



NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2010

MATHEMATICS: PAPER II

Time: 3 hours

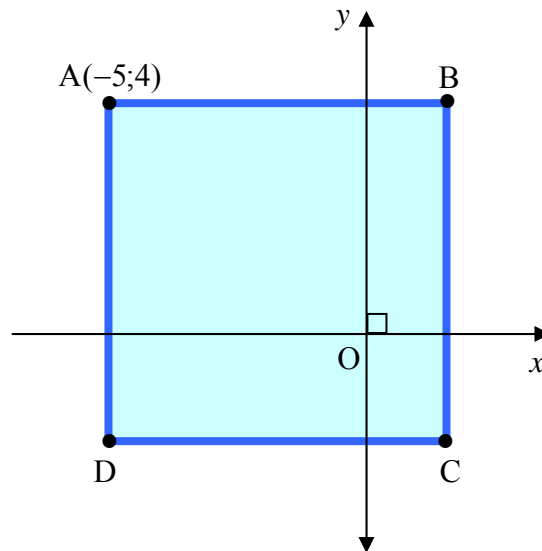
150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 16 pages, a Diagram Sheet (pages i to iv) and an Answer Sheet for Question 3 and an Information Sheet (pages i to iii). Please check that your paper is complete.
 2. Write your examination number in the space provided in your Answer Book, the Diagram Sheet and the Answer Sheet.
 3. Answer ALL the questions. Answer Question 3 on the Answer Sheet provided, and hand this in with your Answer Book.
 4. Please note that diagrams are not necessarily drawn to scale.
 5. A diagram sheet is provided. Any changes made to a diagram must be shown on this sheet, and not on the question paper.
 6. All necessary working details must be shown.
 7. Approved non-programmable and non-graphical calculators may be used, unless otherwise stated.
 8. Ensure that your calculator is in DEGREE mode.
 9. Answers must be rounded off to one decimal digit, unless otherwise stated.
 10. It is in your own interest to write legibly and to present your work neatly.
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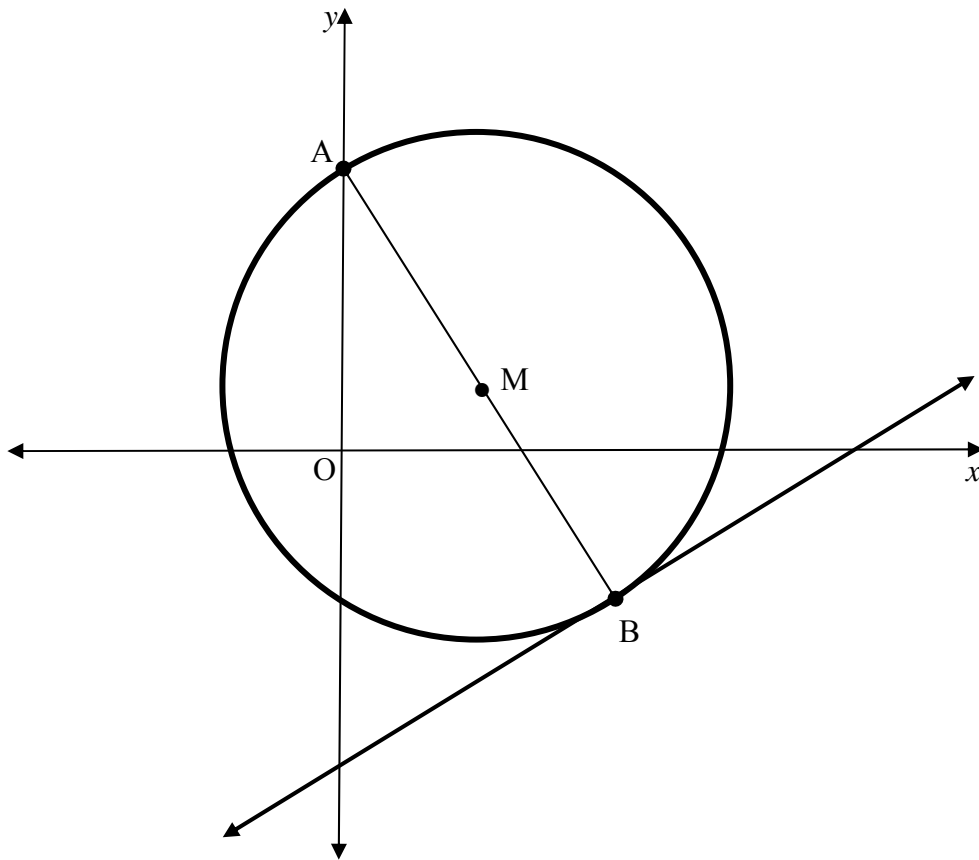
SECTION A**QUESTION 1**

- (a) In the diagram below, square ABCD of side length 6 units is drawn. A has coordinates $(-5;4)$ and AB is parallel to the x -axis.



