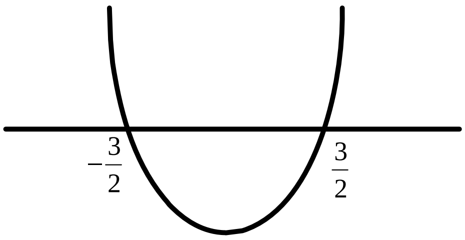
MATRIC MATHEMATICS PAPER 1 MEMORANDUM

QUESTION 1

1.1.1	$\frac{x^2-1}{x+1}=2$ $\therefore x^2-1=2(x+1)$ $\therefore x^2-1=2x+2$ $\therefore x^2-2x-3=0$ $\therefore (x-3)(x+1)=0$ $\therefore x=3$ or $x=-1$ But $x\neq -1$ $\therefore x=3$ is the solution OR $\frac{x^2-1}{x+1}=2$ $\therefore \frac{(x+1)(x-1)}{(x+1)}=2$ $\therefore x-1=2$ provided $x\neq -1$ $\therefore x=3$	✔ standard form = 0 ✔ factorisation ✔ both answers ✔ excluding $x=-1$ (4) ✔ factorisation ✔ simplification ✔ correct answer ✔ excluding $x=-1$

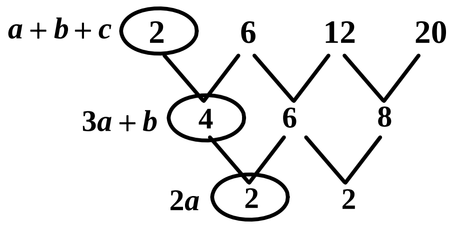
1.1.2	$8-(x-2)(x-3)=0$ $\therefore 8-(x^2-5x+6)=0$ $\therefore 8-x^2+5x-6=0$ $\therefore -x^2+5x+2=0$ $\therefore x^2-5x-2=0$ $\therefore x=\frac{-(-5)\pm\sqrt{(-5)^2-4(1)(-2)}}{2(1)}$ $\therefore x=\frac{5\pm\sqrt{33}}{2}$ $\therefore x=5,37$ or $x=-0,37$ – 1 for inaccurate rounding off for both answers.	✔ simplification ✔ standard form ✔ substitution into formula ✔ correct answers (4)

1.2.1	$4x^2<9$ $\therefore 4x^2-9<0$ $\therefore (2x+3)(2x-3)<0$ $\therefore -\frac{3}{2}<x<\frac{3}{2}$  	✔ factorisation ✔ endpoints ✔ inequality notation (3)
1.2.2	$x\in\{-1;0;1\}$	✔ correct answer (1)
1.3	$3x-y=2$ $\therefore 3x-2=y$ $\therefore 3(3x-2)+9x^2=4$ $\therefore 9x-6+9x^2=4$ $\therefore 9x^2+9x-10=0$ $\therefore (3x+5)(3x-2)=0$ $\therefore x=-\frac{5}{3}$ or $x=\frac{2}{3}$ $\therefore y=-7$ or $y=0$	✔ $3x-2=y$ ✔ substitution ✔ standard form ✔ factorisation ✔ both x-values ✔ $y=-7$ ✔ $y=0$ (7)

QUESTION 2

2.1	Area of triangle 1: $\frac{1}{2}(2cm)(2cm)=(1)(2)cm^2$ Area of triangle 2: $\frac{1}{2}(4cm)(3cm)=(2)(3)cm^2$ a Area of triangle 3: $\frac{1}{2}(6cm)(4cm)=(3)(4)cm^2$ Area of triangle 4: $\frac{1}{2}(8cm)(5cm)=(4)(5)cm^2$ The areas form the following pattern:	✔ determining areas ✔ establishing pattern ✔ obtaining general term ✔ area of 100th triangle (4)

