NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2010

MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 3 hours 150 marks

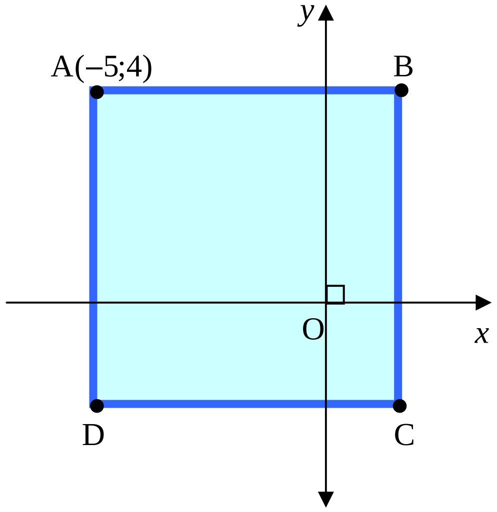
These marking guidelines were used as the basis for the official IEB marking session. They were prepared for use by examiners and sub-examiners, all of whom were required to attend a rigorous standardisation meeting to ensure that the guidelines were consistently and fairly interpreted and applied in the marking of candidates' scripts.

At standardisation meetings, decisions are taken regarding the allocation of marks in the interests of fairness to all candidates in the context of an entirely summative assessment.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines, and different interpretations of the application thereof. Hence, the specific mark allocations have been omitted.

SECTION A

QUESTION 1



(a) (1) B(1;4) a C(1;–2) a (2)

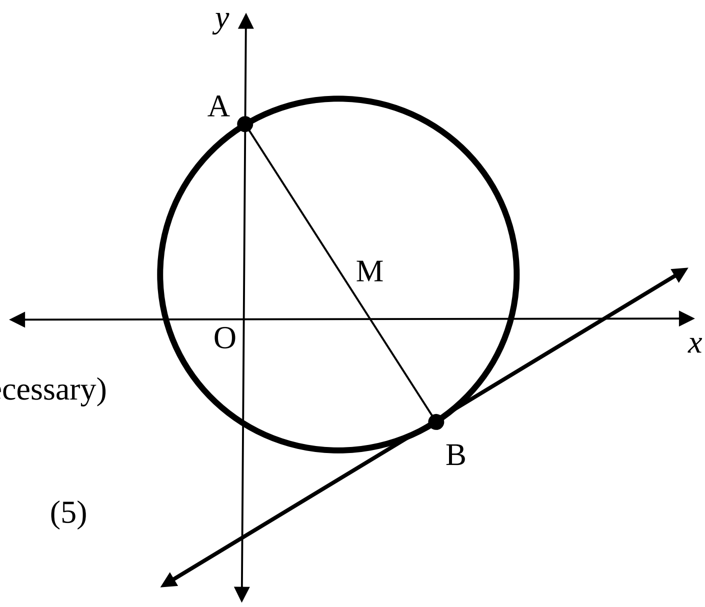
(2) $y = -2$ a or y – co-ord of c ca (1)

(3) $x = 1$ or x co-ord of B and c if both same (1)

(4) Gradient of AC is -1, ca

$\therefore$ $y - 4 = -(x - 1)$ ca

$\therefore$ $y = -x + 5$ ca (3)



(b) (1) M(3;1) a (1)

(2) $(x-3)^2 + (y-1)^2 = 25$

Let $x = 0$ m

$\therefore (0-3)^2 + (y-1)^2 = 25$

$\therefore (y-1)^2 = 16$ ✓a

$\therefore (y-1) = \pm 4$ ca – (not necessary)

$\therefore$ $y = 5$ or $y = -3$ ca

$\therefore$ $A(0;5)$ ca (must be 0; positive) (5)

(3) ✓m $\begin{cases} \dfrac{x_B + 0}{2} = 3 \quad \therefore \quad x_B = 6 \text{ ca} \\ \dfrac{y_B + 5}{2} = 1 \quad \therefore \quad y_B = -3 \text{ ca} \end{cases}$

