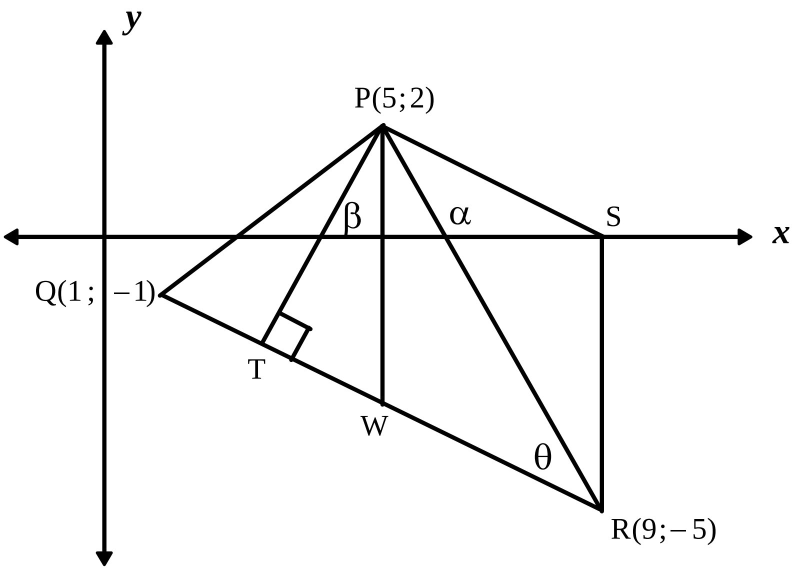
MATRIC MATHEMATICS PAPER 2 MEMORANDUM

QUESTION 1

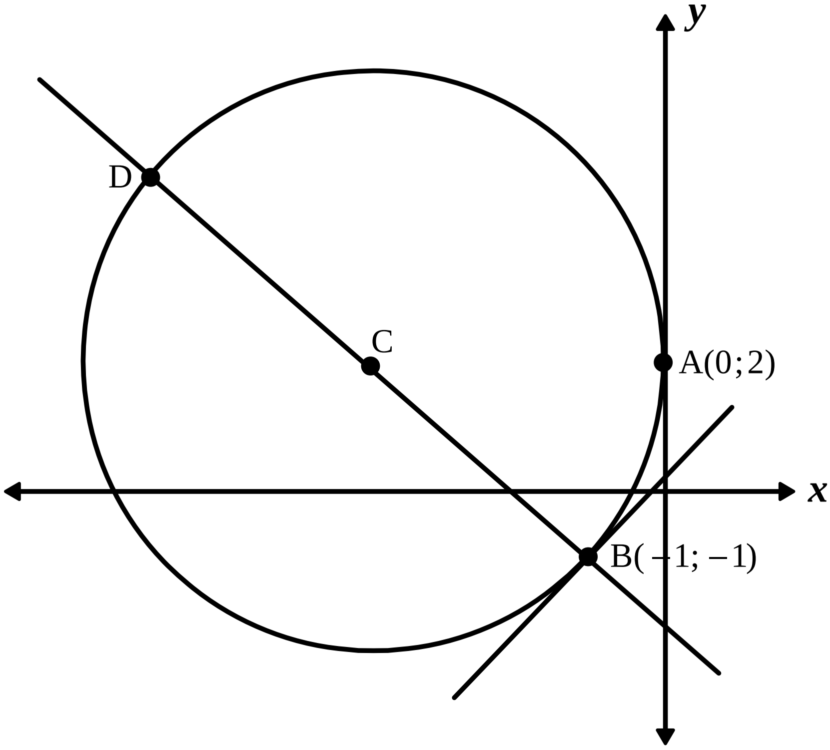


1.1	$W\left(\frac{1+(9)}{2};\frac{-1+(-5)}{2}\right)$ $= W(5\,;-3)$ The equation of PW is $x=5$	✔ midpoint ✔ $x=5$ (2)
1.2	$m_{QR}=\frac{-5-(-1)}{9-1}=\frac{-4}{8}=-\frac{1}{2}$ $\therefore m_{PS}=-\frac{1}{2}$ (PS‖QR) $y-2=-\frac{1}{2}(x-5)$ $\therefore y-2=-\frac{1}{2}x+\frac{5}{2}$ $\therefore y=-\frac{1}{2}x+\frac{9}{2}$	✔ m_{QR} ✔ m_{PS} ✔ correct substitution into formula for equation ✔ $y=-\frac{1}{2}x+\frac{9}{2}$ (4)