	$(1)(2);(2)(3);(3)(4);(4)(5);...........$ Area of triangle *n*: $(n)(n+1)cm^2$ Area of triangle 100: $(100)(100+1)cm^2 = 10100cm^2$	
	OR Area of triangle 1: $\frac{1}{2}(2cm)(2cm)$ Area of triangle 2: $\frac{1}{2}(4cm)(3cm)$ Area of triangle 3: $\frac{1}{2}(6cm)(4cm)$ Area of triangle 4: $\frac{1}{2}(8cm)(5cm)$ The bases are in arithmetic sequence: 2; 4; 6; 8; 10; .... General term is $2+(n-1)2 = 2n$ The heights are in arithmetic sequence: 2; 3; 4; 5; .... General term is $2+(n-1)(1) = n+1$ Therefore, the general term for areas is: Area of triangle *n*: $\frac{1}{2}(2n)(n+1)cm^2$ $= n(n+1)cm^2$ Area of triangle 100: $(100)(100+1)cm^2 = 10100cm^2$ **OR** Area of triangle 1: $\frac{1}{2}(2cm)(2cm) = 2cm^2$ Area of triangle 2: $\frac{1}{2}(4cm)(3cm) = 6cm^2$ Area of triangle 3: $\frac{1}{2}(6cm)(4cm) = 12cm^2$ Area of triangle 4: $\frac{1}{2}(8cm)(5cm) = 20cm^2$	✔ determining areas ✔ general terms of arithmetic sequences ✔ obtaining area of *n*th triangle ✔ area of 100th triangle ✔ determining areas ✔ quadratic pattern ✔ obtaining general term ✔ area of 100th triangle

	The areas form a quadratic number pattern: 2; 6; 12; 20; ....  $2a = 2$ $\quad 3a+b=4$ $\quad a+b+c=2$ $a=1$ $\quad \therefore 3(1)+b=4$ $\quad \therefore 1+1+c=2$ $\therefore b=1$ $\quad \therefore c=0$ Area of triangle *n*: $(n^2+n)\,cm^2$ Area of triangle 100: $\left[(100)^2+100\right]cm^2 = 10100cm^2$	
2.2	$n(n+1)=240$ $\therefore n^2+n-240=0$ $\therefore (n+16)(n-15)=0$ $\therefore n=-16$ or $n=15$ But $n \neq -16$ $\therefore n=15$ The 15th triangle will have an area of $240\,\text{cm}^2$	✔ equating general term to 240 ✔ factorising ✔ obtaining 15 triangles (3)

QUESTION 3

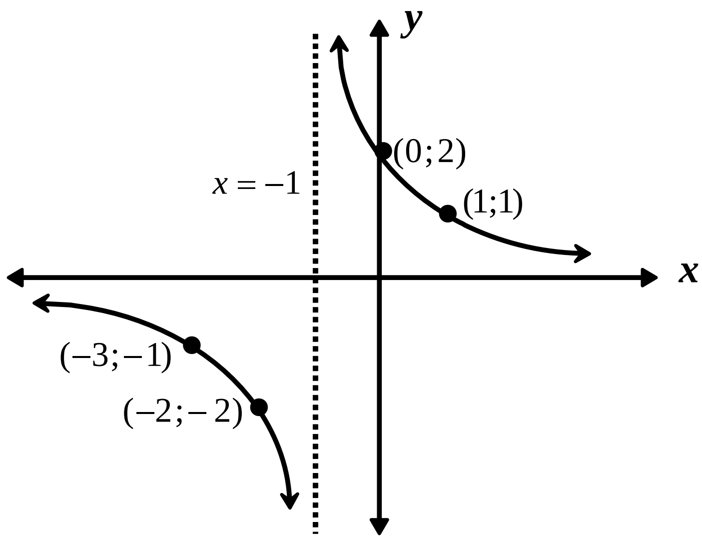
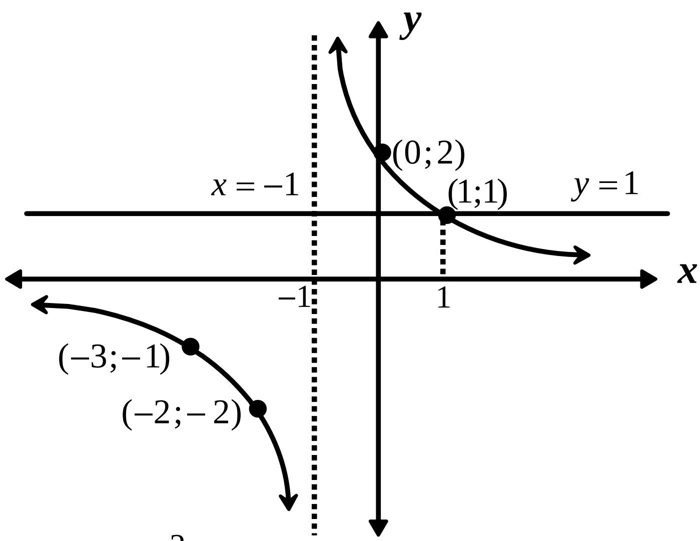
3.1.1	$\frac{1}{181}+\frac{2}{181}+\frac{3}{181}+\frac{4}{181}+.....................+\frac{180}{181}$ $a=\frac{1}{181}$ $\quad d=\frac{1}{181}$ $\quad n=180$ $\therefore S_{180}=\frac{180}{2}\left[\frac{1}{181}+\frac{180}{181}\right]=90[1]=90$ OR $S_{180}=\frac{180}{2}\left[2\left(\frac{1}{181}\right)+(179)\frac{1}{181}\right]=90[1]=90$	✔ correct *a* and *d* ✔ correct *n* ✔ S_n formula ✔ correct answer (4)

