



**MATHEMATICS: PAPER II**

**EXAMINATION NUMBER**

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Time: 3 hours

150 marks

**PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY**

1. This question paper consists of 26 pages and an Information Sheet of 2 pages (i – ii). Please check that your question paper is complete.
2. Read the questions carefully.
3. **Answer ALL the questions on the question paper and hand this in at the end of the examination.**
4. Diagrams are not necessarily drawn to scale.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. All necessary working details must be clearly shown.
7. Round off your answers to one decimal digit where necessary, unless otherwise stated.
8. Ensure that your calculator is in **DEGREE** mode.
9. It is in your own interest to write legibly and to present your work neatly.

**FOR OFFICE USE ONLY: MARKER TO ENTER MARKS**

Q1	Q2	Q4	Q5	Q6	Q7
16	11	21	8	23	23

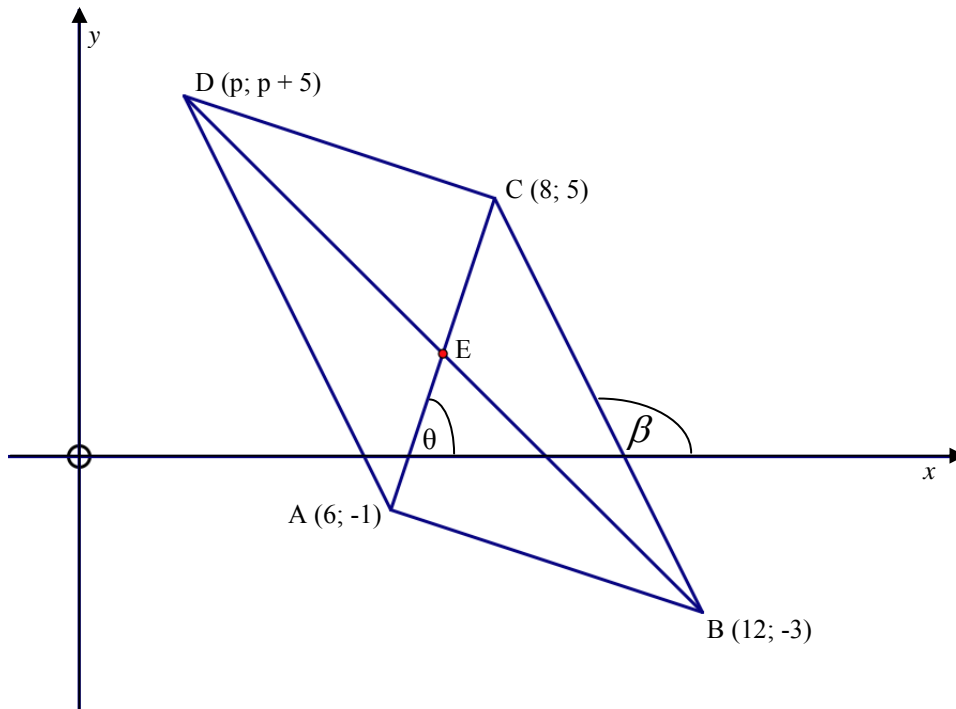
Q3	Q8	TOTAL: Q3 + Q8
24	24	48

<b>TOTAL</b>	<b>/150</b>
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**SECTION A**

**QUESTION 1**

In the diagram below, quadrilateral ABCD has A(6; -1); B(12; -3); C(8; 5) and D(p; p + 5). The diagonals of the quadrilateral ABCD intersect at E. The angle of inclination of AC and BC is  $\theta$  and  $\beta$  respectively.



- (a) Determine the co-ordinates of E, if E is the midpoint of AC.

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(2)

- (b) Determine the equation of line passing through B, E and D in the form  $y = mx + c$ .

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(3)

- (c) Determine the value of  $p$ .

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(2)

(d) Determine, correct to one decimal digit, the size of:

(1)  $\theta$  \_\_\_\_\_ (3)

(2)  $\beta$  \_\_\_\_\_ (3)

(3)  $\hat{A}CB$  \_\_\_\_\_ (1)

(e) (1) Explain why  $\hat{A}EB \neq 90^\circ$ .