$\therefore$ $B(6;-3)$ (3)

(4) $m_{AB} = \dfrac{5-(-3)}{0-6}$ m $= -\dfrac{4}{3}$ ca

product of gradients = –1

$\therefore$ $m_{tan} = \dfrac{3}{4}$ ca

$\therefore$ $y - (-3) = \dfrac{3}{4}(x - 6)$ ca (5)

[21]

QUESTION 2

(a) (1) centre: $(0;5)$ a equation: $x^2 + (y-5)^2 = 9$ ca (2)

(2) centre: $(-5;0)$ a equation: $(x+5)^2 + y^2 = 9$ ca (2)

(3) centre: $(0;10)$ a equation: $x^2 + (y-10)^2 = 36$ ca (2)

(b) (1) $P'(5;-2)$ a a , $Q'(5;1-k)$ a a (4)

(2) Method 1: P(2; 5) Q(5; 1)
P'(5; –2) Q'(5; 1 – k)

$PQ^2 = (P'Q')^2$

$\therefore\ 3^2 + 4^2 = (1 - k + 2)^2$ a m a

$\therefore\ 3 - k = \pm\ 5$

$\therefore\ k = -2$ or $k = 8$ ca

N/A

Method 2: Q' (5; 1 – k)
1 – k = –7
k = 8 (4)