3.1.2	$\left(\frac{1}{2}\right)+\left(\frac{1}{3}+\frac{2}{3}\right)+\left(\frac{1}{4}+\frac{2}{4}+\frac{3}{4}\right)+....+\left(\frac{1}{181}+\frac{2}{181}+....+\frac{180}{181}\right)$ $=\frac{1}{2}+1+1\frac{1}{2}+2+..........+90$ $a=\frac{1}{2} \quad d=\frac{1}{2} \quad T_n=90$ $\therefore \frac{1}{2}+(n-1)\frac{1}{2}=90$ $\therefore 1+n-1=180$ $\therefore n=180$ $\therefore S_{180}=\frac{180}{2}\left[\frac{1}{2}+90\right]=90\left[90\frac{1}{2}\right]=8145$ OR $S_{180}=\frac{180}{2}\left[2\left(\frac{1}{2}\right)+(179)\left(\frac{1}{2}\right)\right]=90\left[90\frac{1}{2}\right]=8145$	✔ simplifying fractions to get series ✔ $\frac{1}{2}+(n-1)\frac{1}{2}=90$ ✔ $n=180$ ✔ substitution into S_n formula to get 8145 (4)
3.2	$ar^5=\sqrt{3}$ $ar^7=\sqrt{27}$ $\therefore \frac{ar^7}{ar^5}=\frac{\sqrt{27}}{\sqrt{3}}$ $\therefore r^2=\sqrt{\frac{27}{3}}$ $\therefore r^2=\sqrt{9}$ $\therefore r^2=3$ $\therefore r=\sqrt{3}$ (terms are positive) $\therefore a(\sqrt{3})^5=\sqrt{3}$ $\therefore a=\frac{\sqrt{3}}{(\sqrt{3})^5}$ $\therefore a=\frac{1}{(\sqrt{3})^4}$ $\therefore a=\frac{1}{(3^{\frac{1}{2}})^4}$ $\therefore a=\frac{1}{9}$	✔ $ar^5=\sqrt{3}$; $ar^7=\sqrt{27}$ ✔ dividing ✔ $r=\sqrt{3}$ ✔ correct working with surds ✔ $a=\frac{1}{9}$ (5)

