ST STITHIANS GIRLS COLLEGE

MATHEMATICS: PAPER 3

GRADE 12

DATE: 23 July 2011 TIME: 2 hours

MARKS: 100

NAME: MEMO **TEACHER:** _____

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of **16** pages, including the front cover.
2. Read the questions carefully.
3. **Answer ALL the questions on the question paper.**
4. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
5. Round off your answers to two decimal digits where necessary.
6. All the necessary working details must be clearly shown.
7. It is in your own interest to write legibly and to present your work neatly.
8. The last four pages can be used for additional working, if necessary. If this space is used, make sure that you indicate clearly which question is being answered.

SECTION A

QUESTION 1

The first term of an Arithmetic Sequence is 8 and the seventh term is 26.

Determine:

(a) the first four terms of the sequence

$T_1 = 8 \quad T_7 = 26$

$26 = 8 + (7-1)d$ ✓

$18 = 6d$ ✓

$d = 3$ ✓ 8 11 14 17 ✓

(4)

(b) a formula for T_k, the k^{th} term of the sequence

$T_k = 3n + 5$ ✓

(2)

(c) a recursive formula for T_{k+1} of the sequence

$T_{k+1} = T_k + 3$ ✓✓

(2)

[8 marks]

QUESTION 2

All answers involving factorials must be calculated, e.g. 4! = 24.

(a) Using the letters in the word 'ANCILLOTTI', determine

(1) the number of ten letter 'words' that can be formed

$$\frac{10!}{2!\,2!\,2!} = 453600$$

(5)

(2) the probability that the new word will start and end on the letter 'T'

$$\frac{8!}{2!\,2!} = 10080 \qquad \frac{10080}{453600} = \frac{1}{45}$$

(5)

(b) A learner writes an Accounting examination and a Mathematics examination.
She believes that she has 40% chance of passing the Mathematics examination, 60% of passing the Accounting examination and 30% chance of passing both.
What is the probability that she passes Mathematics or Accounting or both?

$$P(A \text{ or } M) = 0,6 + 0,4 - 0,3 = 0,7$$

(3)

(c) Four different Mathematics textbooks and three different Geography textbooks are left on the table. You need to place these books on a shelf. If all the Mathematics books must be placed next to each other and all the Geography books must be placed next to each other, in how many different ways can you arrange the books on the shelf?

$$2(4!3!) = 288$$

(4)

[17 marks]

QUESTION 3

The manager of a women's clothing store was curious about the amount of money women of various ages spent monthly on clothing items. She obtained the information below from a representative sample of women who regularly buy from her store.

x (women's ages in years)	18	21	23	25	30	32	36	38	39	45
y (amount spent in rand)	330	300	300	240	250	190	180	310	150	120

(a) Use your calculator to find the regression line which best fits the given data. Round off your answers to two decimal digits where necessary.

$y = A + Bx$ $A = 434,47$ ✓✓

$B = -6,43$ ✓✓

$\therefore y = 434,47 - 6,43x$ (4)

(b) Calculate the correlation coefficient r for the data.

$-0,77$ ✓✓

(2)

(c) Predict how much money a 16-year old girl will spend on clothes at the store.

$y = 434,47 - 6,43(16)$ ✓

$= R331,59$ ✓

(2)

(d) What does the correlation coefficient suggest about the relationship between age and amount spent on clothes per month?

Fairly strong ✓ -ve trend ✓

(2)

[10 marks]

QUESTION 4

A survey was done on 240 of the Saints staff to determine the distances they travel to work each day. The table below shows the results of the survey.

Distance, d (in km)	**Frequency**	**Midpoint**
$0 < d \leq 5$	5	2,5
$5 < d \leq 10$	41	7,5 ✓
$10 < d \leq 15$	77	12,5 ✓
$15 < d \leq 20$	58	17,5 ✓
$20 < d \leq 25$	39	22,5 ✓
$25 < d \leq 30$	17	27,5 ✓
$30 < d \leq 35$	3	32,5 ✓

(a) Complete the table. (3)

(b) Estimate the mean for the data. ✓✓

$(2,5 \times 5) + (7,5 \times 41) + (12,5 \times 77) + (17,5 \times 58)$
$+ (22,5 \times 39) + (27,5 \times 17) + (32,5 \times 3) = 3740$

$\frac{3740}{240} = 15,58$ ✓✓

or $\bar{x} = 15,58$ ✓✓✓
(on calculator)

(4)

(c) Estimate the standard deviation for the data.

s.d. = 6,34 ✓✓✓

(4)

(d) Which is the median interval for this set of data?