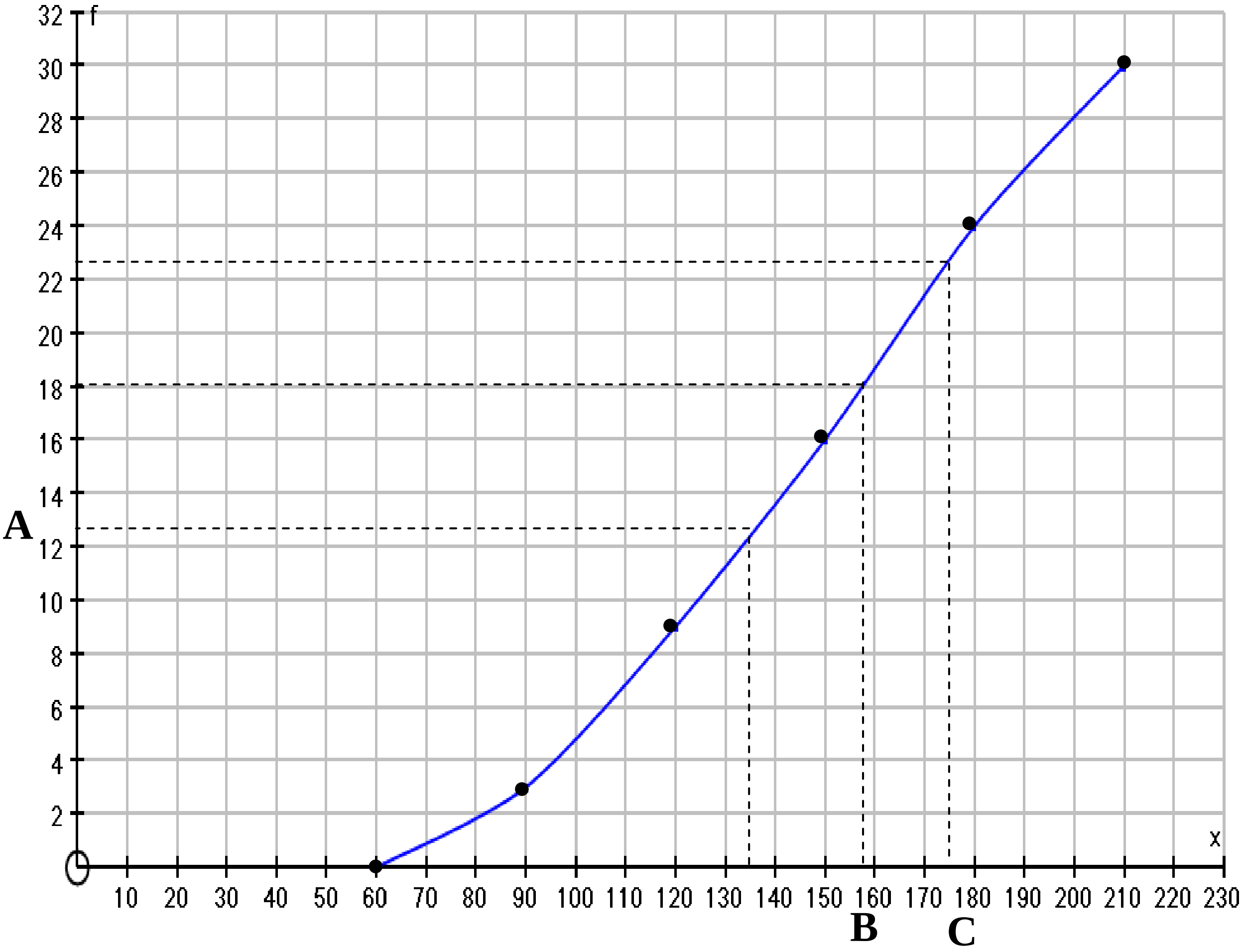
$PQ^2 = 25$ a

$1 - k = -2 - 5$ ca ca

$k = 8$ a

[14]

QUESTION 3



(a) for the point (60;0) ca for the point (90;3) a

for the point (120;9) ca for the point (150;16) ca

for the point (180; 24) ca for the point (210; 30) ca

for the curve through the points ca (7)

(b) (1) dotted lines and A ca (1)

(2) dotted lines and B ca (1)

(3) dotted lines and C ca (1)

[10]

QUESTION 4

(a) $\cos\beta = \cos 232° + \cos 108°$

$\cos\beta = -0{,}924678469$ a

$\therefore \quad \beta = 180° - 22{,}3...° = 157{,}6°$ ca or $\therefore \quad \beta = 180° + 22{,}380079...° = 202{,}4°$ ca (3)

(b) LHS $= \dfrac{1}{1-\sin\theta} - \dfrac{1}{1+\sin\theta}$

$= \dfrac{1+\sin\theta-(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$ a a

$= \dfrac{2\sin\theta}{1-\sin^2\theta}$ ca

$= \dfrac{2\sin\theta}{\cos^2\theta}$ a $= \dfrac{2\left(\dfrac{\sin\theta}{\cos\theta}\right)}{\cos\theta}$ a

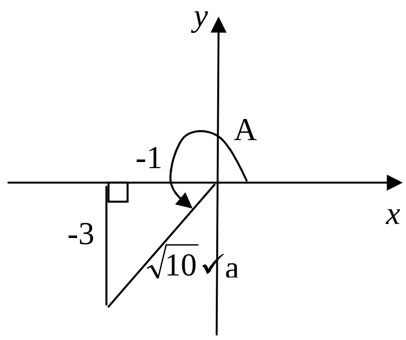
$= \dfrac{2\tan\theta}{\cos\theta} =$ RHS (5)

(c)

$2\sin\dfrac{A}{2}\cos\dfrac{A}{2}$

$= \sin A$ a

$= \dfrac{-3}{\sqrt{10}}$ a ca



(4)

(d) $\dfrac{\cos(x+45°)}{\cos x + \cos(90°+x)}$

$= \dfrac{\cos x.\cos 45° - \sin x.\sin 45°}{\cos x - \sin x}$ a a

ca $\begin{cases} = \dfrac{\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x}{\cos x - \sin x} \text{ a } = \dfrac{\frac{\sqrt{2}}{2}(\cos x - \sin x)}{\cos x - \sin x} \\ = \dfrac{\sqrt{2}}{2} \end{cases}$

(4)