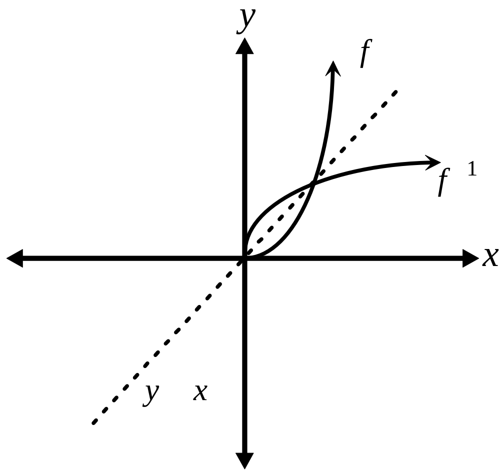
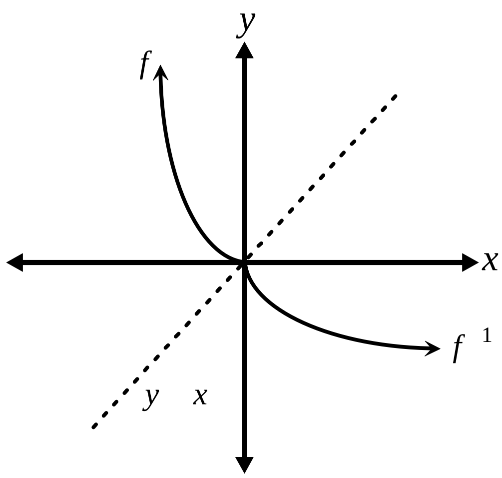
3.3.1	$\sum_{n=1}^{\infty} 2(\frac{1}{2}x)^n$ $= 2(\frac{1}{2}x)^1 + 2(\frac{1}{2}x)^2 + 2(\frac{1}{2}x)^3 + 2(\frac{1}{2}x)^4 + \ldots\ldots\ldots\ldots$ $= x + \frac{1}{2}x^2 + \frac{1}{4}x^3 + \frac{1}{8}x^4 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots$ The series converges for: $-1 < \frac{1}{2}x < 1$ $\therefore -2 < x < 2$	✔ $r = \frac{1}{2}x$ ✔ $-1 < \frac{1}{2}x < 1$ ✔ $-2 < x < 2$ (3)
3.3.2	$a = \frac{1}{2} \quad r = \frac{1}{2}\left(\frac{1}{2}\right) = \frac{1}{4}$ $\therefore S_\infty = \frac{\frac{1}{2}}{1-\frac{1}{4}} = \frac{2}{3}$	✔ a and r ✔ S_∞ formula ✔ $\frac{2}{3}$ (3)
3.4	$4 + 6 + 9 + 13,5 + \ldots\ldots\ldots\ldots$ $a = 4 \quad r = \frac{3}{2} \quad S_n = 2000\ 000$ $\therefore 2000\ 000 = \frac{(4)\left[\left(\frac{3}{2}\right)^n - 1\right]}{\frac{3}{2}-1}$ $\therefore 2000\ 000 = 8\left[\left(\frac{3}{2}\right)^n - 1\right]$ $\therefore 250\ 000 = \left(\frac{3}{2}\right)^n - 1$ $\therefore 250\ 001 = \left(\frac{3}{2}\right)^n$ $\therefore \log_{\frac{3}{2}}(250\ 001) = n$ $\therefore n = 30,65422881$ Malibongwe will be able to pay off the R2000 000 on the last day of March (31 days)	✔ constant ratio ✔ correct substitution into the S_n formula ✔ use of logs ✔ $n = 30,65422881$ ✔ 31 days (5)

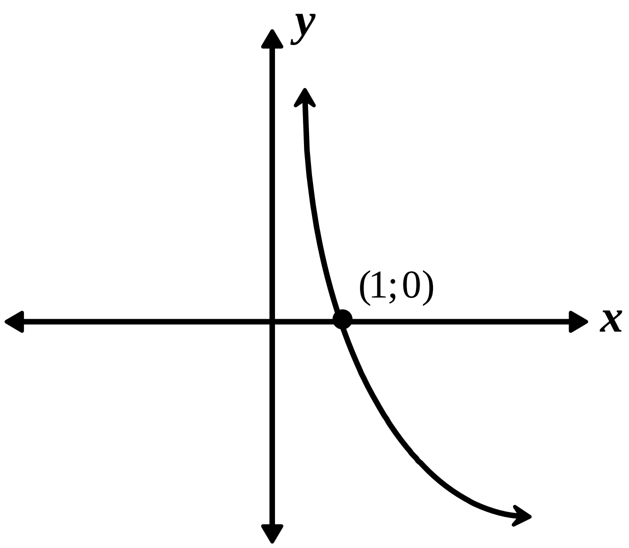
QUESTION 4

4.1.1	vertical: $x = -1$ horizontal: $y = 0$	✔ vertical asymptote ✔ horizontal asymptote (2)