- (1) Write down the coordinates of B and C. (2)
- (2) Write down the equation of CD. (1)
- (3) Write down the equation of BC. (1)
- (4) Determine the equation of the line that is parallel to AC and passes through the point B. Give your answer in the form $y = mx + c$. (3)

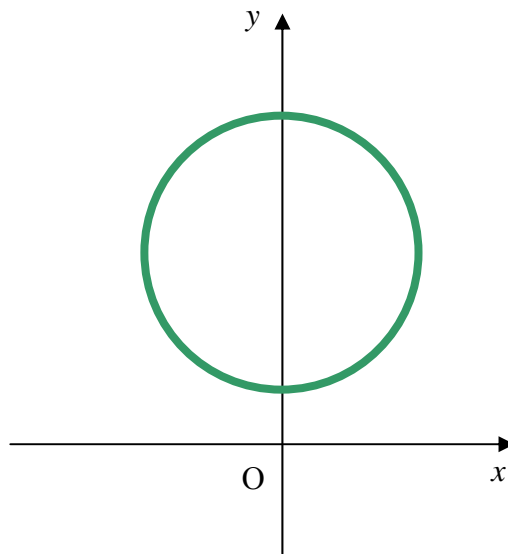
- (b) In the diagram, the circle with equation $(x-3)^2 + (y-1)^2 = 25$ is drawn.
 A is a point on the y-axis. M is the centre of the circle. AB is a diameter of the circle.



- | | | |
|-----|---|-------------|
| (1) | Write down the coordinates of M. | (1) |
| (2) | Determine the coordinates of A. | (5) |
| (3) | Determine the coordinates of B. | (3) |
| (4) | Determine the equation of the tangent to the circle at B. | (5) |
| | | [21] |

QUESTION 2

- (a) In the diagram below, the circle drawn has equation $x^2 + (y - 5)^2 = 9$.



In each case, write down the centre and the equation of the circle obtained if the given circle is:

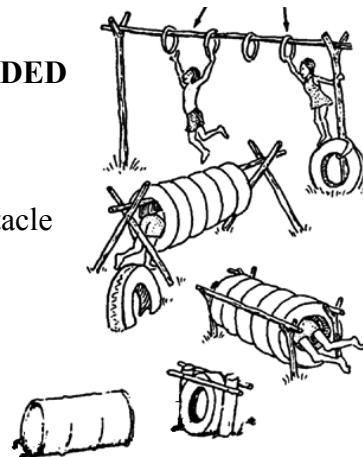
- (1) reflected about the y axis. (2)
 - (2) rotated about the origin through an angle of 90° in an anticlockwise direction. (2)
 - (3) enlarged through the origin by a factor of 2. (2)
- (b) $P(2;5)$ and $Q(5;1)$ are points in the Cartesian Plane. P is rotated clockwise about the origin through 90° to P' and Q is translated downwards by k units to Q' .
- (1) Write down the coordinates of P' and Q' . (4)
 - (2) Determine the value of k if the lengths PQ and $P'Q'$ are equal. (4)
- [14]**

QUESTION 3

ANSWER THIS QUESTION ON THE ANSWER SHEET PROVIDED

A group of 30 children was asked to complete an obstacle course.

