



GRADE 12 STANDARDISATION PROJECT
SEPTEMBER 2014

MATHEMATICS: PAPER I

Time: 3 hours

150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 8 pages and an Information Sheet of 2 pages (i – ii). Please check that your paper is complete.
 2. Read the questions carefully.
 3. Answer all the questions.
 4. Number your answers exactly as the questions are numbered.
 5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
 6. Round off your answers to one decimal digit where necessary.
 7. All the necessary working details must be clearly shown.
 8. It is in your own interest to write legibly and to present your work neatly.
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SECTION A**QUESTION 1**

Simplify the following:

$$(a) \quad \frac{x^3 - 8}{6 - 3x} \quad (3)$$

$$(b) \quad \frac{3^{x+1} + 3^x}{m \cdot 3^x + 3^x \cdot 2^2} - \frac{3m - 12}{m^2 - 16} \quad (5)$$

$$(c) \quad \sqrt{x^2 - 10x + 25} + 5, \quad x \geq 5 \quad (3)$$

[11]**QUESTION 2**Solve for x in terms of m :

$$(a) \quad 3x^2 = m \quad (2)$$

$$(b) \quad (mx + 1)(x - m) = 0 \quad (2)$$

$$(c) \quad 5^{x-3} = m \quad (3)$$

$$(d) \quad -2x < 4 - 10m \quad (2)$$

[9]

QUESTION 3

(a) The first three terms of an arithmetic sequence are:

$$10 - x ; 2x + 3 ; 4x + 1$$

(1) Calculate the value of x . (3)

(2) Find S_{23} . (3)

(b) If $S_n = 2n^2 + 3n$ then:

(1) Calculate T_{15} . (3)

(2) Find T_n in its simplest form. (4)

(c) Calculate the value of $\sum_{n=3}^6 (2n^2 - 1)$. (3)

(d) In a converging geometric series $S_\infty = \frac{40}{3}$ and $T_2 = \frac{5}{2}$; calculate the possible value(s) of the first term in the series. (6)

[22]

QUESTION 4

Given: $f(x) = -3x^3 + 2x$

(a) Calculate $f'(-1)$. (3)

(b) Determine the equation of the tangent to $f(x)$ where $x = -1$. (4)

[7]

QUESTION 5

Philip loves fishing and is planning on developing a campsite to run fishing tours.



- (a) Philip starts investing R500 at the end of every month into an account that offers him an interest rate of 15% per annum compounded monthly. Show that the investment grows to a value greater than R135 000 over a period of 10 years. (4)
- (b) Ten years go by and eventually Philip finds a piece of land on the West Coast of South Africa to set up his fishing campsite. The cost of the piece of land is R1 205 000. He pays a deposit of R135 000. Calculate the monthly payment needed to pay off the loan over a period of 20 years if the rate of borrowing money is 9% per annum compounded monthly. (4)
- (c) Philip takes out the loan in Question (b) above and starts paying it off every month. The business starts to go well and Philip starts to think about paying off his loan quicker.
- (1) Calculate the balance outstanding on the loan at the end of 8 years. (4)
 - (2) Once Philip calculates the balance outstanding on his loan at the end of 8 years, he decides to increase his payments to R25 000 per month. Calculate how many months it will take to pay off the remainder on his loan.
(Round your answer up to find number of months) (4)

[16]**QUESTION 6**

- (a) Given $y = \sqrt{x} + 3\pi^2$, determine $\frac{dy}{dx}$ leaving your answer with positive exponents. (3)
- (b) Given $h(x) = x^3 + 2$ and $g(x) = ax^3 + bx^2 + cx + d$ where $g(x)$ is the function achieved when $y = h(x)$ is shifted 3 units to the right and 4 units upwards. Determine the values of a , b , c and d . (4)
- (c) Given $h(x) = -9x^2$, determine the equation for the inverse of $h(x)$ in the form $y = \dots$ (3)

[10]**75 marks**

SECTION B**QUESTION 7**

Given: $f(x) = (x - 4)^2 - 5$ and $g(x) = -2x + 6$

- (a) Write down the minimum value of $f(x)$. (1)
- (b) For what values of x is $g(x) > 0$? (1)
- (c) For what values of x is $f(x) < g(x)$? (5)
- [7]**