1.3	$m_{PT} = 2$ (PT ⊥ QR) $y - 2 = 2(x - 5)$ $\therefore y - 2 = 2x - 10$ $\therefore y = 2x - 8$	✔ m_{PT} ✔ correct substitution into formula for equation ✔ $y = 2x - 8$ (3)
1.4	$m_{QR} = -\frac{1}{2}$ $y - (-1) = -\frac{1}{2}(x - 1)$ $\therefore y + 1 = -\frac{1}{2}x + \frac{1}{2}$ $\therefore y = -\frac{1}{2}x - \frac{1}{2}$ $\therefore -\frac{1}{2}x - \frac{1}{2} = 2x - 8$ $\therefore -x - 1 = 4x - 16$ $\therefore -5x = -15$ $\therefore x = 3$ $\therefore y = 2(3) - 8 = -2$ $\therefore \text{T}(3; -2)$	✔ correct substitution into formula for equation ✔ $y = -\frac{1}{2}x - \frac{1}{2}$ ✔ $-\frac{1}{2}x - \frac{1}{2} = 2x - 8$ ✔ $x = 3$ ✔ $\text{T}(3; -2)$ (5)
1.5	$\text{QT}^2 = (1-3)^2 + (-1-(-2))^2$ $\therefore \text{QT}^2 = 4 + 1$ $\therefore \text{QT}^2 = 5$ $\therefore \text{QT} = \sqrt{5}$ $\text{TR}^2 = (3-9)^2 + (-2-(-5))^2$ $\therefore \text{TR}^2 = 36 + 9$ $\therefore \text{TR}^2 = 45$ $\therefore \text{TR} = \sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$ $\therefore \frac{1}{3}\text{TR} = \sqrt{5} = \text{QT}$ $\therefore \text{QT} = \frac{1}{3}\text{TR}$	✔ correct substitution to get QT ✔ answer for QT ✔ correct substitution to get TR ✔ answer for TR ✔ establishing that $\text{QT} = \frac{1}{3}\text{TR}$ (5)

1.6	$\tan\alpha = m_{PR}$ $\therefore \tan\alpha = \frac{2-(-5)}{5-9}$ $\therefore \tan\alpha = -1$ $\therefore \alpha = 180° - 45°$ $\therefore \alpha = 135°$ $\tan\beta = m_{PT}$ $\therefore \tan\beta = 2$ $\therefore \beta = 63,43494882°$ Now $T\hat{P}R + \beta = \alpha$ $\therefore T\hat{P}R = \alpha - \beta$ $\therefore T\hat{P}R = 135° - 63,43494882°$ $\therefore T\hat{P}R = 71,56505118°$ $\theta + 90° + 71,56505118° = 180°$ $\therefore \theta = 18,43°$	✔ $\tan\alpha = -1$ ✔ $\alpha = 135°$ ✔ $\beta = 63,43494882°$ ✔ $T\hat{P}R = 71,56505118°$ ✔ $\theta = 18,43°$ (5)

QUESTION 2

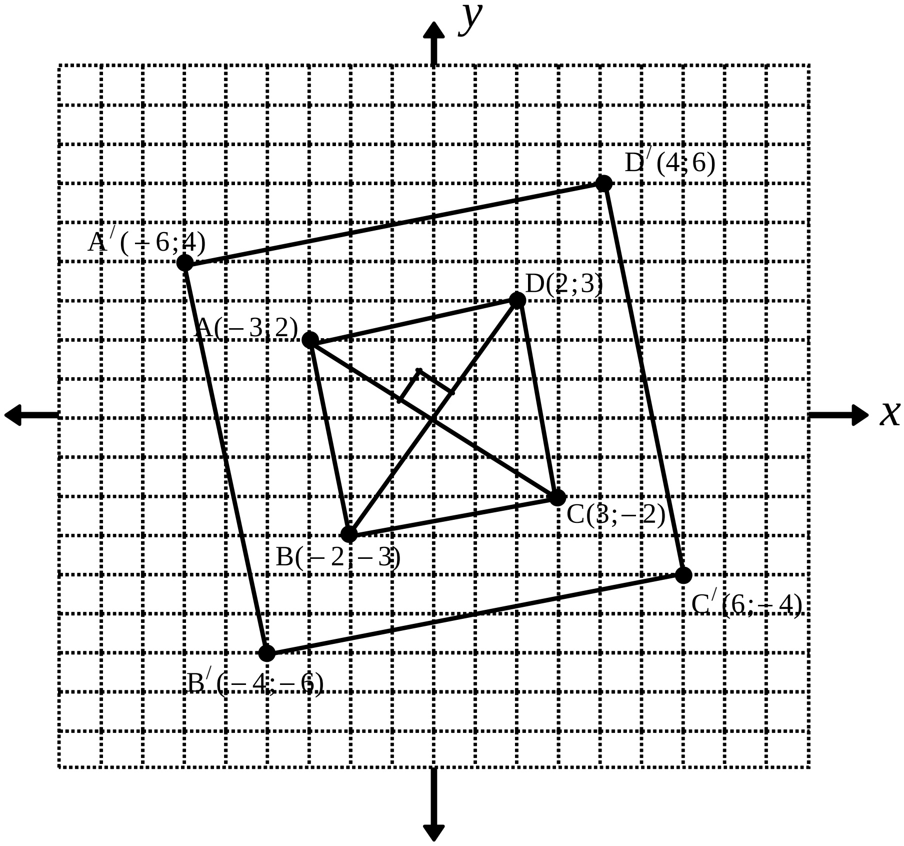


2.1	$m_{AB} = m_{BC}$ $\therefore \frac{2-5}{3-6} = \frac{k+4-2}{2k-3}$ $\therefore 1 = \frac{k+2}{2k-3}$ $\therefore 2k-3 = k+2$ $\therefore k = 5$	✔ $m_{AB} = m_{BC}$ ✔ working out gradients ✔ $k = 5$ (3)