The times (in seconds) taken by the 30 children to complete the obstacle course are given in the table below.



| Time (seconds) | $60 \leq t < 90$ | $90 \leq t < 120$ | $120 \leq t < 150$ | $150 \leq t < 180$ | $180 \leq t < 210$ |
|--------------------|------------------|-------------------|--------------------|--------------------|--------------------|
| Number of children | 3 | 6 | 7 | 8 | 6 |

- (a) Represent the data in the table by a cumulative frequency curve. (7)
- (b) Show on your graph where you would read off ...
 - (1) the number of children that took less than 135 seconds to complete the puzzle (use the letter A). (1)
 - (2) the value of t if 60% of the children took less than t seconds to complete the obstacle course (use the letter B). (1)
 - (3) the 75th percentile (use the letter C). (1)

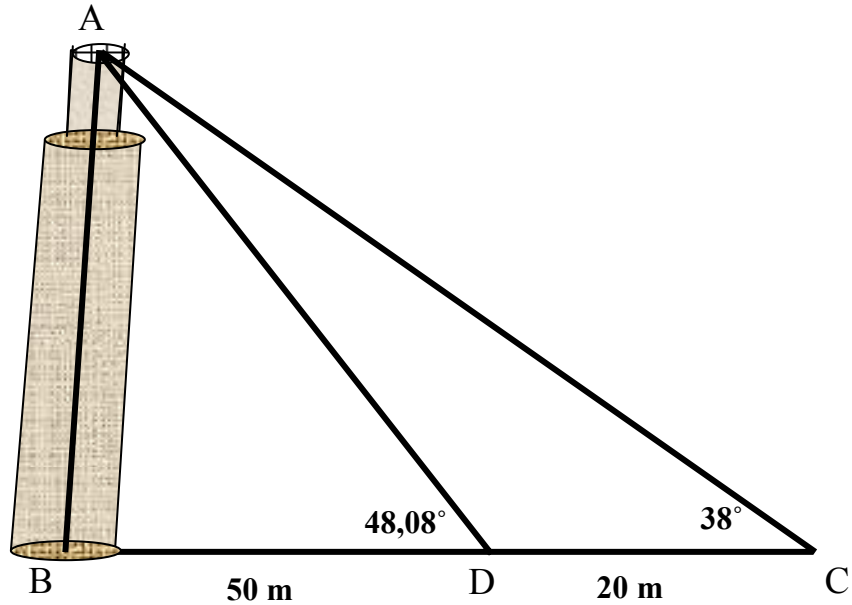
[10]

QUESTION 4

- (a) Solve for β , correct to one decimal digit if $\beta \in [0^\circ; 360^\circ]$ and $\cos \beta = \cos 232^\circ + \cos 108^\circ$.
(Please make sure that your calculator is in degree mode.) (3)
- (b) Prove the identity: $\frac{1}{1 - \sin \theta} - \frac{1}{1 + \sin \theta} = \frac{2 \tan \theta}{\cos \theta}$ (5)
- (c) If $\tan A = 3$ and $\sin A < 0$, determine, without using a calculator and with the aid of a sketch, the value of $2 \sin \frac{A}{2} \cdot \cos \frac{A}{2}$ (4)
- (d) Prove that: $\frac{\cos(x + 45^\circ)}{\cos x + \cos(90^\circ + x)} = \frac{\sqrt{2}}{2}$ (4)
- (e) Prove that $\sqrt{-\cos^2(90^\circ - M) - \cos M \cdot \cos(-M)}$ is non real for all real angles M. (3)
- [19]**