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 \_\_\_\_\_ (1)

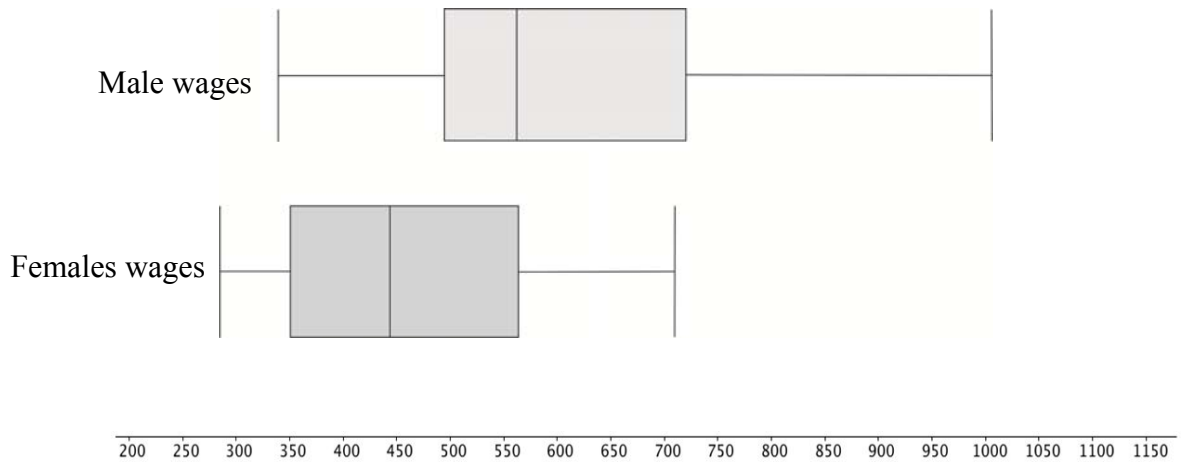
(2) A learner claims that ABCD is a rhombus. Is he correct? Explain.

\_\_\_\_\_  
 \_\_\_\_\_  
 \_\_\_\_\_ (1)

**[16]**

**QUESTION 2**

- (a) The box-and-whisker plots show the wages for women and men for the same type of work.



State whether each of the following statements are true or false.

- (1) 75% of the females earn less than 50% of the males.

True       False

(1)

- (2) Both distributions are positively skewed.

True       False

(1)

- (3) The highest paid male earns approximately 30% more than the highest paid female.

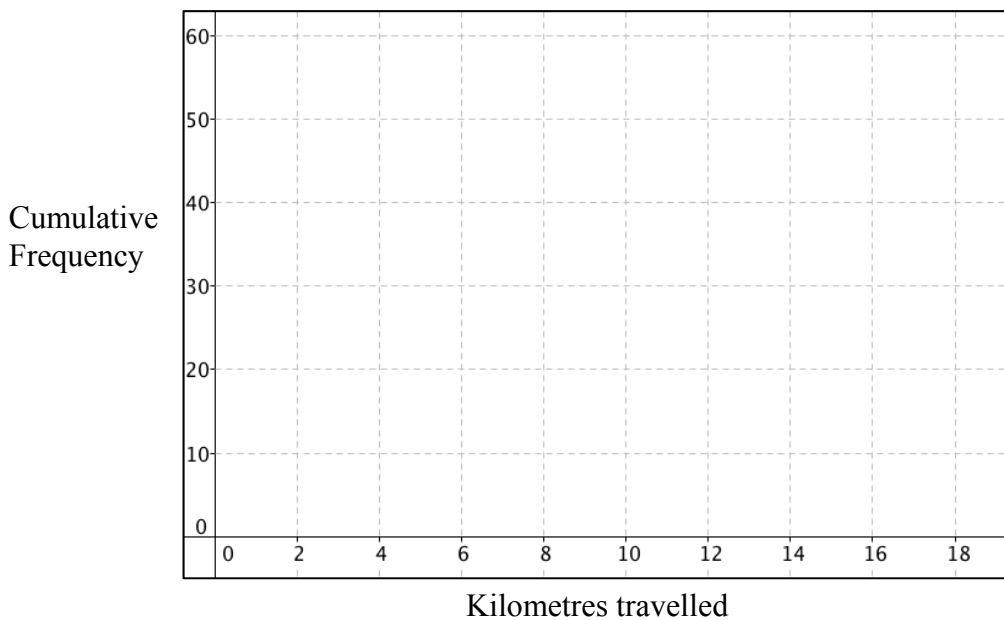
True       False

(1)

- (b) The following information was obtained from a survey done about the distances travelled to school each day by a group of learners.