2.2	$x^2+y^2-4x+6y+4=0$ $\therefore x^2-4x+y^2+6y=-4$ $\therefore x^2-4x+\left[\frac{-4}{2}\right]^2+y^2+6y+\left[\frac{6}{2}\right]^2=-4+\left[\frac{-4}{2}\right]^2+\left[\frac{6}{2}\right]^2$ $\therefore (x-2)^2+(y+3)^2=-4+4+9$ $\therefore (x-2)^2+(y+3)^2=9$ centre $=(2;-3)$ new centre after rotation of 90° clockwise: $(-3;-2)$ new centre after enlargement through origin: $(-6;-4)$ original radius: $r=3$ new radius after enlargement through origin : $r=3\times 2=6$ new circle: $(x+6)^2+(y+4)^2=36$	✓ $(x-2)^2$ ✓ $(y+3)^2$ ✓ $r^2=9$ ✓ new centre $=(-6;-4)$ ✓ new radius: $r=6$ ✓ new circle: $(x+6)^2+(y+4)^2=36$ (6)
2.3.1	$3x+4y=-7$ $\therefore 4y=-3x-7$ $\therefore y=-\frac{3}{4}x-\frac{7}{4}$ $\therefore m_{CB}=-\frac{3}{4}$ $\therefore m_{tangent}=\frac{4}{3}$ $y-(-1)=\frac{4}{3}(x-(-1))$ $\therefore y+1=\frac{4}{3}(x+1)$ $\therefore y+1=\frac{4}{3}x+\frac{4}{3}$ $\therefore y=\frac{4}{3}x+\frac{1}{3}$	✓ $m_{CB}=-\frac{3}{4}$ ✓ $m_{tangent}=\frac{4}{3}$ ✓ substitution into equation of line ✓ $y=\frac{4}{3}x+\frac{1}{3}$ (4)
2.3.2	$C(x;2)$ Substitute $y=2$ into $3x+4y=-7$ $3x+4(2)=-7$ $\therefore 3x=-15$ $\therefore x=-5$ $\therefore C(-5;2)$ $\therefore (x+5)^2+(y-2)^2=r^2$ Now $r=5$ $\therefore (x+5)^2+(y-2)^2=25$	✓ $y_C=2$ ✓ $x=-5$ ✓ $C(-5;2)$ ✓ $r=5$ ✓ $(x+5)^2+(y-2)^2=25$ (5)

2.3.3	$C(-5;2)=\left(\frac{x_D+x_B}{2};\frac{y_D+y_B}{2}\right)$ $\therefore C(-5;2)=\left(\frac{x_D-1}{2};\frac{y_D-1}{2}\right)$ $\therefore -5=\frac{x_D-1}{2}$ and $2=\frac{y_D-1}{2}$ $\therefore -10=x_D-1$ and $4=y_D-1$ $\therefore x_D=-9$ and $y_D=5$ $\therefore D(-9;5)$	✔ correct substitution into midpoint formula ✔ $x_D=-9$ ✔ $y_D=5$ ✔ $D(-9;5)$ (4)