(e) $\sqrt{-\cos^2(90°-M) - \cos M.\cos(-M)}$

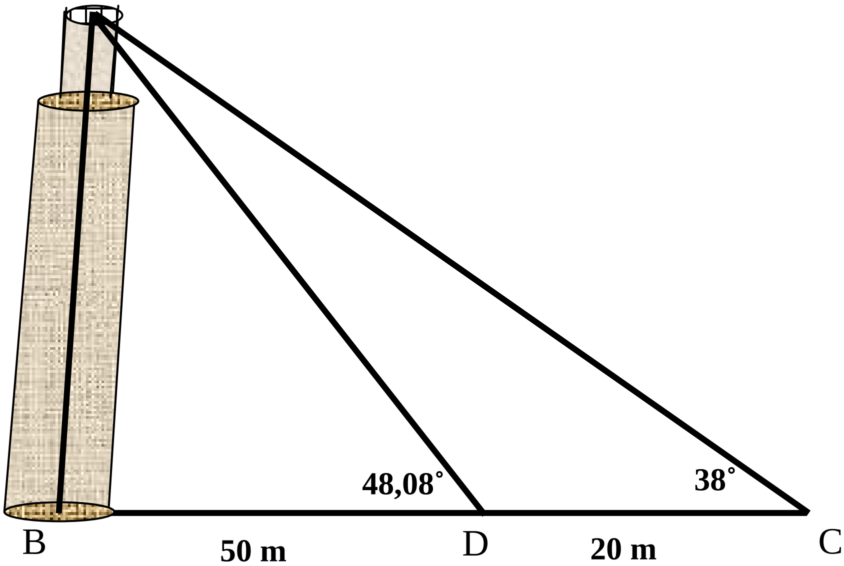
$= \sqrt{-\sin^2 M - \cos^2 M}$ a a

$= \sqrt{-1}$
Which is non-real for all real values of M. } a (3)

[19]

QUESTION 5

(a)



(1) $B\hat{D}C = D\hat{A}C + D\hat{C}A$, exterior angle of triangle a Sum of ∠'s of a Δ a (1)

(2) $\frac{AD}{\sin 38°} = \frac{20}{\sin 10{,}08°}$ m for sine rule
a subs correctly into sine rule.

$\therefore \quad AD = \frac{20 \sin 38°}{\sin 10{,}08°} = 70{,}35$ m ca (3)

(3) $AB^2 = 70{,}35^2 + 50^2 - 2 \times 50 \times 70{,}35 \times \cos 48{,}08°$ m ca

$\therefore \quad AB^2 = 2749{,}09$

$\therefore \quad AB = 52{,}43$ ca (3)

(4) $\frac{AD}{\sin\left(A\hat{B}D\right)} = \frac{AB}{\sin 48{,}08^\circ}$ m for finding $A\hat{B}D$

$\sin\left(A\hat{B}D\right) = \frac{70{,}35 \cdot \sin 48{,}08^\circ}{52{,}43}$ ca into sine/ cosine