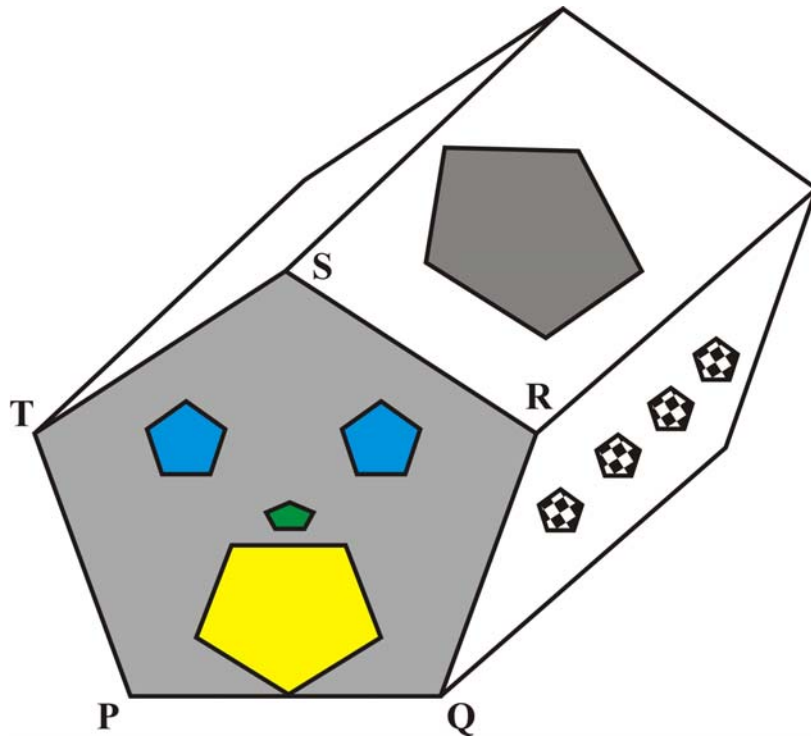
QUESTION 5

- (a) In the diagram below, the leaning tower of Pisa AB is shown. A tourist determines that the angle of elevation of the top of the building increases from 38° to $48,08^\circ$ after walking 20 metres towards the tower from point C to point D. He also determines that point D is 50 metres from the foot of the tower. BDC is horizontal and AB is not vertical.



- (1) Explain why $\hat{D}AC = 10,08^\circ$. (1)
- (2) Determine the straight line distance AD. Give your answer correct to two decimal digits. (3)
- (3) Determine the length of the tower AB correct to two decimal digits. (3)
- (4) Calculate $\hat{A}BH$ if H is a point above the ground and vertically above B. (4)

- (b) In the diagram below, a play house for children in the shape of a pentagonal prism is shown. The base of the prism, PQRST, is a regular pentagon of side 3 metres. The prism is 10 metres long. (All sides and all angles of a regular pentagon are equal).



- (1) Show that $\hat{QRS} = 108^\circ$. (1)
- (2) Determine the area of triangle QRS, correct to two decimal digits. (2)
- (3) If it is given that the area of $\triangle PSQ = 6,92 \text{ m}^2$, determine the volume of the prism correct to the nearest cubic metre. (2)

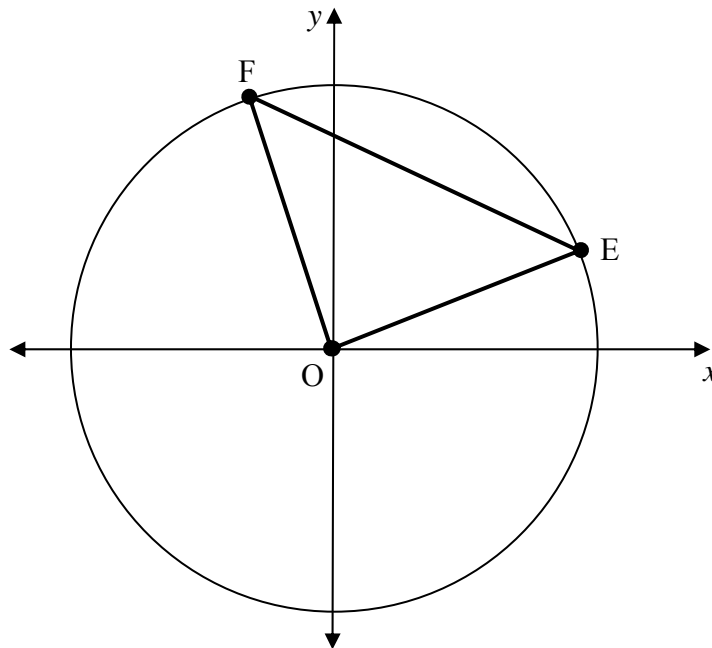
[16]

| |
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| 80 marks |
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SECTION B

QUESTION 6

- (a) In the diagram below, a circle centre O is drawn with E and F points on the circle. E lies on the straight line having equation $y = \frac{1}{4}x$ and F lies on the line having equation $y = -4x$.

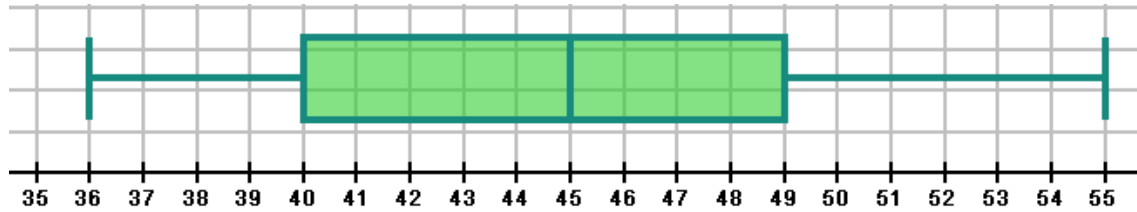


- (1) Explain why $\hat{EOF} = 90^\circ$. (1)
- (2) Determine the inclination of OE, to the nearest degree. (2)
- (3) Determine the inclination of EF, to the nearest degree. (3)
- (b) A circle with centre D is given by $x^2 + y^2 - 10x + 14y = 0$. Determine the co-ordinates of D. (3)
- (c) The lines with equations $3y = -4x - 1$ and $x + y = -1$ are diameters of a circle centre C. Determine the equation of this circle if it passes through the origin. (6)
- [15]**