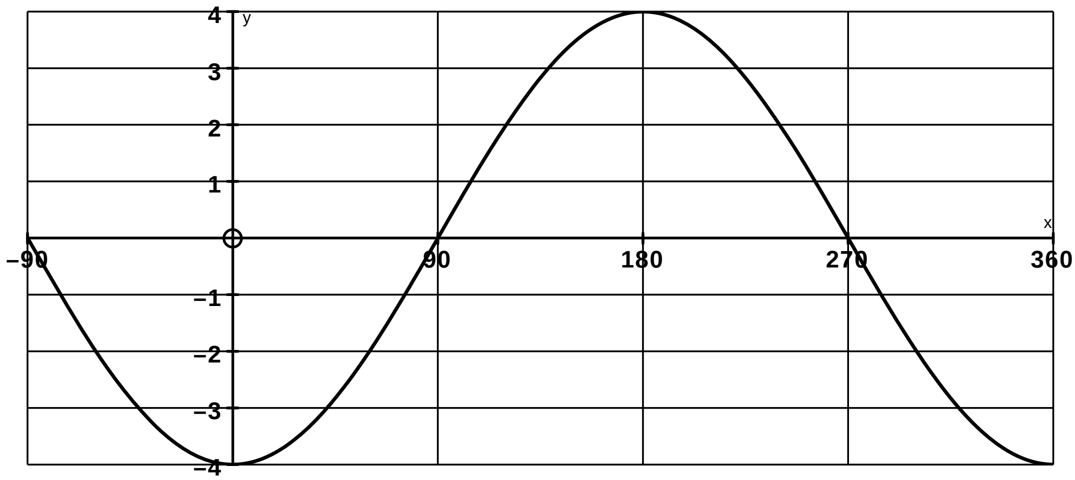
4.1.2		✔ $x = -1$ ✔ $y = 0$ ✔ left branch ✔ coordinates on left branch ✔ right branch ✔ coordinates on right branch (6)
4.1.3	$y = \frac{2}{x+1-3} + 2$ $\therefore y = \frac{2}{x-2} + 2$	✔ denominator: $x - 2$ ✔ +2 (2)
4.1.4	 Therefore $\frac{2}{x+1} \geq 1$ for $-1 < x \leq 1$	✔ $-1 < x$ ✔ $x \leq 1$ (2)

4.2.1	$f(x) = 2x^2$ where $x \geq 0$ OR $f(x) = 2x^2$ where $x \leq 0$	✔ $x \geq 0$ OR $x \leq 0$ (1)

4.2.2	 OR 	✔ f ✔ f^{-1} (2)
4.2.3	$y=\left(\frac{1}{2}\right)^x$ $x=\left(\frac{1}{2}\right)^y$ $\therefore \log_{\frac{1}{2}} x = y$ $\therefore g^{-1}(x)=\log_{\frac{1}{2}} x$	✔ $x=\left(\frac{1}{2}\right)^y$ ✔ $g^{-1}(x)=\log_{\frac{1}{2}} x$ (2)

4.2.4		✔ shape ✔ (1;0) (2)
4.2.5	$\log_{\frac{1}{2}} x < 0$ for $x > 1$	✔ $x > 1$ (1)

QUESTION 5

5.1	$y = -2f(x)$ $\therefore y = -2(2\cos x)$ $\therefore y = -4\cos x$	
		✔ amplitude ✔ domain (2)

5.2	Amplitude is 4	✔ amplitude (1)

5.3	$y = f\left(\frac{x}{2}\right)$ $\therefore y = 2\cos\left(\frac{1}{2}x\right)$ period is $\frac{360°}{\frac{1}{2}} = 720°$	✔ 720° (1)
5.4	$g(x) = f(x) - 2$ $g(x) = 2\cos x - 2$ maximum is 0	✔ max (1)

QUESTION 6

6.1	$y = a(x+p)^2 + q$ $\therefore y = a(x+1)^2 + 6$ Substitute $(0\,;2)$: $\therefore 2 = a(0+1)^2 + 6$ $\therefore 2 = a + 6$ $\therefore 2 - 6 = a$ $\therefore a = -4$ $\therefore f(x) = -4(x+1)^2 + 6$	✔ $y = a(x+1)^2 + 6$ ✔ Substitute $(0\,;2)$ ✔ $a = -4$ ✔ $f(x) = -4(x+1)^2 + 6$ (4)
6.2	Range: $y \in \left(-\infty;6\right]$	✔ $y \in \left(-\infty;6\right]$ (1)
6.3	$y = -4(x+1)^2 + 6 \quad (f)$ $\therefore -y = -4(x+1)^2 + 6 \quad (g)$ $\therefore g(x) = 4(x+1)^2 - 6$	✔ $g(x) = 4(x+1)^2 - 6$ (1)