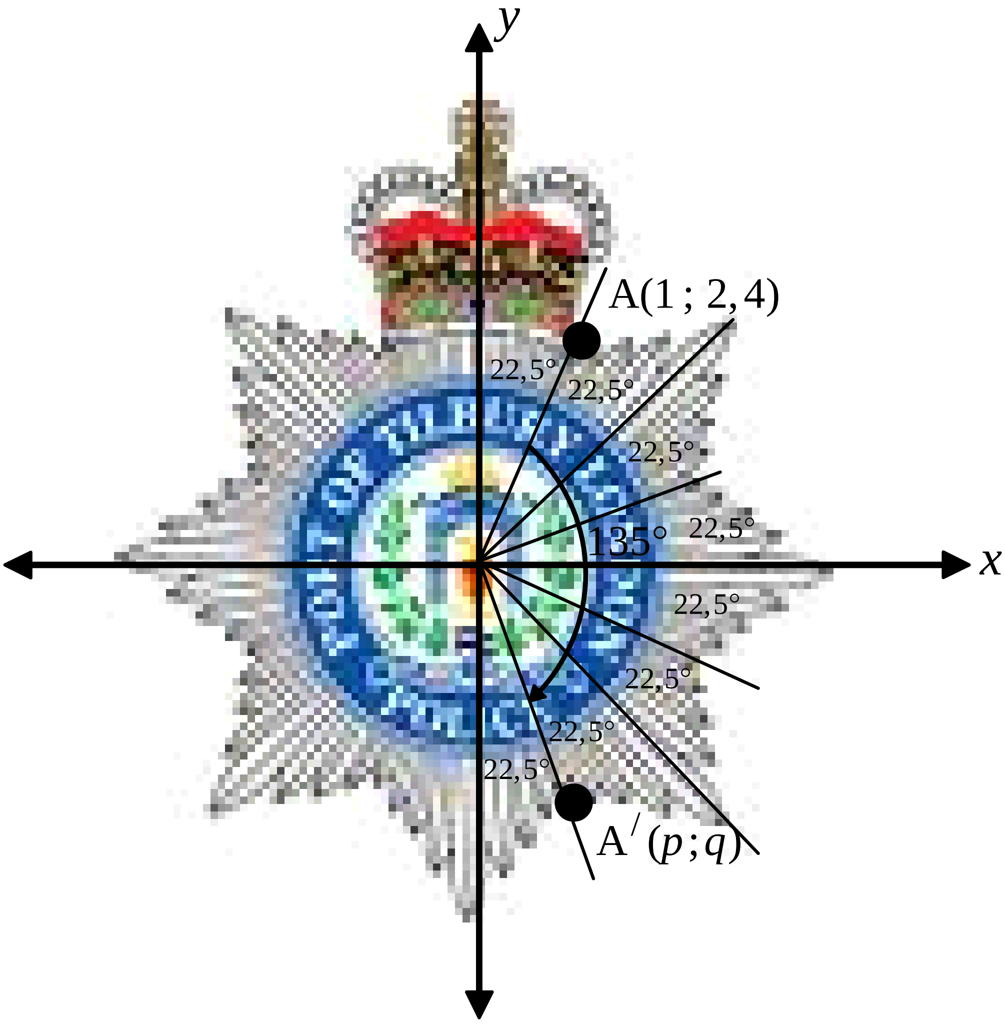
QUESTION 3



3.1	$B(-2;-3)$ $C(3;-2)$ $D(2;3)$	✔ $B(-2;-3)$ ✔ $C(3;-2)$ ✔ $D(2;3)$ (3)
3.2	ABCD is a square since: Diagonals are equal in length Diagonals bisect each other at right angles	✔ square ✔ properties (2)
3.3	$A^{/}(-6;4)$ $B^{/}(-4;-6)$ $C^{/}(6;-4)$ $D^{/}(4;6)$	✔ correct coordinates indicated ✔ joining points to form enlarged square (2)

3.4	$\frac{\text{Area ABCD}}{\text{Area A}^{/}\text{B}^{/}\text{C}^{/}\text{D}^{/}}=\frac{1}{2^2}=\frac{1}{4}$	✔ $\frac{1}{4}$ (1)
3.5.1	E(3;2)	✔ answer (1)
3.5.2	$\frac{\text{Perimeter ABCD}}{\text{Perimeter EFGH}}=\frac{4\times\text{side AB}}{4\times\text{side EF}}=1$ (since AB = EF)	✔ answer (1)
3.6	$(x;y)\rightarrow\left(\frac{1}{2}x;\frac{1}{2}y\right)$ reduction by a factor of $\frac{1}{2}$ $\left(\frac{1}{2}x;\frac{1}{2}y\right)\rightarrow\left(\frac{1}{2}x;-\frac{1}{2}y\right)$ reflection about x – axis $\left(\frac{1}{2}x;-\frac{1}{2}y\right)\rightarrow\left(\frac{1}{2}x;-\frac{1}{2}y-1\right)$ translation of 1 unit downwards $\therefore (x;y)\rightarrow\left(\frac{1}{2}x;-\frac{1}{2}y-1\right)$	✔ reduction ✔ reflection ✔ translation (3)

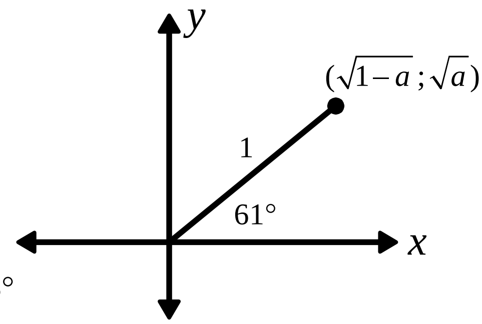
QUESTION 4



4.1	$22,5° \times 6 = 135°$	✓ $22,5°$ ✓ $135°$ (2)
4.2	$x^/ = (1)\cos(-135°) - (2,4)\sin(-135°)$ $\therefore x^/ = 1$ $y^/ = (2,4)\cos(-135°) + (1)\sin(-135°)$ $y^/ = -2,4$ $\therefore A^/(1; -2,4)$	✓ correct substitution into formula for $x^/$ ✓ $x^/ = 1$ ✓ correct substitution into formula for $y^/$ ✓ $y^/ = -2,4$ (4)