Kilometers travelled	Cumulative frequency
$0 < x \leq 2$	12
$2 < x \leq 4$	38
$4 < x \leq 6$	44
$6 < x \leq 8$	45
$8 < x \leq 10$	52
$10 < x \leq 12$	56
$12 < x \leq 14$	58
$14 < x \leq 16$	59
$16 < x \leq 18$	60

- (1) Draw an ogive (cumulative frequency diagram) and estimate the median.



(4)

Estimated median \_\_\_\_\_

(1)

- (2) Estimate from the graph, the percentage of learners who travel between 5 and 13 km per day.

\_\_\_\_\_

\_\_\_\_\_

(2)

- (3) A learner claimed that the interquartile range is a better measure of spread for the data set, than the range. Which of the following is the better reason for the statement?

- (i) More than 60% of the data lies in the interval  $0 < x \leq 4$ .
- (ii) The mean for the data is 5.

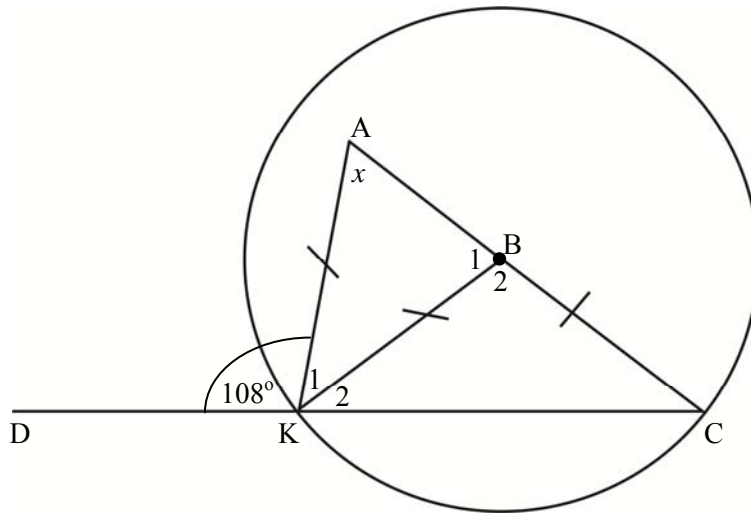
(1)

**[11]**

**QUESTION 3**

In the following questions, show all your working and give valid reasons.

- (a) B is the centre of the circle.  
 $AK = KB = BC$   
 $\hat{AKD} = 108^\circ$   
 $\hat{A} = x$



- (1) Express  $\hat{B}_1$  in terms of  $x$ .

\_\_\_\_\_ (1)

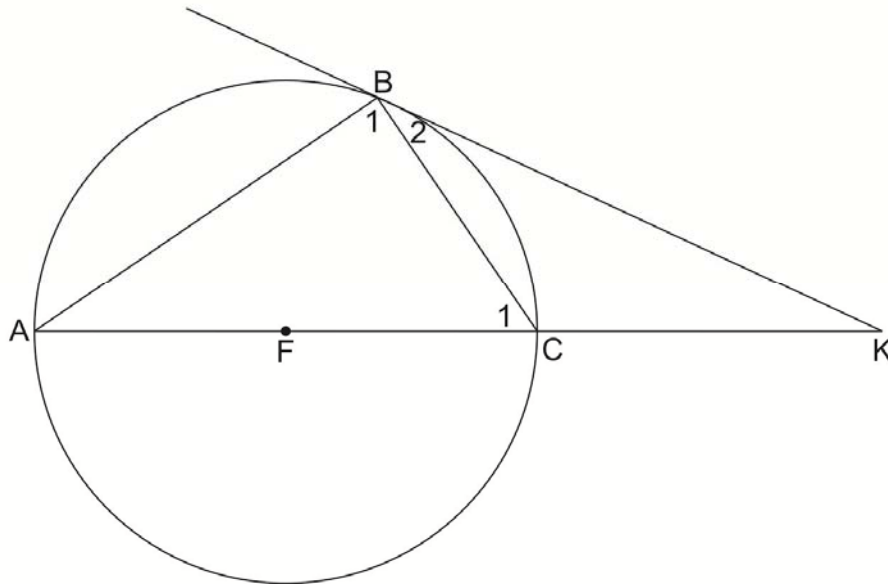
- (2) Show that  $\hat{C} = \frac{x}{2}$ .

