July 2011 Grade 12 Paper 2 Memo

SECTION A

Question 1

a) $\bar{x} = \frac{2106}{11} = 191,5\text{ cm}$ ✓✓ 1dp (-1) (2)

b) $\sigma = 6,7$ ✓✓ (2)

c) i) $184,8 \leq \bar{x} \leq 198,2$ ✓ ✓ (2)
or $[184,8 ; 198,2]$

ii) $\frac{8}{11} \times 100 = 72,7\%$ ✓ ✓ca (2)

[8]

Question 2

a) i) $m_{CAT} = \frac{1}{3}$ (given)
$\therefore \tan\theta = \frac{1}{3}$ ✓
$\therefore C\hat{T}X = 18,43°$ ✓ (2)

ii) $m_{AB} = \frac{12-4}{4} = 2$ ✓
$A\hat{K}O = \tan^{-1}(2)$
$= 63,43°$ ✓
$\therefore T\hat{A}K = 63,43° - 18,43°$ ✓m (ext $\angle$ Δ)
$= 45°$ ✓
$T\hat{A}K = B\hat{A}C = 45°$ ✓ (Vert. opp $\angle$'s) (5)

b) i) $d_{QR} = \sqrt{(9-1)^2 + (-5+1)^2}$ ✓
$= \sqrt{80}$
$= 4\sqrt{5}$ ✓ (2)

ii) $m_{QR} = \frac{-5+1}{9-1}$ ✓ $= \frac{-1}{2}$ ✓

$(1;-1)$ $y-(-1) = \frac{-1}{2}(x-1)$ ✓
$y_{QR} = \frac{-1}{2}x - \frac{1}{2}$ ✓ (4)

b) iii) $m_{QR} = \frac{-1}{2}$
$\therefore m_{PW} = 2$ ✓ $P(5;2)$
$y-2 = 2(x-5)$ ✓m
$y_{PW} = 2x - 8$ ✓ (3)

iv) $2x - 8 = \frac{-1}{2}x - \frac{1}{2}$ ✓
$4x - 16 = -x - 1$
$5x = 15$
$\therefore x = 3$ ✓
$y = 2(3) - 8$
$y = -2$ ✓
$W(3;-2)$ ✓ (4)
(-1) if not as coords.

[20]

Question 3

a) $P(\sqrt{3}; -2)$ given

i) $P'(-\sqrt{3}; -2)$ ✓✓ (2)

ii) $P''(-\sqrt{3}; 2)$ ✓✓ (2)

b) i) $P(-3;2) \rightarrow P'(2;3)$ ✓✓ (2)

ii) $P''(4;6)$ ✓✓ (2)

iii) $(x;y) \rightarrow (2y; -2x)$ ✓ ✓ (2)

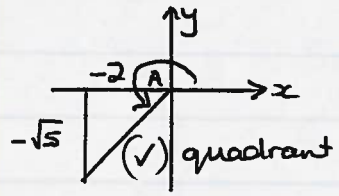
iv) $1 : 2^2$

$1 : 4 \rightarrow$ ✓✓ (2)

[12]

Question 4

a)



$r = 3$ ✓

$$\frac{\sin^2 A}{1+\cos A}$$

$$= \frac{\left(-\frac{\sqrt{5}}{3}\right)^2}{1 - \frac{2}{3}} \checkmark$$

$$= \frac{5}{9} \times \frac{3}{1} \checkmark$$

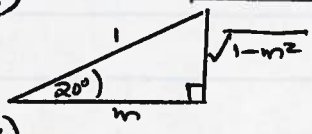
$$= \frac{5}{3} \rightarrow \checkmark \quad (5)$$

b) i) $\sin 110^\circ = \sin 70^\circ$ ✓

$= \cos 20^\circ$ ✓

$= m \rightarrow$ ✓ (3)

ii) $\tan 20^\circ = \frac{\sqrt{1-m^2}}{m}$ ✓✓ ✓



(3)

iii) $\sin 140^\circ = \sin 40^\circ$ ✓

$= \sin 2.20^\circ$ ✓

$= 2\sin 20^\circ \cos 20^\circ$ ✓

$= 2m\sqrt{1-m^2} \rightarrow$ ✓ (4)

Question 4 contd....

