Grade 12 AP Maths Prelim

Section: A

Question: 1

(a) $\frac{(1+3i)^2}{i+1} = \frac{1+6i-9}{1+i} \cdot \frac{1-i}{1-i}$

$= \frac{-8+6i}{1+i} \cdot \frac{1-i}{1-i}$

$= \frac{-8+8i+6i+6}{1+1}$

$= \frac{-2+14i}{2}$

$= -1+7i$

(b) $x = 1-3i \quad x = 1+3i \quad x = -3$

$(x-1+3i)(x-1-3i)(x+3) = 0$

$(x^2-x-3xi-x+1+3i+3xi-3i+9)(x+3) = 0$

$(x^2-2x+10)(x+3) = 0$

$x^3+3x^2-2x^2-6x+10x+30 = 0$

$x^3+x^2+4x+30 = 0$

$2x^2+2x^2+8x+60 = 0$

$a = 2 \longrightarrow \quad b = 8 \longrightarrow$

Question: 2

(a)(i) $\lim_{x \to 3} \frac{2\sqrt{x}-2\sqrt{3}}{(\sqrt{3}-\sqrt{x})(\sqrt{3}+\sqrt{x})}$

$= \lim_{x \to 3} \frac{-2(\sqrt{3}-\sqrt{x})}{(\sqrt{3}-\sqrt{x})(\sqrt{3}+\sqrt{x})}$

$= \lim_{x \to 3} \frac{-2}{\sqrt{3}+\sqrt{x}} = \frac{-2}{2\sqrt{3}} = \underline{\frac{-1}{\sqrt{3}}}$