QUESTION 5

5.1	$\dfrac{\tan(-60°)\cos(-156°)\cos 294°}{\sin 492°}$ $= \dfrac{(-\tan 60°)(\cos 156°)(-\cos 66°)}{(\sin 132°)}$ $= \dfrac{(-\sqrt{3})(-\cos 24°)(-\sin 24°)}{(\sin 48°)}$ $= \dfrac{(-\sqrt{3})(-\cos 24°)(-\sin 24°)}{2\sin 24° \cos 24°}$ $= \dfrac{\sqrt{3}}{2}$	✓ $(-\tan 60°)(\cos 156°)$ ✓ $-\cos 66°$ ✓ $\sin 48°$ ✓ $-\sqrt{3}$ ✓ $-\sin 24°$ ✓ $2\sin 24° \cos 24°$ ✓ $\dfrac{\sqrt{3}}{2}$ (7)
5.2	$\cos^2(180° + x)\left[\tan(360° - x).\cos(90° + x) + \sin(x - 90°).\cos 180°\right]$ $= (-\cos x)^2\left[(-\tan x)(-\sin x) + (-\cos x)(-1)\right]$ $= (\cos^2 x)\left[\left(\dfrac{-\sin x}{\cos x}\right)(-\sin x) + \cos x\right]$ $= (\cos^2 x)\left[\dfrac{\sin^2 x}{\cos x} + \cos x\right]$ $= (\cos^2 x)\left[\dfrac{\sin^2 x + \cos^2 x}{\cos x}\right]$ $= (\cos^2 x)\left[\dfrac{1}{\cos x}\right]$ $= \cos x$	✓ $\cos^2 x$ ✓ $-\tan x$ ✓ $-\sin x$ ✓ $-\cos x$ ✓ -1 ✓ $\dfrac{-\sin x}{\cos x}$ ✓ $\dfrac{\sin^2 x + \cos^2 x}{\cos x}$ ✓ $\dfrac{1}{\cos x}$ ✓ $\cos x$ (9)

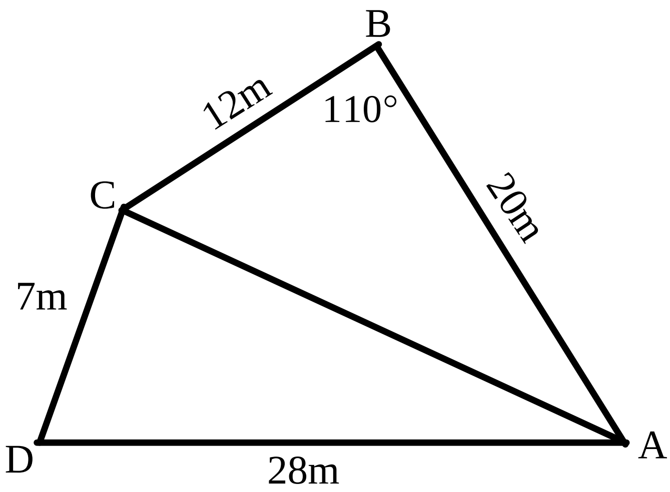
5.3	$\sin 61° = \frac{\sqrt{a}}{1}$ $x^2 + (\sqrt{a})^2 = (1)^2$ $\therefore x^2 = 1 - a$ $\therefore x = \sqrt{1-a}$  $\cos 73° \cos 15° + \sin 73° \sin 15°$ $= \cos(73° - 15°)$ $= \cos 58°$ $= 2\cos^2 29° - 1$ $= 2\sin^2 61° - 1$ $= 2(\sqrt{a})^2 - 1$ $= 2a - 1$	✔ diagram ✔ $x = \sqrt{1-a}$ ✔ $\cos 58°$ ✔ $2\cos^2 29° - 1$ ✔ $2\sin^2 61° - 1$ ✔ $2a - 1$ (6)

QUESTION 6

6.1.1	$\sin(45° + \theta).\sin(45° - \theta)$ $= [\sin 45° \cos\theta + \cos 45° \sin\theta][\sin 45° \cos\theta - \cos 45° \sin\theta]$ $= \left[\frac{\sqrt{2}}{2}\cos\theta + \frac{\sqrt{2}}{2}\sin\theta\right]\left[\frac{\sqrt{2}}{2}\cos\theta - \frac{\sqrt{2}}{2}\sin\theta\right]$ $= \left[\frac{\sqrt{2}}{2}(\cos\theta + \sin\theta)\right]\left[\frac{\sqrt{2}}{2}(\cos\theta - \sin\theta)\right]$ $= \frac{2}{4}(\cos\theta + \sin\theta)(\cos\theta - \sin\theta)$ $= \frac{1}{2}(\cos^2\theta - \sin^2\theta)$ $= \frac{1}{2}\cos 2\theta$	✔ expansion of $\sin(45° + \theta)$ ✔ expansion of $\sin(45° - \theta)$ ✔ $\sin 45° = \cos 45° = \frac{\sqrt{2}}{2}$ ✔ $(\cos^2\theta - \sin^2\theta)$ ✔ $\frac{1}{2}\cos 2\theta$ (5)
6.1.2	$\sin 75°.\sin 15°$ $= \sin(45° + 30°).\sin(45 - 30°)$ $= \frac{1}{2}\cos 2(30°)$ $= \frac{1}{2}\cos 60° = \frac{1}{2}\left(\frac{1}{2}\right) = \frac{1}{4}$	✔ $45° + 30°;\ 45° - 30°$ ✔ $\frac{1}{2}\cos 60°$ ✔ $\frac{1}{4}$ (3)