QUESTION 7

7.1.1	$i_{eff} = \left(1+\frac{0,08}{2}\right)^2 - 1$ $\therefore i_{eff} = 0,0816$	✔ formula ✔ 0,0816 (2)
7.1.2	$\text{P} = 100\ 000(1,0816)^{-4}$ $\therefore \text{P} = \text{R}73\ 069,02$ OR $\text{P} = 100\ 000\left(1+\frac{0,08}{2}\right)^{-8}$ $\therefore \text{P} = \text{R}73\ 069,02$	✔ formula ✔ answer (2)

7.2	$90\ 000 = 200\ 000(1-0,08)^n$ $\therefore \frac{9}{20} = 0,92^n$ $\therefore \log_{0,92}\left(\frac{9}{20}\right) = n$ $\therefore n = 9,576544593$ 9 years and 7 months	✔ correct substitution into formula ✔ use of logs ✔ answer (3)

QUESTION 8

8.1	$2\ 500\ 000 = \frac{x\left[(1,0075)^{361}-1)\right]}{0,0075}$ $\therefore \frac{2\ 500\ 000 \times 0,0075}{\left[(1,0075)^{361}-1)\right]} = x$ $\therefore x = \text{R}1354,67$	✔ correct formula ✔ $n = 361$ ✔ $\frac{0,09}{12} = 0,0075$ ✔ $\text{F} = 2\ 500\ 000$ ✔ answer (5)
8.2	$2\ 500\ 000 = \frac{x\left[1-\left(1+\frac{0,07}{12}\right)^{-240}\right]}{\left(\frac{0,07}{12}\right)}$ $\therefore \frac{2\ 500\ 000 \times \left(\frac{0,07}{12}\right)}{\left[1-\left(1+\frac{0,07}{12}\right)^{-240}\right]} = x$ $\therefore x = \text{R}19\ 382,47$	✔ correct formula ✔ $n = 240$ ✔ $\frac{0,07}{12}$ ✔ $\text{P} = 2\ 500\ 000$ ✔ answer (5)

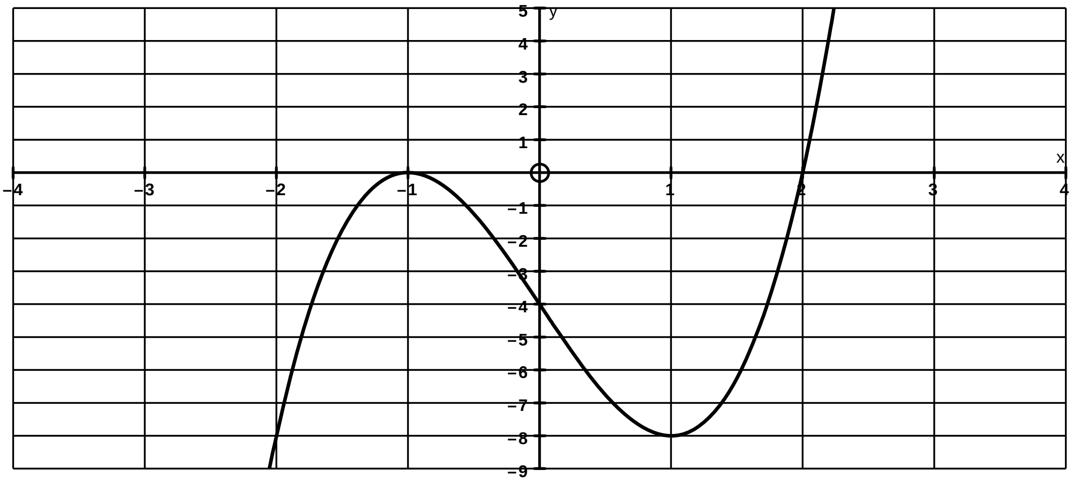
QUESTION 9

9.1	$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ $\therefore f'(x) = \lim_{h\to 0} \frac{-2(x+h)^2+1-(-2x^2+1)}{h}$ $\therefore f'(x) = \lim_{h\to 0} \frac{-2(x^2+2xh+h^2)+1+2x^2-1}{h}$ $\therefore f'(x) = \lim_{h\to 0} \frac{-2x^2-4xh-2h^2+1+2x^2-1}{h}$ $\therefore f'(x) = \lim_{h\to 0} \frac{h(-4x-2h)}{h}$ $\therefore f'(x) = \lim_{h\to 0} (-4x-2h)$ $\therefore f'(x) = -4x-2(0)$ $\therefore f'(x) = -4x$ – 1 for inaccurate notation	✔ $-2(x+h)^2+1$ ✔ $-(-2x^2+1)$ ✔ $-2x^2-4xh-2h^2$ ✔ $\frac{h(-4x-2h)}{h}$ ✔ $-4x$ (5)