(ii) $\lim_{x\to 0} \frac{5\sin 2x}{\sin 4x}$

$= 5 \lim_{x\to 0} \frac{\sin 2x}{2\sin 2x \cos 2x}$

$= \frac{5}{2} \lim_{x\to 0} \frac{1}{\cos 2x}$

$= \frac{5}{2}$

(iii) $\lim_{x\to\infty} \frac{x}{\sqrt{x^2 - x}}$

$= \lim_{x\to\infty} \frac{x}{x\sqrt{1 - \frac{1}{x}}}$

$= \lim_{x\to\infty} \frac{1}{\sqrt{1 - \frac{1}{x}}}$

$= \frac{1}{\sqrt{1-0}}$

$= 1$

(b) $\frac{1+\sec 2A}{\cos^2 A} = \frac{1 + \frac{1}{\cos 2A}}{\cos^2 A}$

$= \frac{\cos 2A + 1}{\cos 2A} \cdot \frac{1}{\cos^2 A}$

$= \frac{2\cos^2 A - 1 + 1}{\cos 2A} \cdot \frac{1}{\cos^2 A}$

$= \frac{2\cos^2 A}{\cos 2A} \cdot \frac{1}{\cos^2 A}$

$= \frac{2}{\cos 2A}$

$= 2\sec 2A = RHS$

Question: 3

$$xy^3 + 3x^2 = xy + 12 \qquad (-2; 1)$$

$$y^3 + 3xy^2\frac{dy}{dx} + 6x = y + x\frac{dy}{dx}$$

$$(3xy^2 - x)\frac{dy}{dx} = y - y^3 - 6x$$

$$\frac{dy}{dx} = \frac{y - y^3 - 6x}{3xy^2 - x}$$

$$= \frac{(1) - (1)^3 - 6(-2)}{3(-2)(1)^2 - (-2)}$$

$$= \frac{12}{-4} = -3 \qquad m_T = -3$$

Question: 4

(a) $f'(x) = 20(2-3x)^{19} \cdot (-3)$

$f''(x) = 20 \cdot 19(2-3x)^{18}(-3)(-3)$

$f'''(x) = 20 \cdot 19 \cdot 18(2-3x)^{17}(-3)(-3)(-3)$

(b) $f^n(x) = \frac{20!}{(20-n)!}(2-3n)^{20-n} \cdot (-3)^n$

(c) (i) $f^{20}(x) = \frac{20!}{0!}(2-3x)^0(-3)^{20} = 20!(-3)^{20}$

(ii) $f^{21}(x) = 0$

Question: 5

(a) $f(0) = 0 - \cos 0 = -1$

$f(1) = 1 - \cos 2 = 1,41$

$\therefore$ Intercept exists as y-values change from $\ominus$ to $\oplus$

(b) $a_{n+1} = a_n - \dfrac{f(a_n)}{f'(a_n)}$ $\qquad f'(x) = 1 + 2\sin 2x$

$= a_n - \dfrac{a_n - \cos 2(a_n)}{1 + 2\sin 2(a_n)}$ $\qquad a_0 = 0,5$

$= 0,514933$ →

Question: 6

(a) Vertical Asymptote $x = -2$

Removable Discontinuity $x = 1$

$m = \lim\limits_{x\to\infty} \dfrac{x^2 + x - 1}{x(x+2)} =$

$= \lim\limits_{x\to\infty} \dfrac{x^2\left(1 + \frac{1}{x} - \frac{1}{x^2}\right)}{x^2\left(1 + \frac{2}{x}\right)}$

$= 1$

$\lim\limits_{x\to 1} \dfrac{x^2+x-1}{x+2}$

$= \frac{1}{3}$

$\left(1; \frac{1}{3}\right)$

Removable Discontinuity

$c = \lim\limits_{x\to\infty} \dfrac{x^2+x-1}{x+2} - x$

$= \lim\limits_{x\to\infty} \dfrac{x^2+x-1-x^2-2x}{x+2}$

$= \lim\limits_{x\to\infty} \dfrac{-x-1}{x+2}$ $\qquad y = x - 1$ →

$= \lim\limits_{x\to\infty} \dfrac{x\left(-1 - \frac{1}{x}\right)}{x\left(1 + \frac{2}{x}\right)} = -1$