6.2	$\sin 2x + 2\sin x + \cos^2 x + \cos x = 0$ $\therefore 2\sin x\cos x + 2\sin x + \cos^2 x + \cos x = 0$ $\therefore 2\sin x(\cos x + 1) + \cos x(\cos x + 1) = 0$ $\therefore (\cos x + 1)(2\sin x + \cos x) = 0$ $\therefore \cos x = -1$ or $2\sin x = -\cos x$ $\therefore \frac{\sin x}{\cos x} = -\frac{1}{2}$ $\therefore \tan x = -0,5$ $x = 0° + k360°$ $x = 153,4° + k360°$ $x = 180° + k360°$ $x = 333,4° + k360°$ OR $x = 0° + k180°$ $x = 153,4° + k180°$ OR $x = 180° + k180°$ $x = 333,4° + k180°$ OR $x = \pm 180° + k360°$ $x = -45° + k180°$	✔ $2\sin x\cos x$ ✔ factorising by grouping ✔ $(\ )(\ ) = 0$ ✔ $\cos x = -1$ ✔ $\tan x = -0,5$ ✔ ✔ general solutions Deduct 1 mark if $k \in$ is not stated. (7)

QUESTION 7

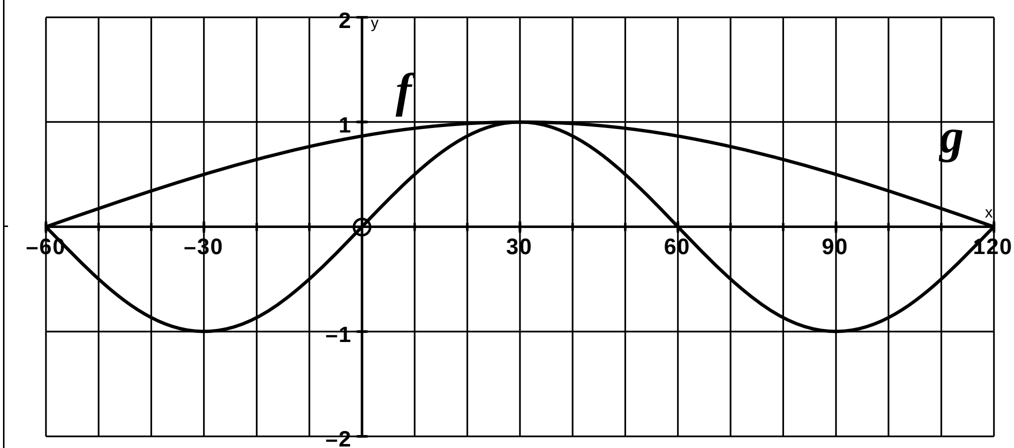


7.1	$AC^2 = (12m)^2 + (20m)^2 - 2(12m)(20m)\cos 110°$ $\therefore AC^2 = 708,1696688$ $\therefore AC = 26,6m$	✔ substitution into cosine rule ✔ answer (2)
7.2	$\frac{\sin B\hat{A}C}{12m} = \frac{\sin 110°}{26,6m}$ $\therefore \sin B\hat{A}C = \frac{12 \times \sin 110°}{26,6m}$ $\therefore \sin B\hat{A}C = 0,4239214831$ $\therefore B\hat{A}C = 25°$	✔ substitution into sine or cosine rule ✔ answer (2)

	OR $(12m)^2 = (20m)^2 + (26,6m)^2 - 2(20m)(26,6m)\cos B\hat{A}C$ $\therefore 1064\cos B\hat{A}C = 963,56m^2$ $\therefore \cos B\hat{A}C = 0,9056015038$ $\therefore B\hat{A}C = 25°$	
7.3	$(26,6m)^2 = (7m)^2 + (28m)^2 - 2(7m)(28m)\cos\hat{D}$ $\therefore 392\cos\hat{D} = 125,44$ $\therefore \cos\hat{D} = 0,32$ $\therefore \hat{D} = 71°$	✔ substitution into cosine rule ✔ $\cos\hat{D} = 0,32$ ✔ answer (3)
7.4	Area ABCD $= \frac{1}{2}(12m)(20m)\sin 110° + \frac{1}{2}(7m)(28m)\sin 71°$ $= 205,4m^2$	✔ $\frac{1}{2}(12m)(20m)\sin 110°$ ✔ $\frac{1}{2}(7m)(28m)\sin 71°$ ✔ answer (3)