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 \_\_\_\_\_  
 \_\_\_\_\_ (3)

- (3) Solve for  $x$ .

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 \_\_\_\_\_  
 \_\_\_\_\_  
 \_\_\_\_\_ (2)

- (b) Circle, centre F, has AC as diameter. AC is produced to K.  
 $\hat{B}_1 + \hat{B}_2 = 118^\circ$  and  $\hat{K} = 34^\circ$ .



Show that BK is a tangent to the circle at B.

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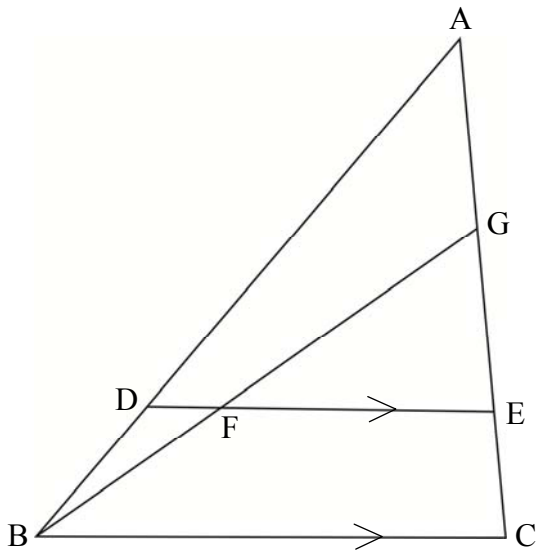
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(5)

- (c) In the triangle ABC, G is the midpoint of AE, DE is parallel to BC and BG intersects DE at F.



- (1) Explain why  $\frac{AD}{DB} = \frac{AE}{EC}$  and  $\frac{GE}{EC} = \frac{GF}{FB}$ .

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(1)

- (2) If  $\frac{AD}{DB} = \frac{4}{1}$ , determine the value of  $\frac{GF}{FB}$ .

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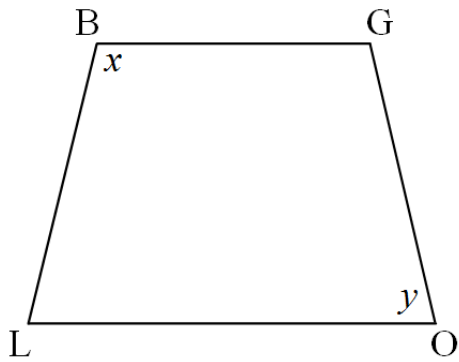


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(3)



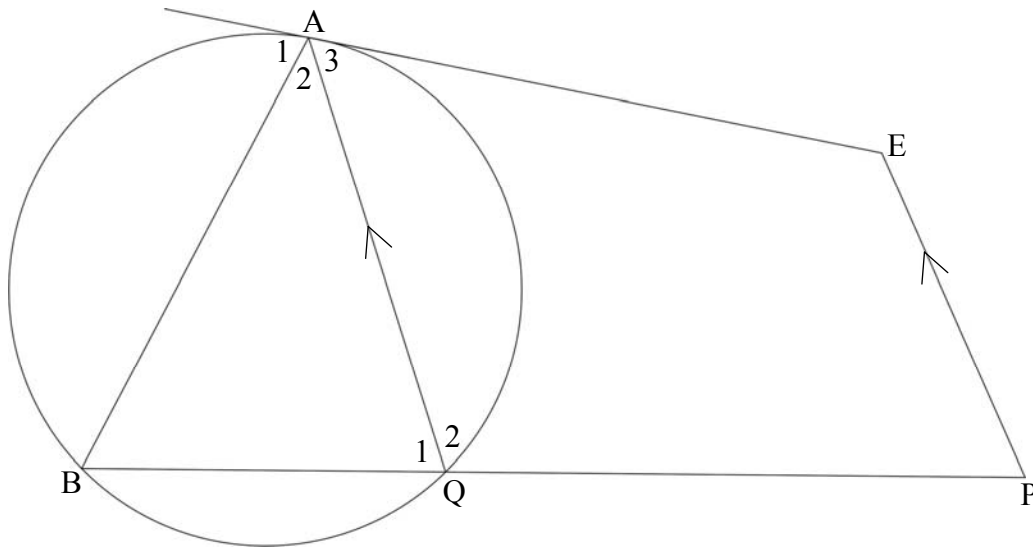
- (d) In quadrilateral BLOG if  $x+y=180^\circ$ , then BLOG is a cyclic quadrilateral.