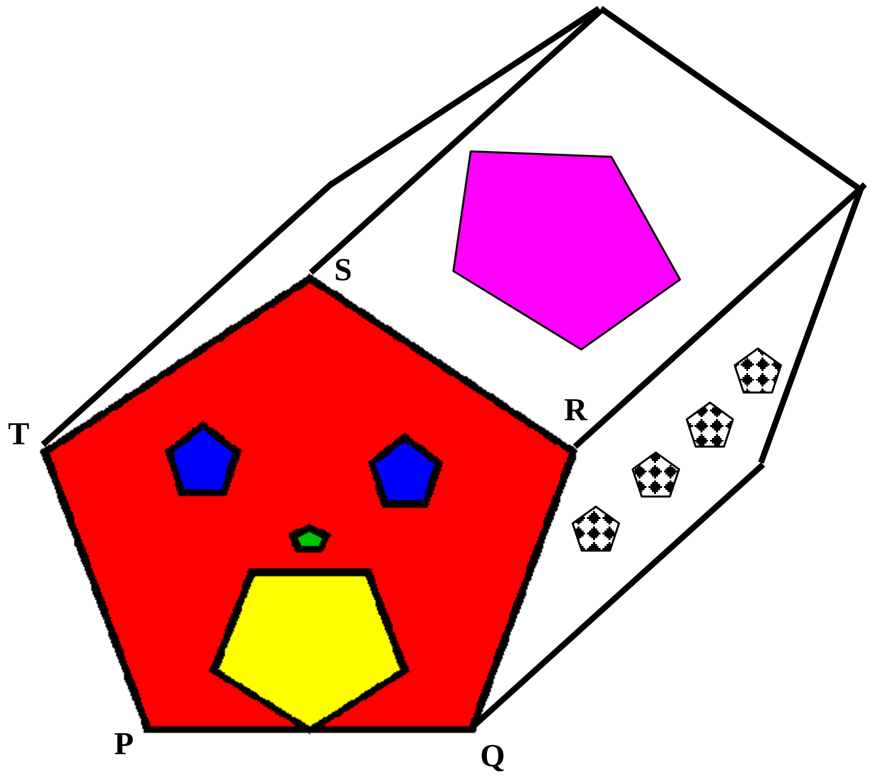
$= 0{,}99839$

$\therefore A\hat{B}D = 86{,}75^\circ$ ca

$\therefore A\hat{B}H = 3{,}25^\circ$ ca

$= 3{,}3^\circ$ (1 d : p.) (4)

(b)



(1) $\frac{540°}{5} = 108°$ a or (1) $\frac{360}{5} = 72°$ $\therefore 180 - 72 = 108°$ a

(2) Area of $\Delta QRS = \frac{1}{2} \times 3 \times 3 \times \sin 108° = 4,28 \text{ cm}^2$ a ca (2)

(3) Volume of prism
= [(2 × 4,28) + 6,92] × 10 ca
= 154,8 m^3 ca (2)

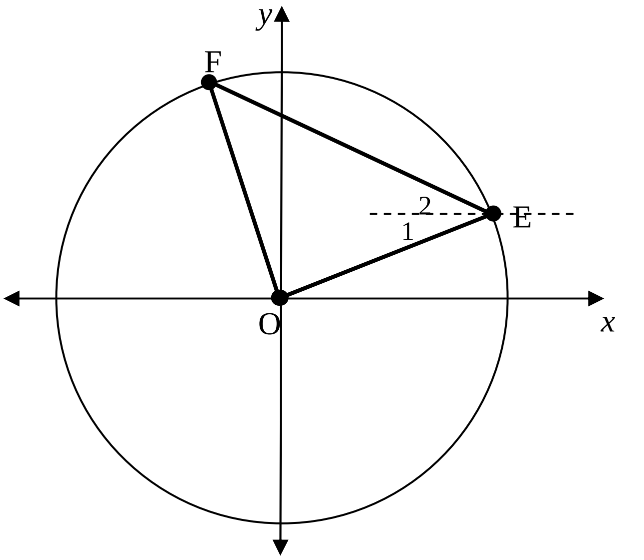
[16]

80 marks

SECTION B

QUESTION 6

(a)



(1) $E\hat{O}F = 90°$. $m_{OE} \times m_{OF} = -1$ a or Transformation rule (1)

(2) $\tan\theta = \frac{1}{4}$ a $\therefore \theta = 14°$ a (2)

(3) ΔFOE is an isosceles triangle.
Therefore, $O\hat{E}F = O\hat{F}E = 45°$ a
$\hat{E}_1 = 14°$ a ; parallel lines, alt angles
Therefore, $\hat{E}_2 = 45° - 14° = 31°$
Therefore the inclination of EF is $149°$ ca (3)

(b) (1) $x^2 + y^2 - 10x + 14y = 0$
$\therefore (x-5)^2 - 25 + (y+7)^2 - 49 = 0$
a
$\therefore (x-5)^2 + (y+7)^2 = 74$
$\therefore$ D (5; –7) (3)
ca ca