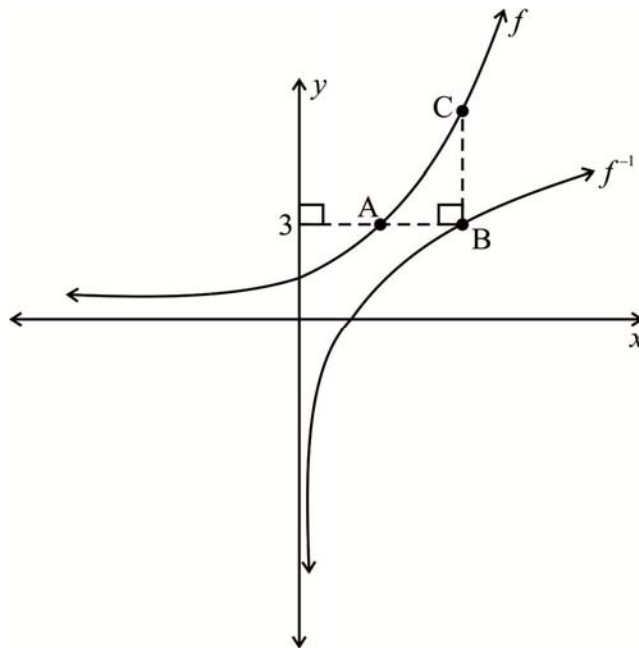
QUESTION 8

Given $g(x) = \frac{3}{x} + 2$

- (a) Determine $g'(x)$ and write with positive exponents. (2)
- (b) Sketch the graph of $g(x)$. (*Label key intercepts*) (3)
- (c) Determine the equation of $h(x)$; an axis of symmetry for $g(x)$ that has a negative gradient. (2)
- (d) A constant k is added to $h(x)$ so that the straight line becomes a tangent to the hyperbola with $x > 0$. Determine the value of k . (5)
- [12]**

QUESTION 9

In the diagram below $f(x) = 2^x$.



- (a) Write down the equation of $f^{-1}(x)$ in the form $y = \dots$ (2)
 - (b) Calculate the length of AB. (5)
 - (c) Calculate the length of CB where $CB \perp AB$. (3)
 - (d) Write down the domain of $f^{-1}(x)$. (1)
- [11]**

QUESTION 10

Give answers in this question correct to 4 decimal digits.

Philip goes deep sea fishing for hake.

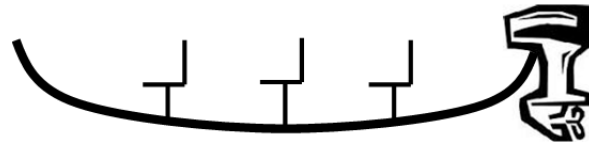


It is known that 47% of hake are male.

- (a) If Philip catches three hake, what is the probability that all three of the hake are male? (2)
 - (b) What is the probability that at least one of the three hake is a male? (3)
- [5]**

QUESTION 11

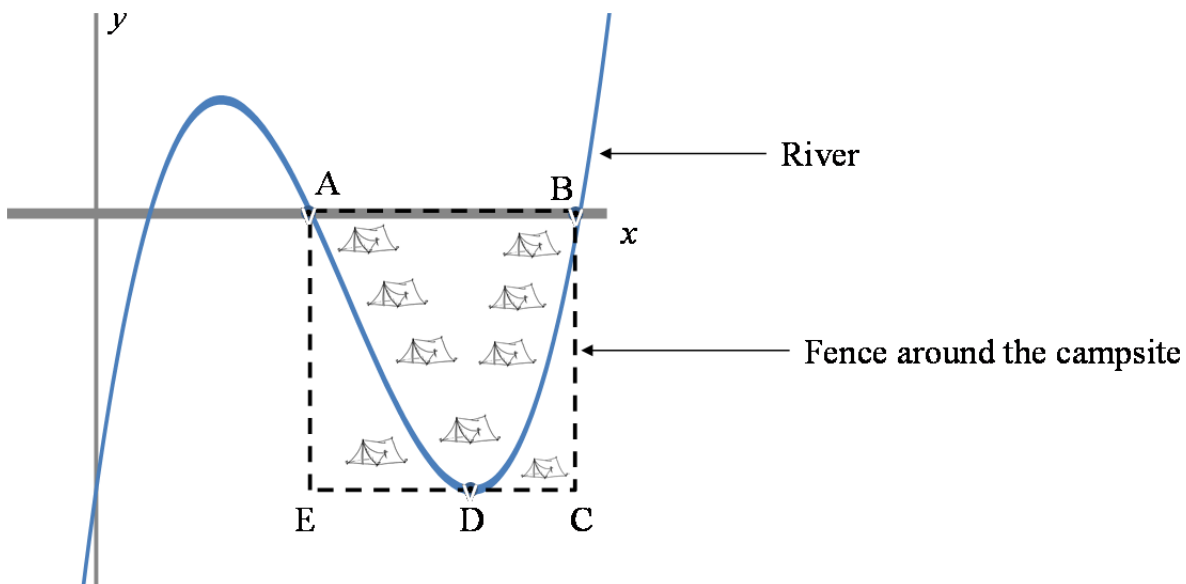
Philip's boat has three seats on it as shown in the diagram below.