QUESTION 8

8.1	$\cos(x-30°) = \sin 3x$ $\therefore \cos(x-30°) = \cos(90°-3x)$ $\therefore x-30° = 90°-3x+k360°$ $\therefore 4x = 120°+k360°$ $\therefore x = 30°+k90°$ $x-30° = 360°-(90°-3x)$ $\therefore x-30° = 270°+3x+k360°$ $\therefore -2x = 300°+k360°$ $\therefore x = -150°+k180°$ $x = 30°$ or $x = 120°$ or $x = -60°$	✔ $\cos(90°-3x)$ ✔ $4x = 120°+k360°$ ✔ $-2x = 300°+k360°$ ✔ $x = 30°+k90°$ ✔ $x = -150°+k180°$ ✔ $x = 30°$ ✔ $x = 120°$ ✔ $x = -60°$ (8)
8.2	see diagram below	$f(x) = \cos(x-30°)$: ✔ shift of $30°$ right ✔ amplitude ✔ range $g(x) = \sin 3x$: ✔ period of $120°$ ✔ amplitude ✔ intercepts with axe (6)



8.3	Points of intersection of the two graphs	✔ correct explanation(1)

8.4	$\cos(x-30°) > \sin 3x$ $\therefore -60° < x < 120°$ where $x \neq 30°$ OR $x \in (-60°;30°)-\{30°\}$	✔ $-60° < x < 120°$ ✔ $x \neq 30°$ (2)

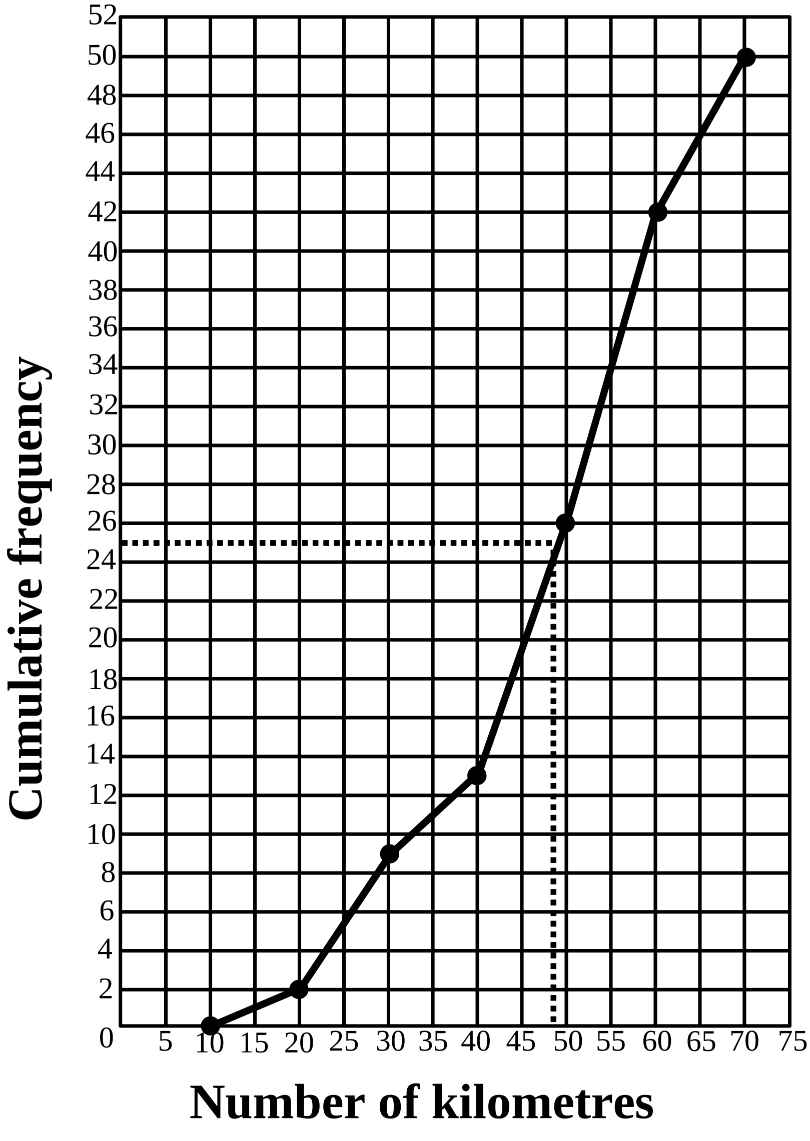
QUESTION 9

9.1

Number of kilometres	Number of motorists	Cumulative frequency
$10 < x \leq 20$	2	2
$20 < x \leq 30$	7	9
$30 < x \leq 40$	4	13
$40 < x \leq 50$	13	26
$50 < x \leq 60$	16	42
$60 < x \leq 70$	8	50

✔ correct second column (1)