(2) $y = -x - 1$ a or any other arrangement of egn.
$\therefore 3(-x-1) = -4x - 1$ ca
$\therefore -3x - 3 = -4x - 1$
$\therefore x = 2$ ca
$\therefore y = -2 - 1$
$y = -3$ ca
$\therefore$ C (2; –3)
$\therefore$ eqn of circle:
$(x-2)^2 + (y+3)^2 = (0-2)^2 + (0+3)^2$
$\therefore \underbrace{(x-2)^2 + (y+3)^2}_{ca} = \underbrace{13}_{ca}$ (6)

[15]

QUESTION 7

(a) (1) Portia's weight = 40 kg a Portia's height = 1,60 m a

Body mass Index = $\frac{40}{(1,6)^2}$

Body Mass Index = 15,625
Therefore, Portia's body mass index is not normal. } ca (3)

(2) Dino's body mass Index = $\frac{\text{weight}}{1,4^2}$ a

$\therefore \quad 20,1 < \frac{\text{Dino's weight}}{1,96} < 25$ ca

$\therefore \quad 39,396 < \text{Dino's weight} < 49$

Therefore, Dino's maximum weight is 49 kg. ca (3)

(b) (1)

$\sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}}$ a . The x values are more spread out. ca (2)

(2) B a (1)

(c) (1) mean $= \frac{x+3+2x+x-1+6}{4} = \frac{4x+8}{4} = x+2$ m a

Therefore the variance =

$\frac{(x+3-x-2)^2 + (2x-x-2)^2 + (x-1-x-2)^2 + (6-x-2)^2}{4}$ a correct deviation a

$= \frac{1+x^2-4x+4+9+x^2-8x+16}{4}$

$= \frac{2x^2-12x+30}{4} = \frac{1}{2}(x^2-6x+15)$ ca (4)

(2) Variance = $= \frac{1}{2}(5^2-6(5)+15) = 5$ a

Therefore, standard deviation $= \sqrt{5}$ a (2)

[15]

QUESTION 8

(a) $\sin^2\beta + \sin 2\beta = 1$ if $\cos\beta \neq 0$

$\therefore\quad \sin^2\beta + 2\sin\beta\cos\beta = 1$ a

$\therefore\quad 2\sin\beta\cos\beta - \cos^2\beta = 0$ a

$\therefore\quad 2\sin\beta - \cos\beta = 0$ since $\cos\beta \neq 0$ ca

$\therefore\quad \tan\beta = \frac{1}{2}$ ca

$\therefore\quad \beta = 26{,}6° + k.180$ where k is an integer. a ca ca (7)

(b) (1) Method 1:

$\cos^2 x + \cos^2 y$
$= 1 - \sin^2 x + 1 - \sin^2 y$ a
$= 2 - (\sin^2 x + \sin^2 y)$
$= 2 - \frac{3}{5} = \frac{7}{5}$
a

Method 2:

$\sin^2 x + \cos^2 x + \sin^2 x + \sin^2 y = 2,$ a
therefore $\cos^2 x + \cos^2 y = 2 - \frac{3}{5} = \frac{7}{5}$ a (2)

(2) $\cos 2x + \cos 2y$
$= (\cos^2 x - \sin^2 x) + (\cos^2 y - \sin^2 y)$ a
$= \frac{4}{5}$ a (2)

(c) (1) $x = 180°$ or $x \approx 250°$ (accept any approv. solution) (2)
a a

(2) $x = 180° + k.360°$ or $x \approx 250° + k.360°$ where k is an integer (2)
a a

[15]