(b) $f(x) = \dfrac{x^2+x-1}{x+2}$

$$f'(x) = \frac{(2x+1)(x+2) - (x^2+x-1)(1)}{(x+2)^2}$$

$$= \frac{2x^2+5x+2-x^2-x+1}{x^2+4x+4}$$

$$= \frac{x^2+4x+3}{x^2+4x+4}$$

$$0 = x^2+4x+3$$

$$= (x+1)(x+3)$$

$x = -1$ $\qquad$ $x = -3$

$f(-1) = -1$ $\qquad$ $f(-3) = -5$

$(-1; -1)$ $\qquad$ $(-3; -5)$

$$f''(x) = \frac{(2x+4)(x^2+4x+4) - (x^2+4x+3)(2x+4)}{(x^2+4x+4)^2}$$

$f''(-1) = 2$ $\qquad$ $f''(-3) = -2$

$\therefore$ Local Min $\qquad$ Local Max

(c) x-int: $y = 0$

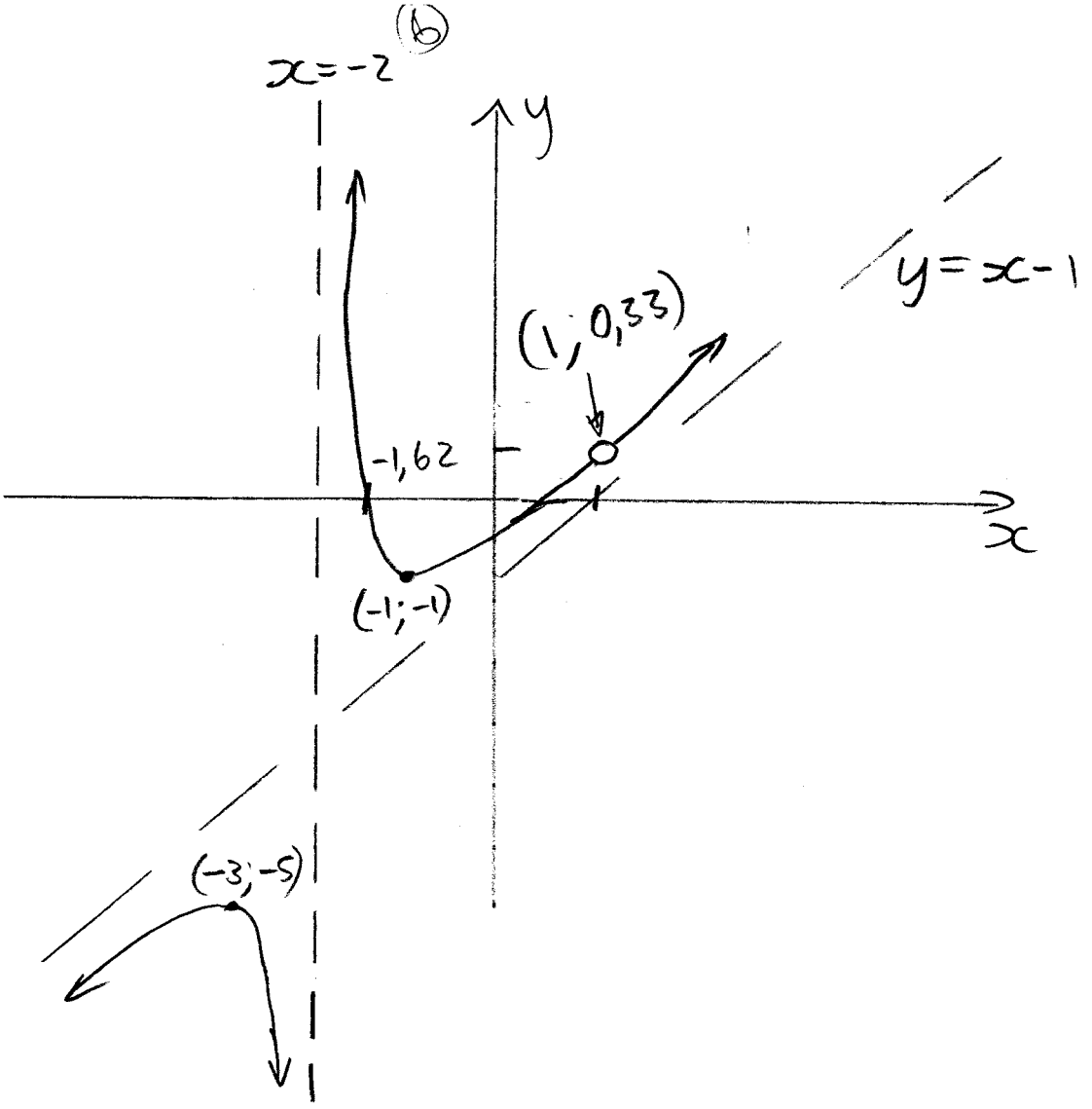
$$x^2+x-1 = 0$$

$$x = \frac{-1+\sqrt{5}}{2} \text{ or } x = \frac{-1-\sqrt{5}}{2}$$

$$= 0{,}62 \qquad = -1{,}62$$

y-int: $(0; -\frac{1}{2})$

(d)



Question: 7

$$x_i = 2 + i\left(\frac{4}{n}\right) = 2 + \frac{4}{n}i$$

$$f(x_i) = \left(2+\frac{4}{n}i\right)^2 - 4 = 4 + \frac{16}{n}i + \frac{16}{n^2}i^2 - 4 = \frac{16}{n}i + \frac{16}{n^2}i^2$$

$$\text{Area} = \lim_{n\to\infty}\left[\frac{4}{n}\sum_{i=1}^{n}\frac{16}{n}i + \frac{16i^2}{n^2}\right]$$

$$= \lim_{n\to\infty}\left[\frac{64}{n^2}\left(\frac{n^2}{2}+\frac{n}{2}\right) + \frac{64}{n^3}\left(\frac{n^3}{3}+\frac{n^2}{2}+\frac{n}{6}\right)\right]$$

$$= \lim_{n\to\infty} 32 + \frac{32}{n} + \frac{64}{3} + \frac{32}{n} + \frac{32}{3n^2}$$