$\frac{240+1}{2} = 120,5$ ✓ ∴ $10 < d \leq 15$ ✓

(2)

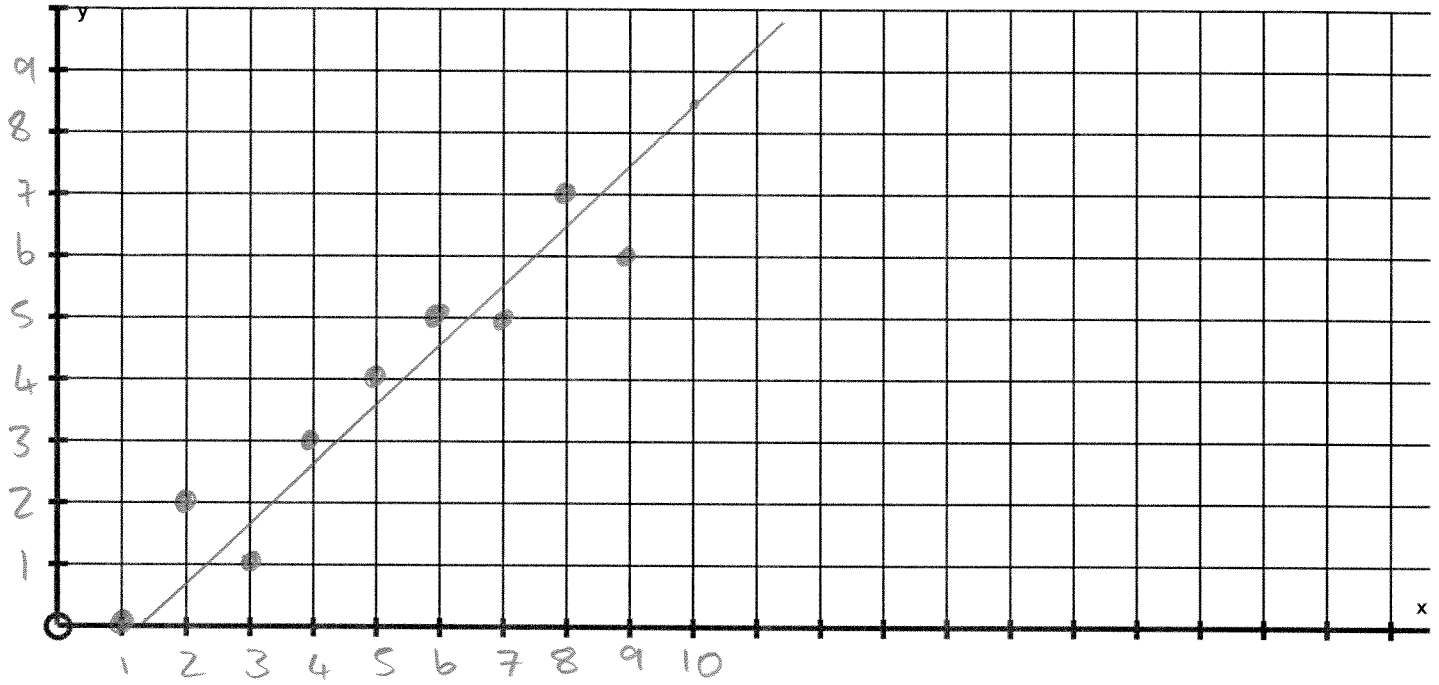
[13 marks]

QUESTION 5

The following number of goals were scored by a soccer team in one season:

MATCH	1	2	3	4	5	6	7	8	9
GOALS	0	2	1	3	4	5	5	7	6

(a) Draw a scatter plot of the data. (4)



(b) Is the correlation between the variables positive, zero or negative?

+ve

(2)

(c) Draw a line of best fit on the graph. (rough estimate) (2)

(d) Estimate how many goals will be scored in the 10th match.

Between 8 and 9

(2)

(e) Discuss whether this is a valid method to determine how many goals will be scored.

There are other factors that will rather have an influence on the number of goals. No relationship exists between matches played and the number of goals. This is an example of misleading statistics.