9.2	$y=\left(2\sqrt{x}-\frac{1}{3x}\right)^2$ $\therefore y=4x-\frac{4\sqrt{x}}{3x}+\frac{1}{9x^2}$ $\therefore y=4x-\frac{4x^{\frac{1}{2}}}{3x}+\frac{1}{9}x^{-2}$ $\therefore y=4x-\frac{4}{3}x^{-\frac{1}{2}}+\frac{1}{9}x^{-2}$ $\therefore \frac{dy}{dx}=4-\frac{4}{3}\times-\frac{1}{2}x^{-\frac{3}{2}}+\frac{1}{9}\times-2x^{-3}$ $\therefore \frac{dy}{dx}=4+\frac{2}{3}x^{-\frac{3}{2}}-\frac{2}{9}x^{-3}$ $\therefore \frac{dy}{dx}=4+\frac{2}{3x^{\frac{3}{2}}}-\frac{2}{9x^3}$ – 1 for inaccurate notation	✔ $4x-\frac{4}{3}x^{-\frac{1}{2}}+\frac{1}{9}x^{-2}$ ✔ ✔ ✔ $4+\frac{2}{3x^{\frac{3}{2}}}-\frac{2}{9x^3}$ (4)

QUESTION 10

10.1	$f(x)=ax^3+bx$ $\therefore f'(x)=3ax^2+b$ $\therefore f'(-1)=3a(-1)^2+b$ $\therefore f'(-1)=3a+b$ Now $y=x+4$ $m_t=1$ $1=3a+b$ Now at $x=-1$ $y=-1+4=3$ $\therefore$ Substitute $(-1;3)$ into the equation of f: $f(-1)=a(-1)^3+b(-1)$ $\therefore 3=-a-b$ $\therefore a+b=-3$ Solving simultaneously: $a=2$ and $b=-5$	✔ $f'(x)=3ax^2+b$ ✔ $m_t=1$ ✔ $1=3a+b$ ✔ $a+b=-3$ ✔ $a=2$ ✔ $b=-5$ (6)
10.2.1	y-intercept: $(0;-4)$ x-intercepts: $0=2x^3-6x-4$ $\therefore 0=x^3-3x-2$ $\therefore 0=(x+1)(x^2-x-2)$ $\therefore 0=(x+1)(x-2)(x+1)$ $\therefore x=-1$ or $x=2$ $(-1;0)$ $(2;0)$	✔ y-intercept ✔ $0=2x^3-6x-4$ ✔ $(x+1)(x^2-x-2)=0$ ✔ $(x+1)(x-2)(x+1)$ ✔ x-intercepts (5)