(2)

c) i) $\sin(\theta + 60^\circ) + \cos(\theta + 150^\circ)$

$= \sin\theta\cos 60^\circ + \cos\theta \sin 60^\circ + \cos\theta \cos 150^\circ - \sin\theta \sin 1$✓

$= \frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta$ ✓ $+ \cos\theta(-\cos 30^\circ) - \sin\theta \sin 3$✓

$= \frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta - \frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta$ ✓

$= 0 \rightarrow$ ✓ (6)

ii) $$\frac{4\sin 25^\circ . \cos 25^\circ}{\cos 40^\circ}$$

$$= \frac{2 . 2\sin 25^\circ \cos 25^\circ}{\cos 40^\circ} \checkmark$$

$$= \frac{2\sin 50^\circ}{\cos 40^\circ} \checkmark = \frac{2\cos 40^\circ}{\cos 40^\circ} \checkmark$$

$= 2 \rightarrow$ ✓ (4)

d) $$\frac{\cos x(-\tan x)}{\cos x(\tan x)} - \frac{(-\sin x)^2}{1}$$ ✓ ✓ ✓

$= 1 - \sin^2 x$ ✓

$= \cos^2 x \rightarrow$ ✓ (7)

e) i) LHS: $\cos 2\theta + 3\cos\theta - 1$

$2\cos^2\theta - 1 + 3\cos\theta - 1$ ✓

$2\cos^2\theta + 3\cos\theta - 2$ ✓ = RHS. (2)

ii) $2\cos^2\theta + 3\cos\theta - 2 = 0$

$(2\cos\theta - 1)(\cos\theta + 2) = 0$ ✓ ✓

$\cos\theta = \frac{1}{2}$ ✓ or $\cos\theta \neq -2$ ✓ No solution

$\theta = \pm 60^\circ + k360^\circ$; $k \in Z$ ✓

iii) $\theta \in \{60^\circ; -60^\circ; -300^\circ\}$ ✓ ✓ (3)

Question 6 **[15]**

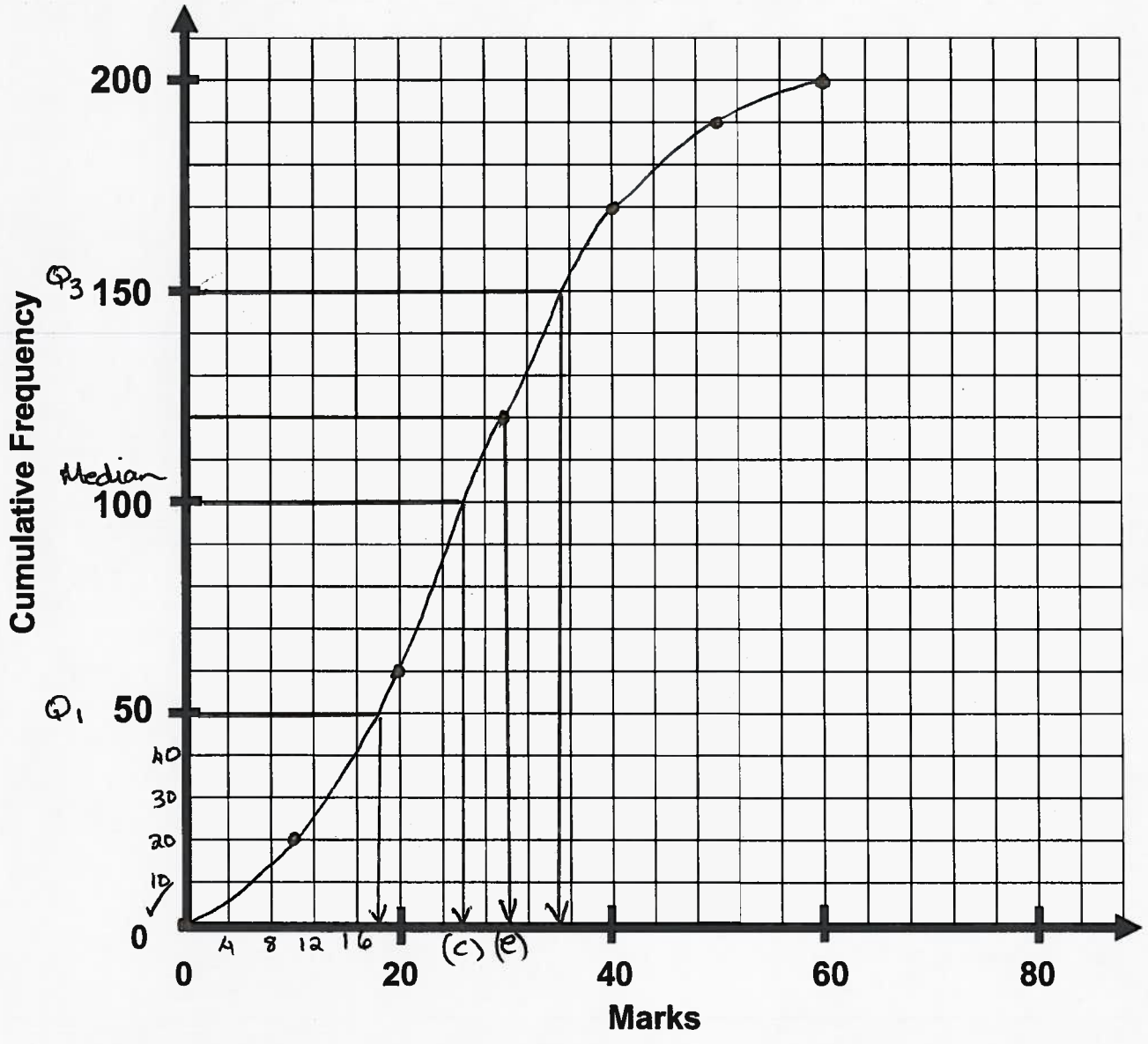
The following frequency table shows the distribution of the marks of 200 students in a Mathematics Examination.