(3)

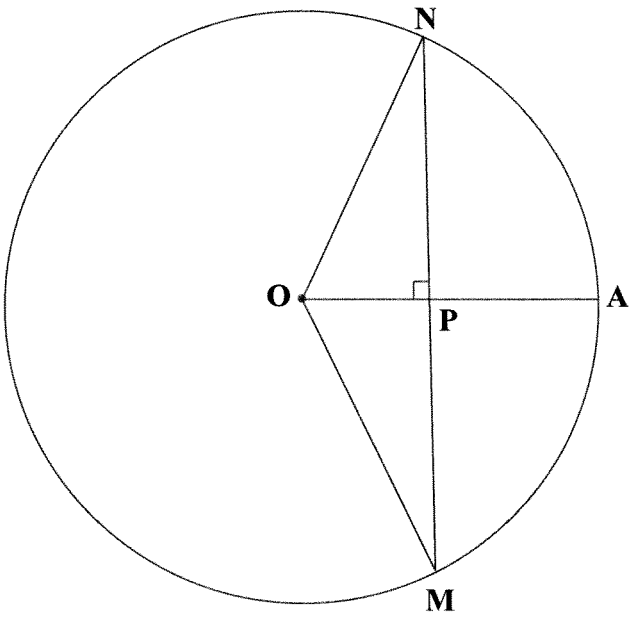
[13 marks]

SECTION B

REASONS MUST BE GIVEN UNLESS OTHERWISE STATED

QUESTION 6

In the diagram, O is the centre of circle NAM and $OPA \perp MPN$.
$MN = 48$ *units*
$OP = 7$ *units*



Calculate, with reasons, the length of PA.

$MP = NP = 24$ ✓ (Radius ⊥ chord) ✓

$OM^2 = 24^2 + 7^2$ (Pythag) ✓

$OM = 25$ ✓

$OM = OA$ ✓ (Radii) ✓

$\therefore PA = 25 - 7 = 18$ ✓

(7)

[7 marks]

QUESTION 7

Please note that the sketch is NOT drawn to scale.

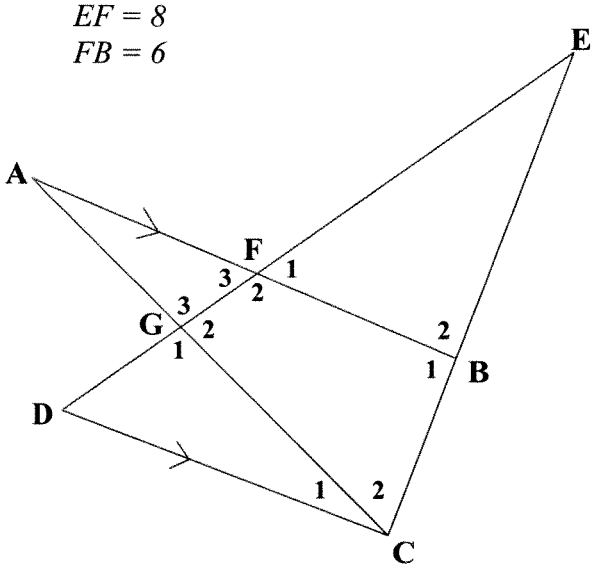
In the diagram, ΔABC is such that F is on AB and G is on AC. CB produced meets GF produced at E.

$DGFE$ is a straight line.

$BFA \,//\, DC$ $\quad\quad$ $AB = 20$

$BC = 10$ $\quad\quad$ $EF = 8$

$EB = 5$ $\quad\quad$ $FB = 6$



(a) Determine, with reasons, the numerical value of $\frac{EF}{ED}$

$\frac{EB}{EC} = \frac{EF}{ED}$ (Line // to one side Δ)

$\frac{EF}{ED} = \frac{5}{15} = \frac{1}{3}$

(3)

(b) Calculate the length of ED.

$\frac{1}{3} = \frac{8}{ED}$ ✓

$ED = 24$ ✓

(2)

(c) Complete, without stating reasons: $\Delta EFB /// \Delta \ldots$

$\Delta EFB /// \Delta EDC$

(1)

(d) Hence, calculate, giving reasons, the length of DC.

$\frac{DC}{6} = \frac{15}{5}$ ✓ (Line // to one side Δ) ✓

$DC = 18$ ✓

(3)

(e) Prove that $\frac{AF}{CD} = \frac{FG}{DG}$ (Hint: Use similar triangles)

In ΔAFG & ΔCDG

① $\hat{A} = \hat{C}_1$ (Alt ∠'s $AB \parallel DC$) ✓

② $\hat{G}_2 = \hat{G}_1$ (Vert Opp ∠'s) ✓

③ $\hat{F}_3 = D$ (3rd ∠ Δ)