Give two more conditions for a quadrilateral to be cyclic. Illustrate each of your answers with a suitable sketch for explanation.

(4)

- (e) AE is a tangent to the circle. B and Q are points on the circle and BQ is produced to P. PE // QA.



- (1) Prove that  $\hat{A}_1 = \hat{P}$ .

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(4)

- (2) Explain why ABPE is a cyclic quadrilateral.

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(1)  
[24]

**QUESTION 4**

(a)  $4 \tan \theta + 5 = 0$  and  $\theta \in [0^\circ; 180^\circ]$ .

Determine, without the use of a calculator, the value of

$$\sqrt{41} \cos \theta - 4 \sin(-150^\circ) \cdot \cos 180^\circ$$

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(6)

(b) If  $\tan 50^\circ = k$ , evaluate  $\frac{4 \cos^2 25^\circ - 2}{2 \sin 25^\circ \cdot \cos 25^\circ}$  in terms of  $k$ .

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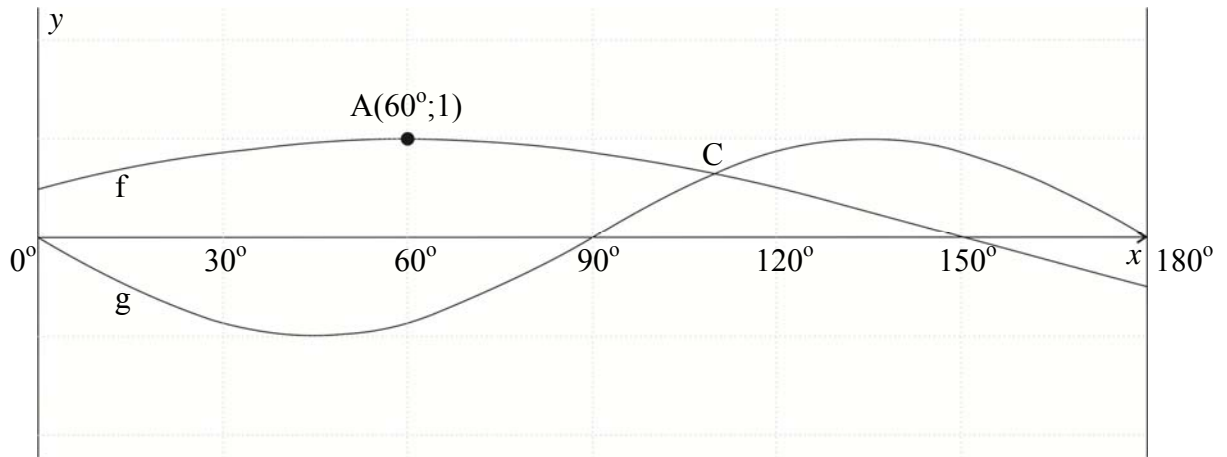
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(4)

- (c) The figure shows the graph  $f(x) = \cos(x + \theta)$  and  $g(x) = -\sin 2x$  for  $x \in [0^\circ; 180^\circ]$ .  
 A is  $(60^\circ; 1)$



- (1) Write down the range of  $g$ .

\_\_\_\_\_ (1)

- (2) Determine the value of  $\theta$ .

\_\_\_\_\_ (1)

- (3)  $C(x; y)$  is the point of intersection of the two graphs. Solve for  $x$ .

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 \_\_\_\_\_ (7)

- (4) For which values of  $x$  is  $f(x).g(x) > 0$ ?