a) Complete the cumulative frequency table. (1)

Mathematics Mark		Frequency	Cumulative frequency
0 - 10	5	20	20
11 – 20	15,5	40	60
21 – 30	25,5	60	120
31 – 40	35,5	50	170
41 – 50	45,5	20	190
51 – 60	55,5	10	200

b) Draw the cumulative frequency polygon **on the grid provided** below: (3)

DISTRIBUTION OF MARKS



c) Estimate the median mark using the graph. (2)

≈ 26 ✓✓

d) Estimate the interquartile range, using the graph. (3)

$\hat{Q}_1 = 18$ ✓

$\hat{Q}_3 = 35$ ✓ Be lenient

$IQR = 35 - 18 = 17$ ✓

e) The top 40% of students will be allowed to enroll for Advanced Mathematics.

Determine the cut-off mark. (2)

$\frac{x}{200} = 40\%$ $\therefore x = 80$

$\therefore 200 - 80 = 120$

$\therefore$ Cut-off mark is 30

f) Determine the following for the distribution of marks.

(i) the mean $\bar{x} = \frac{5490}{200} = 27,45$ ✓ (1)

(ii) the standard deviation

$\sigma = 12,97$ (1)

g) The teacher found that the marks were too low. He added 20 to each mark.

Determine the following for the new set of scores:

(i) the mean $\bar{x} = 47,75$ (f(i) answer + 20) (1)

(ii) the standard deviation

$\sigma = 12,97$ (1)

SECTION B.

Question 5

a) $a = 1$ ✓

$k = 3$ ✓

$p = 90°$ ✓ (3)

b) $y = -\sin x$ ✓ (1)

c) $g(x) = \cos(x + 90°)$

$g(x)' = \cos(x + 150°)$ ✓✓ (ca) (2)

d) $90° < x < 150°$ ✓ (2)

(-1) if ≤

Question 6

On Answer Sheet

Question 7

a) $\tan\alpha = \frac{DR}{RY}$ ✓

$\therefore DR = RY \tan\alpha$ ✓ (2)

b) $\hat{Y_1} = \theta$ ✓ (Isos Δ)

$\therefore \hat{W_1} = 180° - 2\theta$ ✓ (∠'s of Δ) (2)

c) $\frac{RY}{\sin W_1} = \frac{m}{\sin\theta}$ ✓

$RY = \frac{m \sin(180° - 2\theta)}{\sin\theta}$ ✓

$= \frac{m \sin 2\theta}{\sin\theta}$ ✓ (3)

d) $RY = \frac{m \cdot \sin 2\theta}{\sin\theta}$

$= \frac{m\, 2\sin\theta\cos\theta}{\sin\theta}$ ✓

$= 2m\cos\theta$ ✓ (2)

e) $RD = RY \cdot \tan\alpha$

$= 2m\cos\theta \cdot \tan\alpha$ ✓ (1)

f)

$RD = 2\sqrt{2} \cdot \cos 75° \cdot \tan 30°$

$= 2\sqrt{2}\left(\frac{1}{\sqrt{3}}\right)\cos(45° + 30°)$ ✓ ✓

$= \frac{2\sqrt{2}}{\sqrt{3}}\left[\cos 45° \cos 30° - \sin 45° \sin 30°\right]$ ✓

$= \frac{2\sqrt{2}}{\sqrt{3}}\left[\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}\right]$ ✓

$= 1 - \frac{1}{\sqrt{3}}$ ✓ (5)

[15]