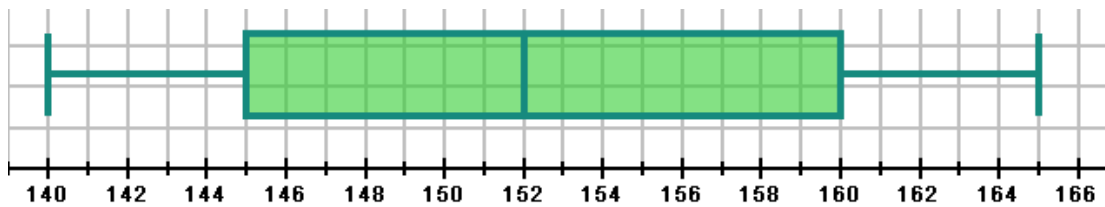
QUESTION 7

- (a) The heights (cm) and weights (kg) of a group of 23 students in a class are summarised by the box and whisker plots below.

Box and whisker plot for weights of 23 students



Box and whisker plot for heights of 23 students

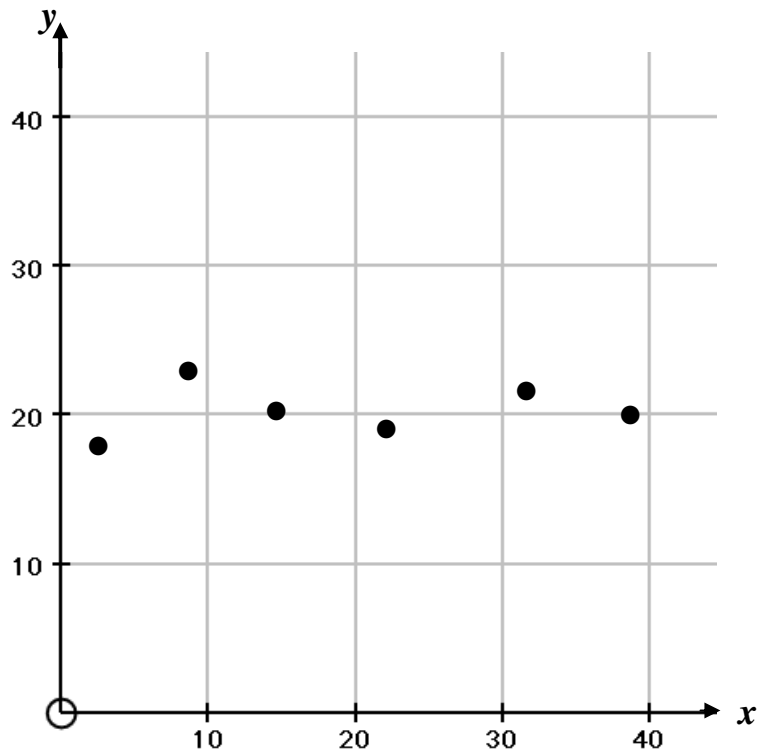


Given:
$$\text{Body mass index} = \frac{\text{body weight}}{(\text{body height})^2} \text{ (unit: kg/m}^2\text{)}$$

A body mass index lying between 20,1 kg/m² and 25 kg/m² is considered normal.