$\therefore \Delta AFG \,|||\, \Delta CDG$ (AAA) ✓

$\therefore \frac{AF}{CD} = \frac{FG}{DG}$ (||| Δ's) ✓

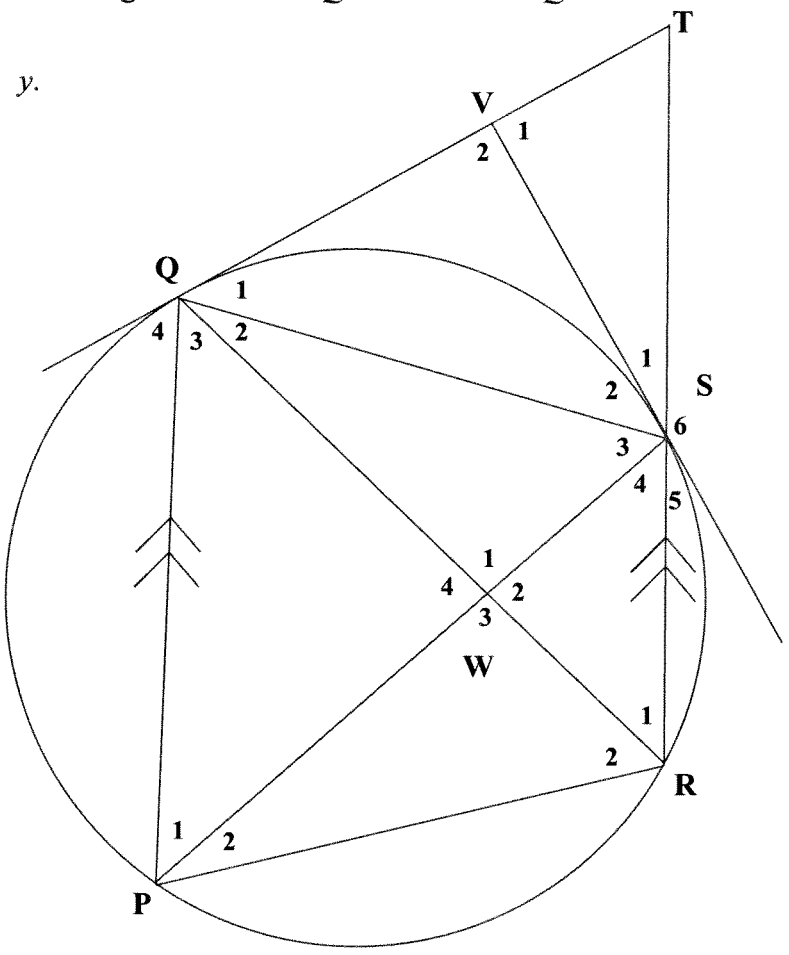
(4)

[13 marks]

QUESTION 8

In the diagram PQ and RS are two chords of the circle such that $PQ \,//\, RS$. The tangent to the circle at Q meets RS produced at T and the tangent at S meets QT at V. PS and QR intersect at W.
QS and PR are drawn.

Let $\hat{Q}_1 = x$ *and* $\hat{R}_2 = y$.



Prove that:

(a) $\hat{V}_1 = 2\hat{R}_1$

$QV = VS$ ✓ (Tan to a point) ✓

$\therefore Q_1 = S_2 = x$ ✓ (Isos Δ) ✓

$R_1 = S_2 = x$ ✓ (Tan chord Thm) ✓

$V_1 = Q_1 + S_2 = 2x$ (Ext $\angle$ Δ = Sum Opp Int $\angle$'s) ✓

$\therefore V_1 = 2R_1$ ✓

(8)

(b) QVSW is a cyclic quadrilateral.

$\hat{P}_1 = \hat{R}_1 = x$ ✓ ($\angle$'s in same segment) ✓

$\hat{W}_1 = 2\hat{P}_1 = 2x$ ✓ ($\angle$ at centre = 2 × $\angle$ at circum)

$\hat{W}_1 = 2x = \hat{V}_1$ ✓

$\therefore$ QVSW is cyclic (Ext $\angle$ = Opp Int $\angle$) ✓

(6)

(c) $\hat{P}_1 + \hat{T} = P\hat{R}T$

$\hat{Q}_4 = y$ ✓ (Tan Chord Thm) ✓

$\hat{T} = \hat{Q}_4 = y$ ✓ (Corr $\angle$'s PQ ∥ TR) ✓

$\therefore T\hat{R}P = x + y$

$\hat{P}_1 + \hat{T} = x + y$ ✓

$\therefore T\hat{R}P = \hat{P}_1 + \hat{T}$

(5)

[19 marks]