Question 8

a) i) $x^2 + y^2 + 2x - 4y - 3 = 0$

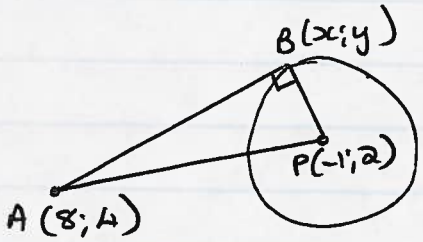
$x^2 + 2x + 1 + y^2 - 4y - 4 - 3 - 1 - 4 = 0$

$(x+1)^2 + (y-2)^2 = 8$ ✓ ✓ ✓

$P(-1; 2)$ (3)

ii) $r = \sqrt{8}$ ✓ (1)

iii)



$d_{AP} = \sqrt{(8+1)^2 + (4-2)^2}$ ✓

$= \sqrt{85}$ ✓

$d_{PB} = \sqrt{8}$ (radius)

$\therefore AB^2 = (\sqrt{85})^2 - (\sqrt{8})^2$ ✓✓ (Pythag)

$AB = \sqrt{77}$ ✓ (5)

b) $y = 3x + 2$

$m_{tan} = 3$

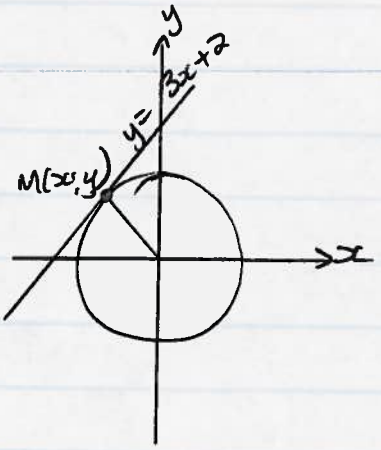
$m_{normal} = -\frac{1}{3}$ ✓

$\therefore y_{radius} = -\frac{1}{3}x$

$\therefore -\frac{1}{3}x = 3x + 2$ ✓

$-x = 9x + 6$

$x_M = -\frac{3}{5}$ ✓



$\therefore x^2 + y^2 = k$

$\left(-\frac{3}{5}\right)^2 + \left(\frac{1}{5}\right)^2 = k$

$\therefore k = \frac{2}{5}$ ✓ (5)

$y = 3x + 2$

$= 3\left(-\frac{3}{5}\right) + 2$

$y_M = \frac{1}{5}$ ✓

$M\left(-\frac{3}{5}; \frac{1}{5}\right)$

Question 9

[5] (5)

$(x;y) \rightarrow \left(\frac{x\sqrt{3}-y}{2}; \frac{x+y\sqrt{3}}{2}\right)$

$A\ (2;3) \rightarrow A'\left(\frac{2\sqrt{3}-3}{2}; \frac{2+3\sqrt{3}}{2}\right)$

$\frac{2\sqrt{3}-3}{2} = 2\cos\theta - 3\sin\theta$ ✓(subst) ... (1)

$2\sqrt{3}-3 = 4\cos\theta - 6\sin\theta$

(1) × 3

$6\sqrt{3}-9 = 12\cos\theta - 18\sin\theta$ .... (3)

(3) − (4) $6\sqrt{3}+4 = 12\cos\theta + 8\sin\theta$ .... (4)

$-13 = -26\sin\theta$

$\frac{1}{2} = \sin\theta$ ✓

$\therefore \theta = 30°$ ✓ (5)

$\frac{2+3\sqrt{3}}{2} = 3\cos\theta + 2\sin\theta$ ✓(subst) ... (2)

$2+3\sqrt{3} = 6\cos\theta + 4\sin\theta$

(2) × 2

$4+6\sqrt{3} = 12\cos\theta + 8\sin\theta$ .. (4)

✓m (elimination or subst)

{Only answer given may 1/5}

Question 10

a) $\hat{A} = \hat{B} = \frac{180^\circ - 30^\circ}{2} = 75^\circ$ ✓

$\frac{AO}{\sin 75^\circ} = \frac{6}{\sin 30^\circ}$ ✓

$AO = \text{radius} = 11,6\ cm$ ✓ (3)

b) Area $\Delta AOB = \frac{1}{2} \cdot 6 \cdot (11,6) \cdot \sin 75^\circ$ ✓

$= 33,6\ cm^2$ ✓ (2)

c) Area $\odot = \pi (11,6)^2$

$= \frac{3364}{25}\pi\ cm^2$ ✓

$\frac{360^\circ}{30^\circ} = 12$ sectors

Area 12 Δ's $= 33,6 \times 12$

$= 403,2$ ✓

Area $\odot$ − area 12 Δ's

$= \frac{3364}{25}\pi - 403,2$

$= 19,5\ cm^2$ ✓

1 shaded portion $= \frac{19,5}{12}$

$= 1,6\ cm^2$ ✓ (4)

[9]