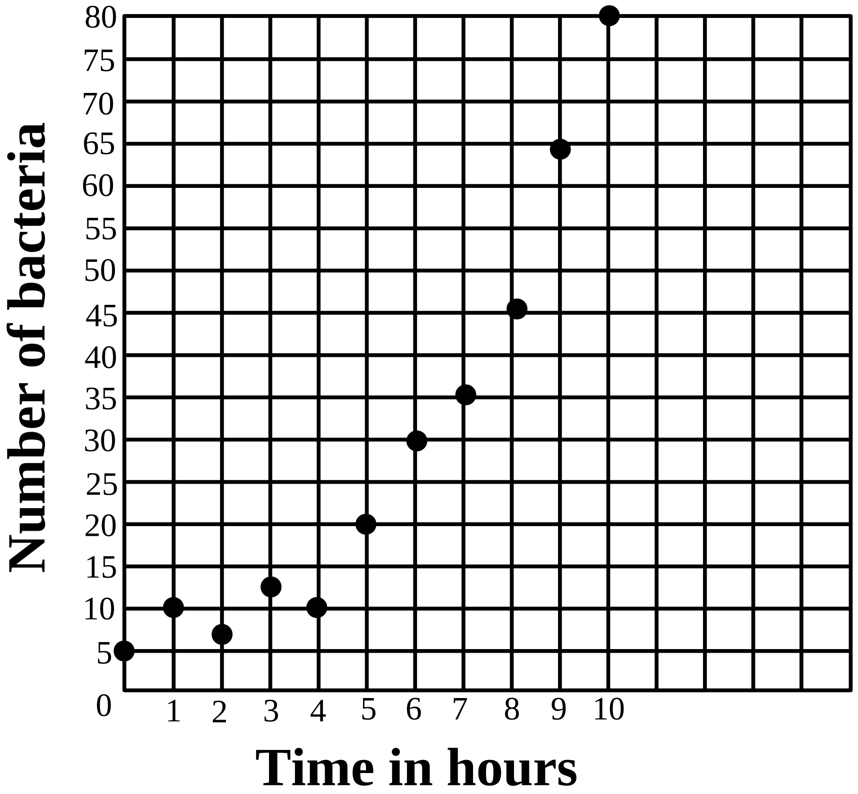
9.2



✔ endpoints of class intervals
✔ cumulative frequencies
✔ joining points
(3)

9.3	median lies in the interval $48 \leq x \leq 49$	✔ median in the allowable interval (1)

QUESTION 10

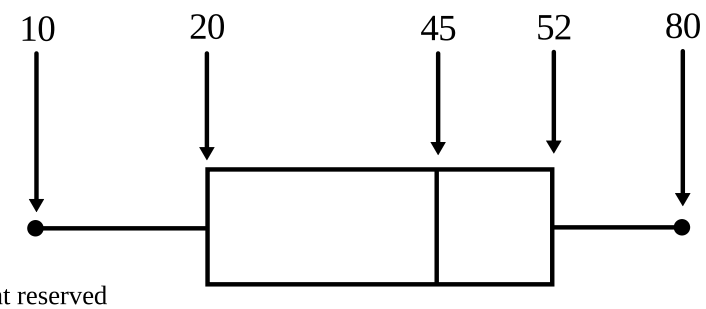
10.1		✔ ✔ plotting of points (2)
10.2	quadratic or exponential	✔ answer (1)

QUESTION 11

23	**25**	**22**	**28**	**27**
20	**18**	**17**	**24**	**25**

11.1	$\overline{x} = 22,9$	✔ ✔ answer (2)
11.2	standard deviation $= 3,5$	✔ ✔ answer (2)
11.3	$\left(\overline{x} - s ; \overline{x} + s\right)$ $= \left(22,9 - 33,5 ; 22,9 + 33,5\right)$ $= (19,4\ ;\ 26,4)$ 4 temperatures lie outside the first standard deviation interval	✔ $(19,4\ ;\ 26,4)$ ✔ 4 temperatures (2)

QUESTION 12



10	**20**	**20**	***x***	**45**	***y***	**51**	**53**	**80**

minimum: 10 maximum: 80 median: 45 Lower quartile: $20 = \frac{20+20}{2}$ Upper quartile: $52 = \frac{51+53}{2}$ Mean: $\frac{10+20+20+x+45+y+51+53+80}{9} = 40$ $\therefore \frac{x+y+279}{9} = 40$ $\therefore x+y+279 = 360$ $\therefore x+y = 81$ Now $20 < x < 45$ and $45 < y < 51$ Therefore let $x = 34$ and $y = 47$ Therefore the set of nine numbers are: 10; 20; 20; 34; 45; 47; 51; 53; 80	✔ min, median and max ✔ $T_2 = T_3 = 20$ ✔ $T_7 = 51$ $T_8 = 53$ ✔ working with the mean ✔ value for x and y ✔ nine set of numbers Accept variations for T_7, T_8, x and y BUT make sure that the mean of all nine numbers is 40. (6)