$$= \frac{160}{3} = 53{,}3\ u^2$$

Question: 9

(a) $\sin 2x = \cos x$

$\cos\left(\frac{\pi}{2} - 2x\right) = \cos x$

$\frac{\pi}{2} - 2x = \pm x + k2\pi$

$\frac{\pi}{2} - 2x = x + k2\pi$ $\qquad$ $\frac{\pi}{2} - 2x = -x + k2\pi$

$-3x = -\frac{\pi}{2} + k2\pi$ $\qquad$ $-x = -\frac{\pi}{2} + k2\pi$

$x = \frac{\pi}{6} + k2\pi$ $\qquad$ $x = \frac{\pi}{2} + k2\pi$

$k \in \mathbb{Z}$

$x = \frac{\pi}{6}$ →

(b) $\text{Area} = \int_0^{\frac{\pi}{6}} \sin 2x \, dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos x \, dx$

$= \left[-\frac{\cos 2x}{2}\right]_0^{\frac{\pi}{6}} + \left[\sin x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$

$= -\frac{1}{4} + \frac{1}{2} + 1 - \frac{1}{2}$

$= \frac{3}{4} u^2$

Question: 9

(a) $\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + c$

(b) $\int x^2 \sqrt{x^3+1} \, dx$

$= \frac{1}{3} \int 3x^2 (x^3+1)^{\frac{1}{2}} dx = \frac{(x^3+1)^{\frac{3}{2}}}{3 \cdot \frac{3}{2}} = \frac{2(x^3+1)^{\frac{3}{2}}}{9} + c$

(c) $\int \cos 3\theta \sin 2\theta \, d\theta$

$= \int \sin 2\theta \cos 3\theta \, d\theta$

$= \frac{1}{2} \int \sin(2\theta+3\theta) + \sin(2\theta-3\theta) \, d\theta$

$= \frac{1}{2} \int \sin 5\theta - \sin\theta \, d\theta$

$= -\frac{1}{2} \frac{\cos 5\theta}{5} + \frac{1}{2}\cos\theta + C$

(d) $\int x\sqrt{x-4} \, dx$

$v' = \sqrt{x-4}$ $\quad v = \frac{2}{3}(x-4)^{3/2}$

$u = x$ $\quad u' = 1$

$\int x\sqrt{x-4} \, dx = \frac{2x(x-4)^{3/2}}{3} - \int \frac{2}{3}(x-4)^{3/2} \, dx$

$= \frac{2x}{3}(x-4)^{3/2} - \frac{2}{3}\frac{(x-4)^{5/2}}{\frac{5}{2}} + C$

$= \frac{2x}{3}(x-4)^{3/2} - \frac{4}{15}(x-4)^{5/2} + C$

Question: 10

$\lim_{x\to 1} \frac{a}{x} = \lim_{x\to 1} b - 2x = f(1)$ $\quad a = b-2$

$b = a+2$

$f'(x) = -\frac{a}{x^2}$ $\quad f'(x) = -2$ $\quad b = 4$

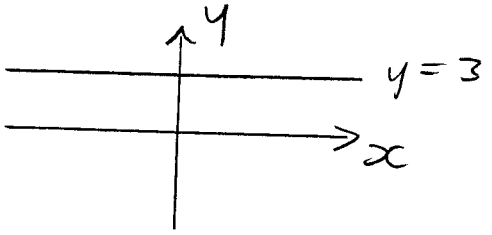
$-a = -2$

$a = 2$

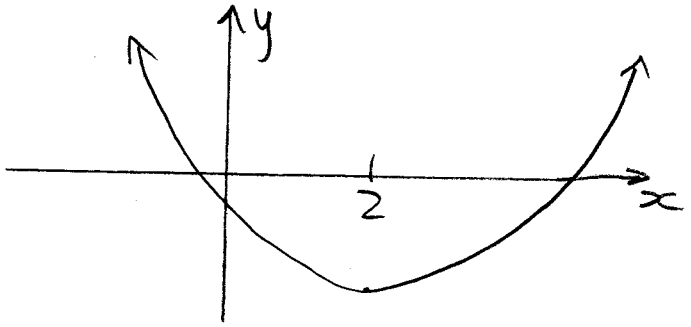
(b) (i) $y' = -\frac{6}{-2}x - 6$

$y' = 3x - 6$

$h'(x) = 3$



(ii)