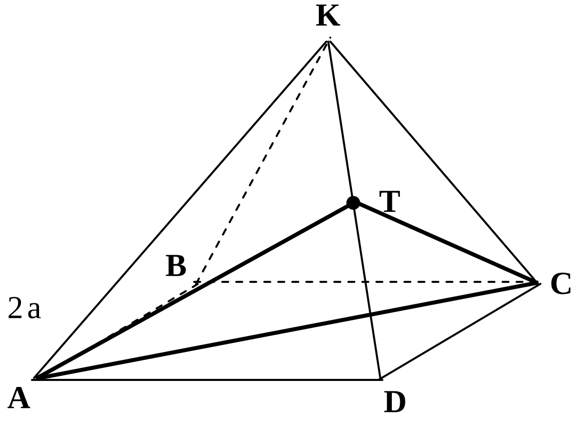
QUESTION 9

(a) (1) ΔKDC is an equilateral triangle because all sides are equal. a

$CT^2 + TD^2 = CD^2$ m

$\therefore\quad CT^2 + \frac{1}{4} = 1$

$\therefore\quad CT = \frac{\sqrt{3}}{2}$ a (3)



(2) (i) $AT = CT = \frac{\sqrt{3}}{2}$ ca (1)

(ii) $AC^2 = AD^2 + DC^2 = 1 + 1 = 2$ a

$\therefore\quad AC = \sqrt{2}$ a (2)

(iii) The angle formed by the faces KAD and KDC is the angle $\hat{ATC}$.

$AC^2 = AT^2 + TC^2 - 2 \times AT \times TC.\cos A\hat{T}C$ a

$\therefore 2 = \frac{3}{4} + \frac{3}{4} - 2 \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} \cos A\hat{T}C$ ca

$\therefore \cos A\hat{T}C = -\frac{1}{3}$ ca $\therefore A\hat{T}C = 109{,}5°$ ca (4)

(b) (1) α = 180°

d = 149,6 (1 – 0,0167 × cos180°) = 152 000 000 km correct answer with units (3)
a ca a

(2) 150 = 149,6 (1 – 0,0167 cos α) a

$\cos\alpha = \frac{150}{149,6} - 1$

–0,0167

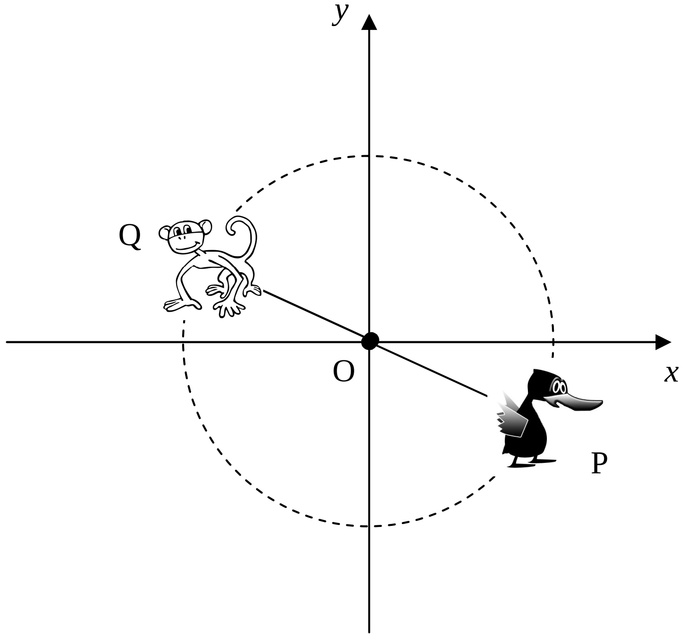
cos α) = – 0,1601ca

α = 99,2° ca

or 260,8° ca (4)

[16]

QUESTION 10



(a) Q(–8;4) (2)
a a

(b) P and Q meet after 3 seconds. That is, after the duck has rotated about the origin by 75°. a
a a m

$x' = 8\cos 75° - (-4)\sin 75° = 5{,}9$ ca

$y' = -4\cos 75° + 8\sin 75° = 6{,}7$ ca

Therefore the point is (5,9 ; 6,7) (6)

[8]

70 marks

Total: 150 marks