $\dfrac{3}{3}$

(b) $\displaystyle\sum_{k=3}^{5}(-1)^k\frac{2}{k}$

$= (-1)^3\times\tfrac{2}{3} + (-1)^4\times\tfrac{2}{4} + (-1)^5\times\tfrac{2}{5}$ **M**

$= -\tfrac{2}{3} + \tfrac{1}{2} - \tfrac{2}{5}$ **A**

$= -\dfrac{17}{30}$ **A**

Expanding

$-2 + 1$

$-\dfrac{47}{30}$ (3)

(c) $y = (x^3 - p^2)(x^2 + 1)$

$= x^5 + x^3 - p^2x^2 - p^2$ **M A**

$\dfrac{dy}{dx} = \underbrace{5x^4 + 3x^2}\ \underbrace{-2p^2x}_{\textbf{A}}$

Expanding

–1 if –2p appears in answer

(4)

$(3x^2)(2x)\ \dfrac{1}{4}$

$\dfrac{dy}{dx} = (3x^2)(x^2 + 1) + (x^3 - p^2)(2x)$

$= 3x^4 + 3x^2 + 2x^4 - 2p^2x$

$= 5x^4 + 3x^2 - 2p^2x$

(d) $f(x) = x^3 - \frac{3}{x^3}$

$= x^3 - 3x^{-3}$ **A**

$f'(x) = 3x^2 + 9x^{-4}$ **A M** — Finding derivative

Slope $= f'(3) = 3 \times 3^2 + \frac{9}{3^4}$ **M** — Substitution

$= 27 + \frac{1}{9}$

$= 27\frac{1}{9}$ or $\left(\frac{244}{9}\right)$ or $(27{,}1)$ **CA**

Answer only $\frac{5}{5}$ for correct answer

(5)

[18]

QUESTION 3

(a) $5{,}7(1+i)^7 = 6$ **A** — Correct values in position

$(1+i)^7 = \frac{20}{19} \,/\, \frac{6}{5{,}7}$

$1+i = \sqrt[7]{\frac{20}{19}}$ or $\left(\frac{6}{5{,}7}\right)^{\frac{1}{7}}$ **M** — Rooting

$1 + i = 1$

$i = 0 \quad \frac{2}{4}$

$= 1{,}007354526$ **A**

$i \approx 0{,}7\%$ **CA**

(4)

(b) 2009: $\frac{400}{10{,}825}$ **M** — Conversion

$=$ € 36,95 150115 **A**

2010: $\frac{2100}{10{,}516}$

$=$ € 199,69 57018 **A**

% increase: $\frac{162{,}7442006}{36{,}95150115} \times 100$ **M** — % increase

$= 440{,}4\%$ **A**

(5)

$\frac{2100 - 400}{400} \times 100 \qquad \frac{2}{5}$

425%

(c) (1) $F_v = \dfrac{500\left[\left(1+\dfrac{0,085}{12}\right)^{360}-1\right]}{\dfrac{0,085}{12}}$ — **360** A; $\dfrac{0,085}{12}$ A

$= 825\ 352,8556$ **M** Sub. in Fv of Annuity

$\approx$ R 825 352,86 **A** (4)

(2) $\dfrac{600\left[\left(1+\dfrac{0,085}{12}\right)^{n}-1\right]}{\dfrac{0,085}{12}} = 825352,86$ **M** Fv of Annuity = Ans

$(1,007...)^n - 1 = 9,743749042$

$(1,007...)^n = 10,7437...$ **A**

$n = \log_{1,007...}(10,7437...)$ **M** Introducing Logs

$= 336,3844621$ **A**

i.e. 28 years **CA** (5)

[18]

QUESTION 4

(a) A = 8
B = 16
C = 12 **A** (1)

(b) X = $2n$ **A**

Y = n^2 **A**

Black tri's:	0	2	6	12
1st diff.		2	4	6
2nd diff.			2	2

M Finding differences

$Z = 0 + (n-1)2 + \dfrac{(n-1)(n-2)}{2} \times 2$ **A**

$= 2n - 2 + n^2 - 3n + 2$

$= n^2 - n$ **A** (5)

ALTERNATIVELY

$T_1 = 0 \times 1$
$T_2 = 1 \times 2$
$T_3 = 2 \times 3$
$T_4 = 3 \times 4$ **M**

$Z = T_n = (n-1)n$ **A** **A**

$2a = d \quad a = 1$
$T_0 = c = 0$ **M**
$T_n = 1n^2 + bn + 0$ **A**
$T_1 = 1 + b = 0$
$b = -1$ **A**

(c) $n^2 - n = 1260$ **M**	Setting $T_n = 1260$
$n^2 - n - 1260 = 0$	Answer from b
$(n-36)(n+35) = 0$ OR $n = \dfrac{1 \pm \sqrt{1 + 4141260}}{2}$ **A**	
$n = 36$ or $n = -35$ **A**	$\therefore$ Showing that T_{3t}
×N.V.	gives 1 260 $\frac{4}{4}$
i.e. The 36th diagram **A**	(4)
	[10]