- (a) How many different ways can Philip, Harry and Siphos sit in the seats? (1)
 - (b) Calculate the number of ways that the three friends can be seated such that Philip and Siphos are next to each other. (2)
 - (c) The name of the boat is HAMMERHEAD. In how many unique ways can the letters from the word be arranged? (3)
 - (d) The letter R and the letter D **cannot** be placed next to each other. Determine the number of unique arrangements that satisfy this restriction. (4)
- [10]**

QUESTION 12

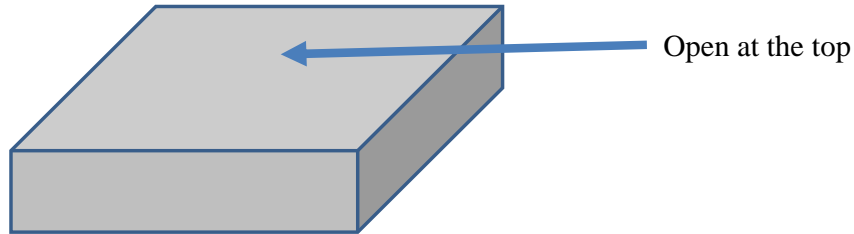
Below is an aerial view of Philip's rectangular shaped campsite enclosed by the fence that runs through the points A, B, C, D and E. There is a river that runs through the piece of land as indicated on the diagram below and point D is a turning point.



- (a) The river is modelled by $f(x) = x^3 + mx^2 + px - 36$ and D is the point $(7; -36)$. Show that $m = -14$ and $p = 49$. (8)
 - (b) Calculate the area of Philip's campsite. (*Diagram is not drawn to scale*) (5)
 - (c) Philip is building a road that has an equation of $y = -8x + k$ and is a tangent to the river. He has to build a bridge where the road crosses the river inside his campsite. Determine the co-ordinates of the central point of this bridge. (7)
- [20]**

QUESTION 13

Philip is designing large fish troughs in the shape of rectangular prisms with an open top as shown in the diagram below. The table shows some standard sized troughs and their surface areas.



Fish Trough	Surface Area
1	$2[1^2 + 3(1)] + 2[1^2 + 5(1)] + (1 + 3)(1 + 5) = 44 \text{ m}^2$
2	$2[2^2 + 3(2)] + 2[2^2 + 5(2)] + (2 + 3)(2 + 5) = 83 \text{ m}^2$
3	$2[3^2 + 3(3)] + 2[3^2 + 5(3)] + (3 + 3)(3 + 5) = 132 \text{ m}^2$
4	$2[4^2 + 3(4)] + 2[4^2 + 5(4)] + (4 + 3)(4 + 5) = 191 \text{ m}^2$
5	$2[5^2 + 3(5)] + 2[5^2 + 5(5)] + (5 + 3)(5 + 5) = 260 \text{ m}^2$
m	$A + B + C = D \text{ m}^2$

- (a) Find the value of $A + B + C$ in terms of m .
(Place your answer into simplest form.) (5)
 - (b) Philip's fish troughs are all built using the surface area formula of $A + B + C$.
What is the surface area of a trough so that the trough can hold $1\,512 \text{ m}^3$ of water if filled right to the top? (5)
- [10]**

75 marks

Total: 150 marks