- (1) Portia 's weight is the lower quartile weight and her height is the upper quartile height. Determine whether or not Portia 's body mass index is normal. Show all necessary calculations. (3)
- (2) Dino is the shortest student in the class. It is known that his body mass index is normal. Determine Dino 's maximum weight. Show all necessary calculations. (3)

(b) Study the scatter plot below and then answer the questions that follow:

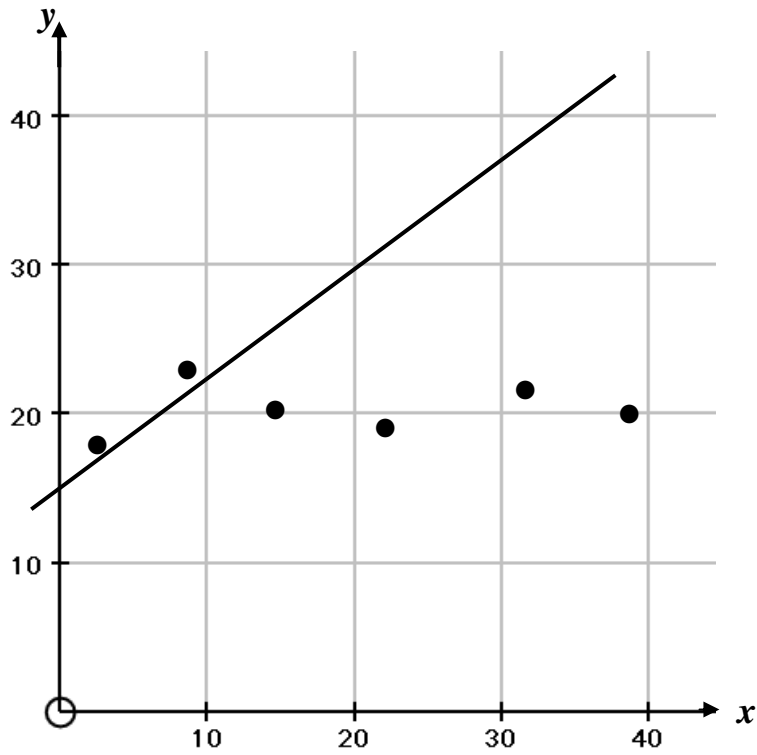


(1) Which one of the following is greater? Give a reason for your choice.

$$\sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}} \quad \text{OR} \quad \sqrt{\frac{\sum_{i=1}^n (y_i - \bar{y})^2}{n}} \quad (2)$$

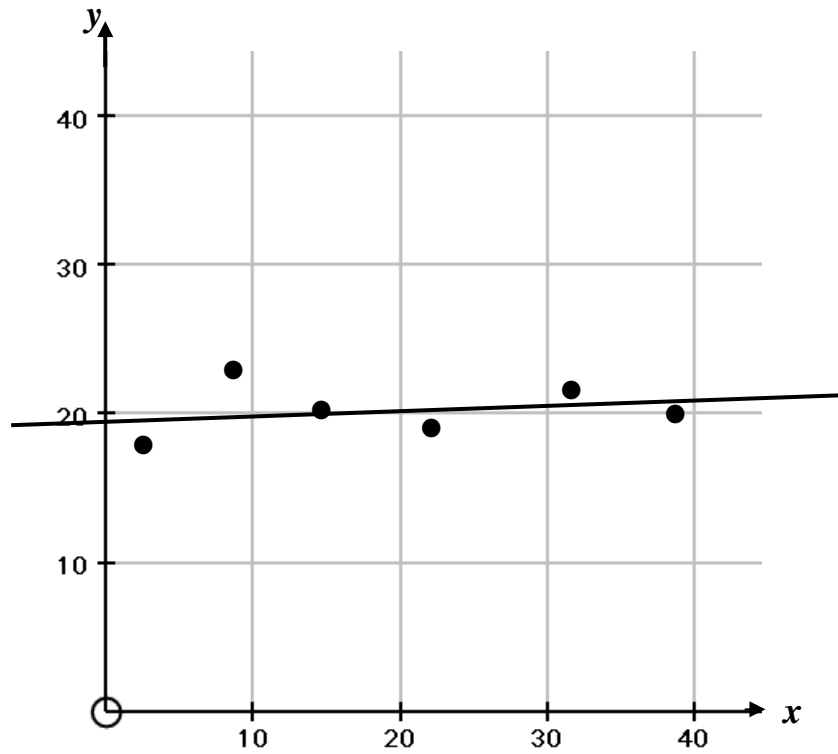
(2) Which of the following is the better line of best fit? (A or B).