\_\_\_\_\_ (2)  
**[21]**

**72 marks**

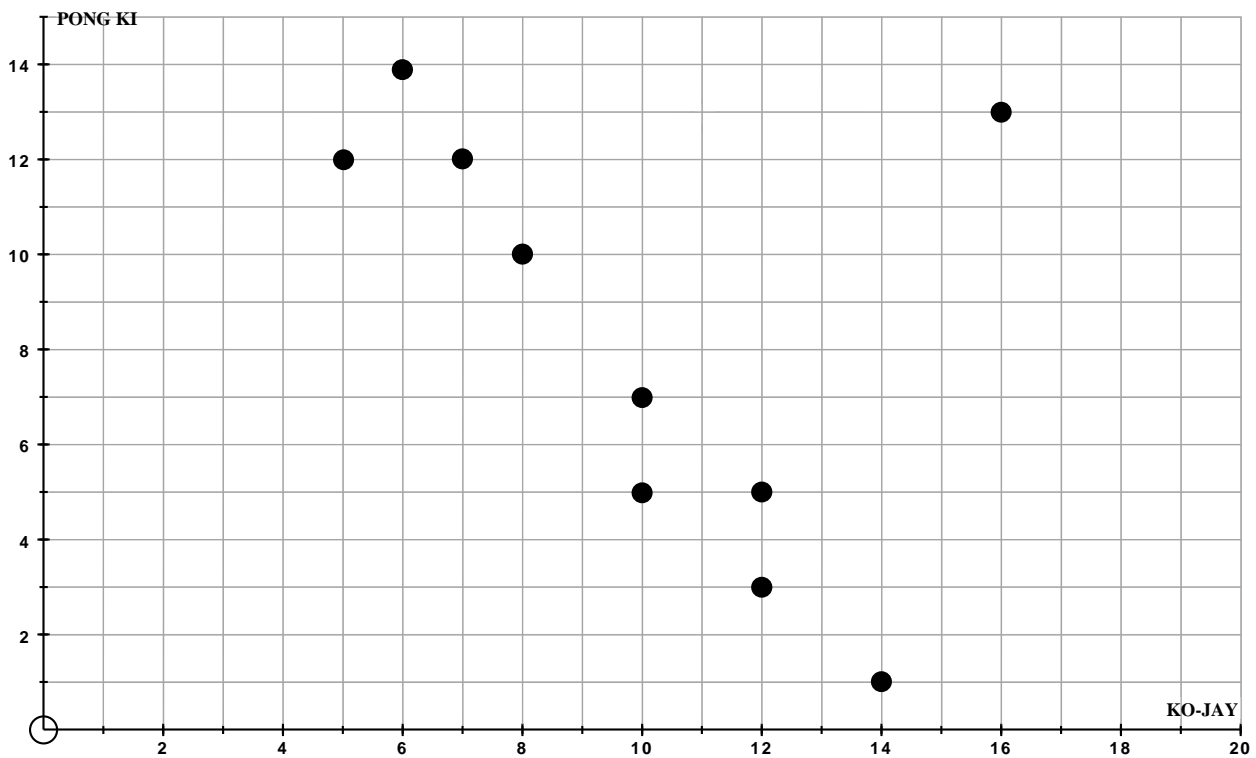
**SECTION B**

**QUESTION 5**

'KO-JAY' and 'PONG KI' are two computer games. The table below shows the number of wins obtained by some friends.

Name	Vuyo	Chi	Thebe	Steve	Shiqi	James	Elize	Morag	Piet	Alice
Number of wins at KO-JAY ( $x$ )	14	8	10	6	12	7	10	16	5	12
Number of wins at PONG KI ( $y$ )	1	10	5	14	3	12	7	13	12	5

These results are plotted below.



(a) Discuss the trend of the data collected

\_\_\_\_\_ (1)

(b) Identify an outlier in the data.

\_\_\_\_\_ (1)

- (c) Find the equation of the best fit line for the number of wins at PONG KI ( $y$ ) against the number of wins at KO-JAY ( $x$ ) in the form  $y = a + bx$ . Determine your values for  $a$  and  $b$  rounded to 4 decimal digits. Do not include the outlier identified in (b).

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(3)

- (d) Use the best fit line to estimate the number of games of PONG KI that Sarah won, if she won 6 games of KO-JAY.

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(2)

- (e) The value of the correlation coefficient is  $r = -0,9622$ . Explain clearly what can be deduced from this result.