QUESTION 5

On Answer Sheet

[12]

SECTION B

QUESTION 6

(a)	(1)	$x < -7$ or $1 < x < 3$	A A	≤ 1 mark deducted (3; 1) pin (2)
	(2)	$-7 < x < 1$	A A	(2)
(b)	(1)	$k(1) \times m(1)$ $= 0 \times m(1)$ $= 0$	A	(1)
	(2)	$k(m(3))$ $= k(0)$ $= 4$	M A	$k(3) \times m\,(3)$ no mark (2)
(c)	(1)	Ave. Grad. $= \dfrac{4-0}{0-1}$ $= -4$	M A	(2)
	(2)	$x - 3$	A	(1)

[10]

QUESTION 7

(a) 8 + 16 + 24 + 32 + ... (n terms)

(1)	A.S. $a = d = 8$	A	
	$S_n = \dfrac{n}{2}[2 \times 8 + (n-1)8]$	M	Sub. into S_n of A.S.
	$= \dfrac{n}{2}[16 + 8n - 8]$		
	$= \dfrac{n}{2}[8n + 8]$		
	$= 4n^2 + 4n$	A	(3)
(2)	$S_n = 4n^2 + 4n + 1 - 1$	M	Completion of square
	$= (2n+1)^2 - 1$	A	$(2n-1)^2 - 1$ M $= 4n^2 - 4n + 1 - 1$ $= 4n^2 - 4n$ A
	$2n + 1$ is an odd number, $n \in N$ $\therefore$ Statement is true.	CA	(3)

(b) (1) G.P. $a = 1024$, **A** $r = -\frac{1}{4}$ **A**

$y = T_{20}$ **M** — Sub. into G.P. formula

$= 1024\left(-\frac{1}{4}\right)^{19}$ **A** (4)

(2) $S_\infty = \dfrac{1024}{1-\left(-\frac{1}{4}\right)}$ **M** — Sub. in Sum to Infinity

$= 819,2$ **A** (2)

If $r = -4$

$S_\infty = \dfrac{1024}{5} = 204,8$

NB: Answers only with no working but 819,2 ½

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QUESTION 8

(a) $y \geq -\dfrac{x}{2} + 30$ **A A**

$y \leq -3x + 150$ **A A**

$y \leq -x + 70$ **A A**

$x + 2y \geq 60$

$3x + y \leq 150$

$x + y \leq 70$

(6)

(b) 50 large flags **A** (1)

(c) $P = 8x + 4y$ **A**

$4y = -8x + P$

$y = -2x + \dfrac{P}{4}$ **M** — Converting to std. form

Max. Profit at D(40 ; 30) **A M** — Sub. into Profit

$P = 8 \times 40 + 4 \times 30$

$= \text{R}440$ **CA** (5)

(d) New $P = 4x + 4y$ **A**

$4y = -4x + P$ **M** — Converting to std. form

$y = -x + \frac{P}{4}$

Objective function has same gradient as one of the constraints. **A**

At E: $y = -x + 70$ and $y = -\frac{x}{4} + 50$

$-x + 70 = -\frac{x}{4} + 50$ **M** — Equating

$-4x + 280 = -x + 200$

$-3x = -80$

$x = \frac{80}{3}$ $(26,7)$

$y = -\frac{80}{3} + 70$

$= \frac{130}{3}$ $(43,3)$

$27 \leq x \leq 40$ $x \ \varepsilon \ N$

$30 \leq y \leq 43$ $y \ \varepsilon \ N$ with $y = -x + 70$ **A**

ALTERNATIVELY: Any point on ED

i.e. 14 solutions: Give any 3.

(27; 43), (28, 42), (29; 41), (30; 40), (31; 39),
(32; 38), (33; 37), (34; 36), (35; 35), (36; 34), **A**
(37; 33), (38; 32), (39; 31), (40; 30)

(6)

[18]

QUESTION 9

(a) $R(x) = x^2\left(\frac{C}{2} - \frac{x}{3}\right)$

(1) $R(C) = C^2\left(\frac{C}{2} - \frac{C}{3}\right)$ **M** Substitution $x = c$

$= \frac{C^3}{6}$ **A**

$2R\left(\frac{C}{2}\right) = 2\left(\frac{C}{2}\right)^2\left(\frac{C}{2} - \frac{1}{3}.\frac{C}{2}\right)$ **M** Substitution $x = \frac{c}{2}$

$= 2\cdot\frac{C^2}{4}\left(\frac{3C - C}{6}\right)$

$= 2\cdot\frac{C^2}{4}.\frac{C}{3}$

$= \frac{C^3}{6}$ **A**

$= R(C)$ (4)

(2) $R(x) = x^2\left(\frac{C}{2} - \frac{x}{3}\right)$

$= \frac{Cx^2}{2} - \frac{x^3}{3}$ **A** Setting $(x - x^2 = 0$

$R'(x) = Cx - x^2$ Der. $x = \text{c}$ or $x = 0$

$R''(x) = C - 2x = 0$ **A** **M** Der. = 0 $\frac{3}{5}$

$2x = C$

$x = \frac{C}{2}$ **A** (5)

(b) $y = 4x^2 + x^{-1}$ Der.