LINE A



OR

LINE B



(1)

(c) Consider the following set of four positive whole numbers:

$$x + 3 ; 2x ; x - 1 ; 6$$

(1) Show that the variance can be given by $\frac{1}{2}(x^2 - 6x + 15)$. (4)

(2) Determine the standard deviation if $x = 5$. (2)

[15]

QUESTION 8

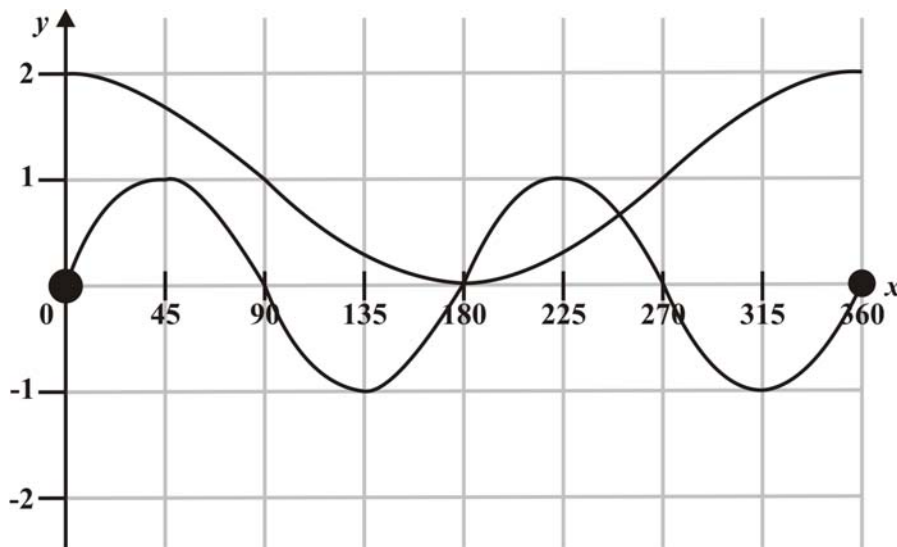
(a) Determine the general solution of $\sin^2 \beta + \sin 2\beta = 1$ if $\cos \beta \neq 0$. (7)

(b) Given: $\sin^2 x + \sin^2 y = \frac{3}{5}$.

(1) Show that $\cos^2 x + \cos^2 y = \frac{7}{5}$. (2)

(2) Determine the value of $\cos 2x + \cos 2y$. (2)

(c) In the diagram below, the curves of $y = \sin 2x$ and $y = \cos x + 1$ are sketched for the interval $[0^\circ; 360^\circ]$.



(1) Use your graph to give the approximate solution to $\sin 2x = \cos x + 1$ in the interval $x \in [0^\circ; 360^\circ]$. (2)

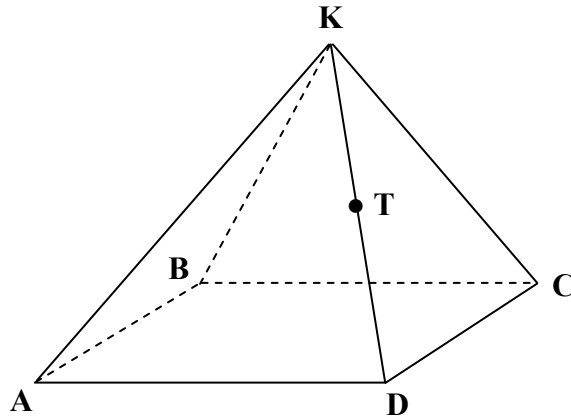
(2) Hence, write down the general solution to $\sin 2x = \cos x + 1$. (2)

[15]

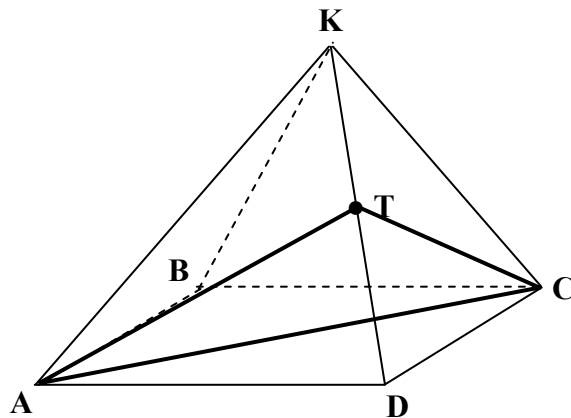
QUESTION 9

- (a) In the diagram below a square pyramid $KABCD$ is shown.
All edges of the square pyramid are 1 unit long.

T is a point on edge KD so that $KT = TD = \frac{1}{2}$.