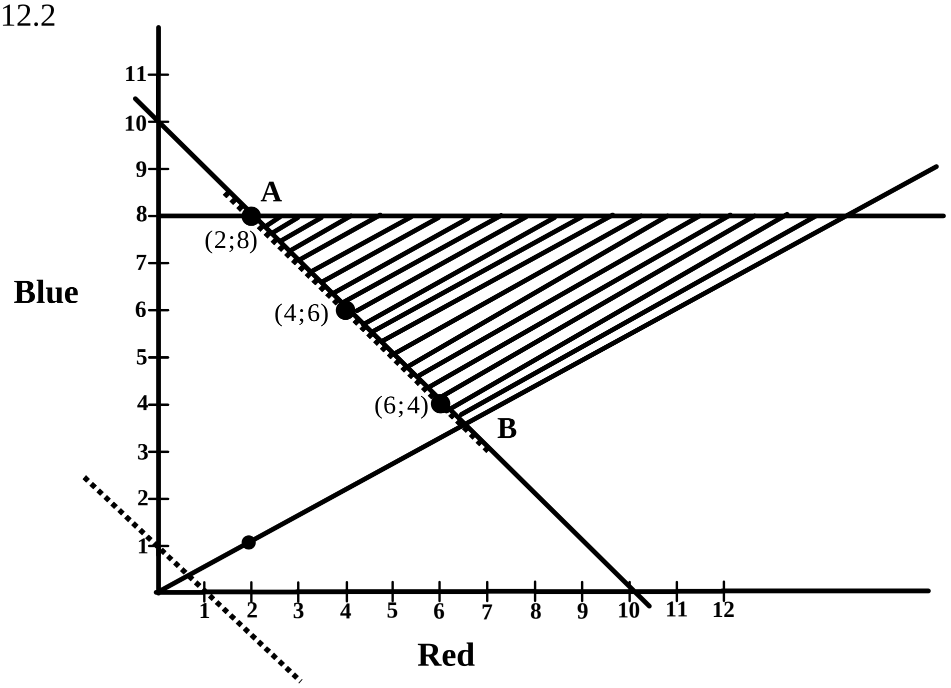
10.2.2	$f(x)=2x^3-6x-4$ $\therefore f'(x)=6x^2-6$ $\therefore 0=6x^2-6$ $\therefore 0=x^2-1$ $\therefore x=\pm 1$ $f(1)=-8$ $f(-1)=0$ Turning points are $(1;-8)$ and $(-1;0)$	✔ $f'(x)=6x^2-6$ ✔ $0=6x^2-6$ ✔ $x=\pm 1$ ✔ $(1;-8)$ and $(-1;0)$ (4)
10.2.3		✔ intercepts with the axes ✔ turning points ✔ shape (3)
10.2.4	$f'(x)=6x^2-6$ $\therefore f''(x)=12x$ $\therefore 0=12x$ $\therefore x=0$ $f(0)=-4$ Point of inflection at $(0;-4)$	✔ $f''(x)=12x$ ✔ $x=0$ ✔ $(0;-4)$ (3)
10.2.4	$p>0$ or $p<-8$	✔ $p>0$ ✔ $p<-8$ (2)

QUESTION 11

11.1	$A=(2x)(3x)+2(y)(3x)+2(y)(2x)$ $\therefore 200=6x^2+6xy+4xy$ $\therefore 200=6x^2+10xy$ $\therefore 100=3x^2+5xy$ $\therefore 100-3x^2=5xy$ $\therefore \frac{100}{5x}-\frac{3x^2}{5x}=y$ $\therefore y=\frac{20}{x}-\frac{3x}{5}$	✓ $6x^2+6xy+4xy$ ✓ $200=$ ✓ arriving at answer (3)
11.2	$V=(2x)(3x)(y)$ $\therefore V=(2x)(3x)\left(\frac{20}{x}-\frac{3x}{5}\right)$ $\therefore V=(6x^2)\left(\frac{20}{x}-\frac{3x}{5}\right)$ $\therefore V=120x-\frac{18x^3}{5}$	✓ $V=(2x)(3x)(y)$ ✓ $V=120x-\frac{18x^3}{5}$ (2)
11.3	$V(x)=120x-\frac{18}{5}x^3$ $\therefore V'(x)=120-\frac{18}{5}\times 3x^2$ $\therefore 0=120-\frac{54}{5}x^2$ $\therefore 0=600-54x^2$ $\therefore 54x^2=600$ $\therefore x^2=\frac{600}{54}$ $\therefore x^2=\frac{100}{9}$ $\therefore x=\frac{10}{3}$	✓ $V'(x)$ ✓ $V'(x)=0$ ✓ $x=\frac{10}{3}$ (3)

QUESTION 12

12.1	$x+y\geq 10$ $y\geq \frac{1}{2}x$ $y\leq 8$	✓ $x+y\geq 10$ ✓ $y\geq \frac{1}{2}x$ ✓ $y\leq 8$ (3)
12.2	see next page	

12.3	$C = 40x + 40y$ $\therefore 40x + 40y = C$ $\therefore 40y = -40x + C$ $\therefore y = -1x + \frac{C}{40}$	✔ $C = 40x + 40y$ ✔ search line on diagram ✔ $(2;8)$ ✔ $(4;6)$ ✔ $(6;4)$ (5)
12.2		✔ $x + y \geq 10$ ✔ $y \geq \frac{1}{2}x$ ✔ $y \leq 8$ ✔ feasible region (4)