Stat. Pt. : $\frac{dy}{dx} = 8x - x^{-2} = 0$ **M** Der. = 0

$8x = \frac{1}{x^2}$ Table of values

$x^3 = \frac{1}{8}$

$x = \frac{1}{2}$ Only one solution **A**

ALTERNATIVELY:

$\frac{dy}{dx} = 8x - x^{-2}$

$= 8x - \frac{1}{x^2}$

$= \frac{8x^3 - 1}{x^2}$

$= \frac{(2x-1)(4x^2 + 2x + 1)}{x^2}$ **M**

$= 0$ when $x = \frac{1}{2}$ **A**

At $x = \frac{1}{4}$ $\frac{dy}{dx} = 8 \times \frac{1}{4} - \left(\frac{1}{4}\right)^{-2}$ **M** Substitution

$= 2 - 16$

$= -14 < 0$ **A**

At $x = \frac{3}{4}$ $\frac{dy}{dx} = 8 \times \frac{3}{4} - \left(\frac{3}{4}\right)^{-2}$

$= 6 - \frac{16}{9}$

$= \frac{38}{9} > 0$

$\therefore$ Local minimum **CA** (6)

ALTERNATIVELY:

$\frac{d^2y}{dx^2} = 8 + 2x^{-3}$

$= 8 + \frac{2}{x^3}$ 2nd Der. **A**

When $x = \frac{1}{2}$ $\frac{d^2y}{dx^2} = 8 + 2 \div \left(\frac{1}{2}\right)^3$ **M** Substitution

$= 2 + 2 \times 8$

$= 24 > 0$

$\therefore$ Local minimum **CA**

No working $x = \frac{1}{2}$

Local min $\frac{6}{6}$

Only $x = \frac{1}{2}$

Or only local min. $\frac{3}{6}$

[15]

QUESTION 10

Table on calc. in steps of 5 or x. Answer given between 65 + 68 $\frac{6}{7}$

(a)

$$f(x) = \frac{1}{2450}(x-50)(x-100)^2$$

$$= \frac{1}{2450}(x-50)(x^2-200x+10000)$$ **M** Expanding

$$= \frac{1}{2450}(x^3-200x^2+10000x-50x^2+10000x-500000)$$

$$= \frac{1}{2450}(x^3-250x^2+20000x-500000)$$ **A**

$$f'(x) = \frac{1}{2450}(3x^2-500x+20000) = 0$$ **A M** Der. = 0

$$(3x-200)(x-100) = 0 \quad \text{OR} \quad x = \frac{500 \pm \sqrt{500^2 - 240000}}{6}$$

$$x = \tfrac{200}{3} \quad \text{or} \quad x = 100$$ **A**

$$f(x) = \frac{1}{2450}\left(\tfrac{200}{3}-50\right)\left(\tfrac{200}{3}-100\right)^2$$ **M** Substitution

$$= 7{,}558578987$$ **CA**

$\therefore$ Highest point : (66,7 ; 7,6) (7)

(b)

$$y = ax^2 + bx$$

$$\frac{dy}{dx} = 2ax + b$$ **A**

At O : $m = b$ **M** Equating gradient = 2

$\therefore\ b = 2$ **A**

At R : $2a \times 20 + 2 = -3$ **A M** Equating gradient = –3

$$40a = -5$$

$$a = -\frac{1}{8}$$ **A** (6)

[13]

QUESTION 11

(a) A M

	Dist. (km)	**Speed (km.h^{-1})**	**Time (hours)**
There	d	165	$\frac{d}{165}$
Back	d	110	$\frac{d}{110}$
Return	$2d$	?	$\frac{d}{165}+\frac{d}{110}$

Total distance/ total time

$$\text{Speed} = 2d \div \left(\frac{d}{165}+\frac{d}{110}\right)$$ A

$$= 2d \div \frac{2d+3d}{330}$$ M

Simplifying

$$= 2d \times \frac{330}{5d}$$

$$= 132 \text{ km.h}^{-1}$$ A

(5)

(b) Let $x = 5\,967\,564\,928$ M

$$\sqrt{x^2-(x-2)(x+2)}$$ A

$$= \sqrt{x^2-(x^2-4)}$$

$$= \sqrt{4}$$ A

$$= 2$$ A

(4)

[9]

$$\sqrt{8^2-6\times 0}$$
$$=8$$

$$\sqrt{28^2-26\times 30}$$

$$=\sqrt{4}$$
$$=2$$ $\frac{2}{4}$

$$\sqrt{3^2-1\times 5}$$
$$=2$$ $\frac{3}{4}$

$+$ 1 more example $\frac{4}{4}$