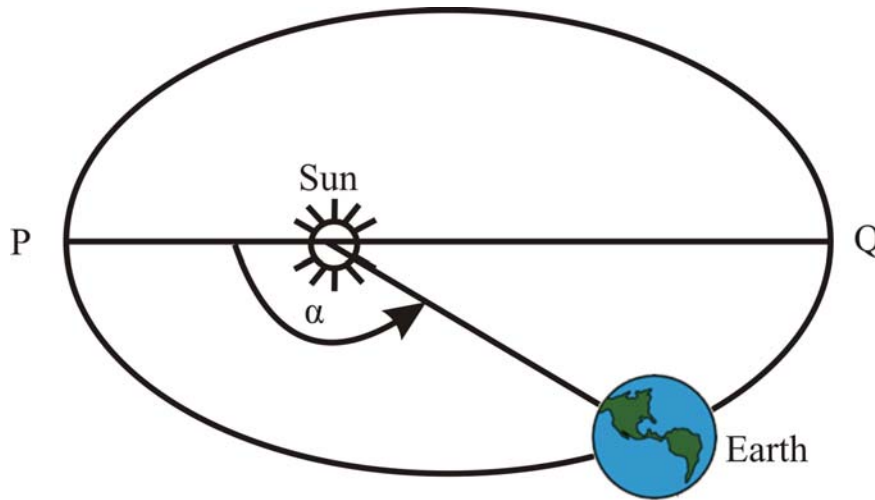


- (1) Explain why $\triangle KDC$ is an equilateral triangle and hence determine the length of CT , leaving your answer in surd form. (3)
- (2) In the diagram below, the sides of $\triangle ATC$ have been drawn in.



- (i) Write down the length of AT . (1)
- (ii) Determine the length of AC . (2)
- (iii) Determine the angle formed by the faces KAD and KDC (\hat{ATC}).
Give your answer correct to one decimal digit. (4)

- (b) In the diagram below, the earth's orbit around the sun is shown. P is the point on the orbit of the earth closest to the sun and Q is the point on the orbit furthest from the sun.



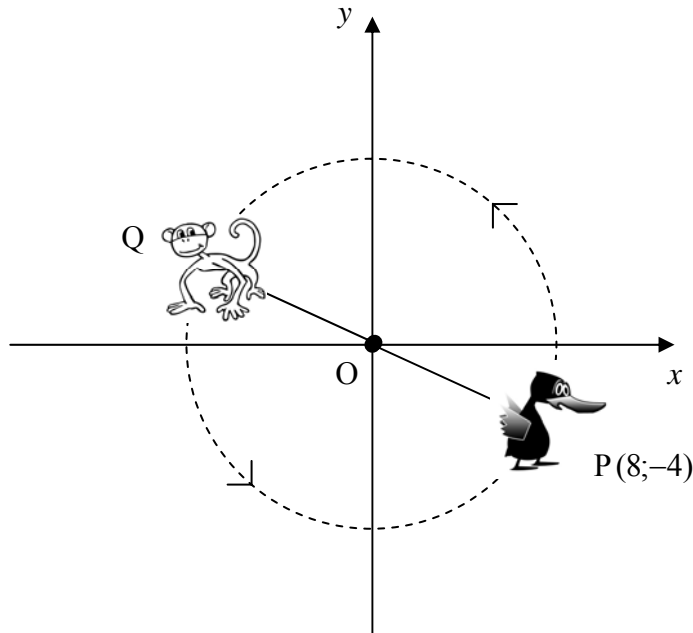
The distance d , in millions of kilometres, of the earth from the sun, is approximated by the formula $d = 149,6(1 - 0,0167 \cos \alpha)$, where α is the angle in degrees formed as shown in the diagram.

- (1) Determine the distance from Q to the sun. (3)
 - (2) Determine the angle α when the distance of the earth from the sun is 150 million km. (4)
- [17]**

QUESTION 10

In the diagram below, a monkey and a duck are placed at Q and P respectively. PQ is a diameter of the circle centre the origin.

P and Q rotate in an ANTICLOCKWISE direction about the origin at different speeds. P is the point $P(8;-4)$.



- (a) Write down the coordinates of Q. (2)

- (b) Determine the coordinates of the point where P and Q will first meet if the duck rotates about the origin at a rate of 25° per second and the monkey rotates about the origin at a rate of 85° per second. (6)

[8]

70 marks

Total: 150 marks