(c) $72 = \frac{1}{2}r^2\theta$

$72 = \frac{1}{2}r^2\left(\frac{36-2r}{r}\right)$

$72 = 18r - r^2$

$r^2 - 18r + 72 = 0$

$(r-6)(r-12) = 0$

$r = 6$ or $r = 12$

$\ell + 2r = 36$

$r\theta + 2r = 36$

$r\theta = 36 - 2r$

$\theta = \frac{36-2r}{r}$

Question: 11

$$E'(V) = \frac{(3kLV^2)(V-2) - kLV^3}{(V-2)^2}$$

$0 = 3kLV^3 - 6kLV^2 - kLV^3$

$0 = 2kLV^3 - 6kLV^2$

$0 = 2V^2(V-1)$

$V \neq 0$ or $V = 1\ m/s$

Section: B

Question: 1

95471 different kinds

$24 \times 24 \times ? = 576 \times 10 = 5760 < 95\ldots$

$= 576 \times 10^2 = 57600 < 95\ldots$

$= 576 \times 10^3 = 576000 > 95\ldots$

∴ 2 capital letters and 3 digits.

Question: 2

$P(C \text{ & } A) = \frac{1}{3}$

$P(I \text{ & } A) = \frac{1}{4}$

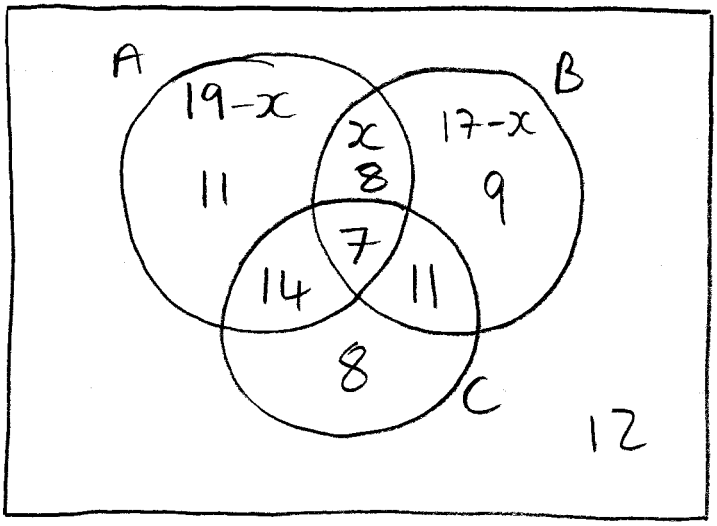
5 clubs.

$P(\text{Any Club}) = \frac{1}{5}$.

$4\left(\frac{1}{5} \cdot \frac{1}{4}\right) + \left(\frac{1}{5} \cdot \frac{1}{3}\right) = \frac{4}{15} = 26,7\%$.

Question: 3

(a)



$19 - x + x + 17 - x + 14 + 18 + 8 = 68$

$x = 8$

(b) 12

(c) 8

(d) $P(\text{At least 2}) = \frac{40}{80}$

$= \frac{1}{2}$

Question: 4

(a) $\frac{12!}{3!2!2!2!} = 9979200$

(b) $\frac{10!}{2!2!2!} = 453600$

Question: 5

$P(R) = 0,4 \quad P(R') = 0,6$

(a) $P(x=7) = {}^{10}C_7 \times (0,4)^7 \times (0,6)^3$
$= 0,042$

(b) $P(x \geq 2) = 1 - P(x < 2)$
$= 1 - [P(x=0) + P(x=1)]$
$= 1 - [{}^{10}C_0 (0,4)^0 \cdot (0,6)^{10} + {}^{10}C_1 (0,4)^1 (0,6)^9$
$= 0,954$

Question: 6

$P(16 < X < 32) = P\left(\frac{16-25}{5} < z < \frac{32-25}{5}\right)$
$= P(-1,8 < z < 1,4)$
$= P(0 < z < 1,8) + P(0 < z < 1,4)$
$= 0,4641 + 0,4192 = 0,8833$
$0,8833 \times 40 = 35$ Candidates

Question: 7

(c) $\frac{1}{n}$