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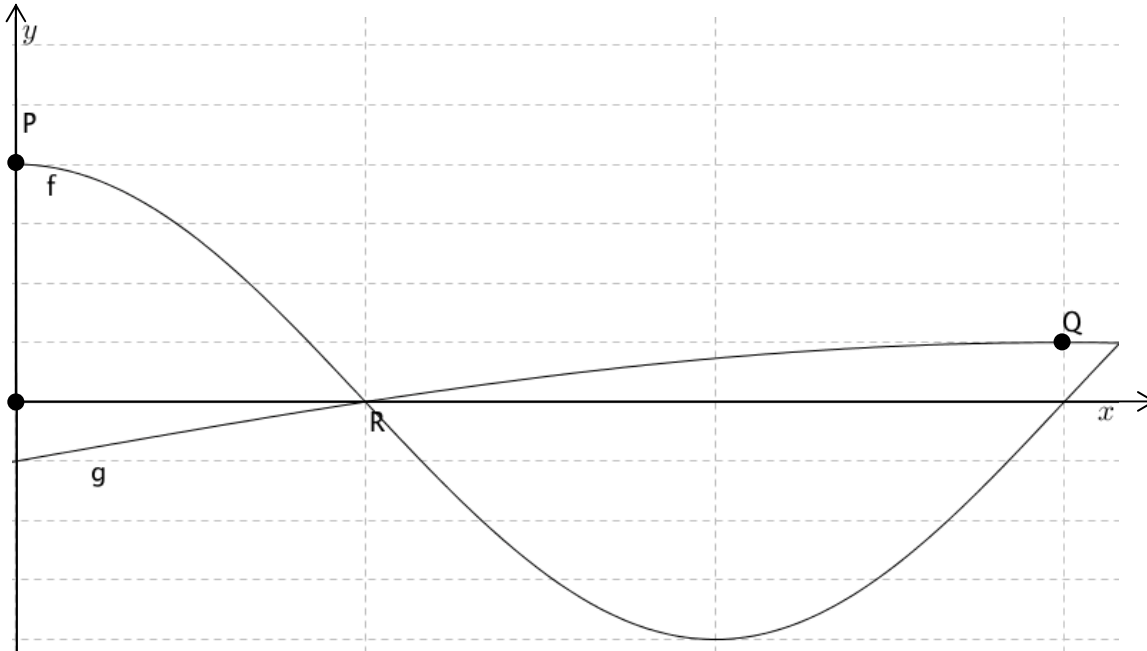


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(1)  
**[8]**

**QUESTION 6**

- (a) The diagram shows part of the graphs of  $f(x) = 4 \cos 3x$  and  $h(x) = \sin(x + b)$ .  
 Points P and Q are the respective maximum points on these graphs.  
 The graphs intersect on the  $x$  axis at R.



- (1) Write down the co-ordinates of R.

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\_\_\_\_\_ (2)

- (2) Write down the value of  $b$ .

\_\_\_\_\_ (1)

- (3) Write down the period of  $f$ .

\_\_\_\_\_ (1)

- (4) If  $f$  is shifted horizontally by  $30^\circ$  to the right, write down the resulting graph.

\_\_\_\_\_ (1)

- (5) If  $h$  is shifted vertically by 2 units, write down the amplitude of the resulting graph.

\_\_\_\_\_ (1)

(b) (1) Prove that  $\frac{\cos 3x}{\cos x} = 2 \cos 2x - 1$ .

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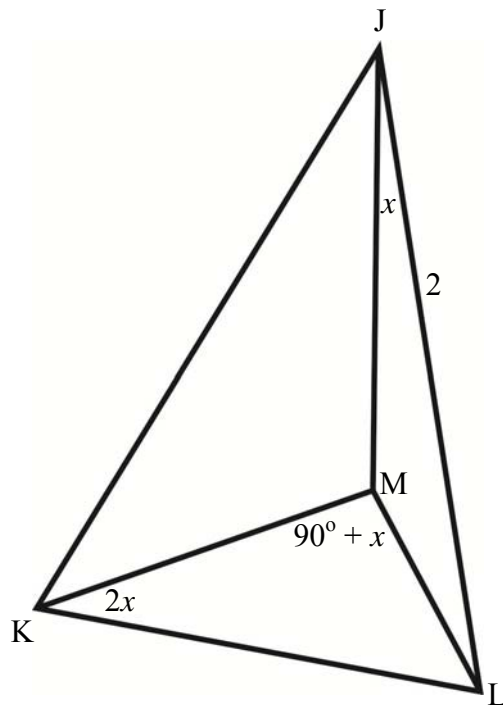
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(4)

(2) JM is a vertical tower and points K and L are in the same horizontal plane as point M, the foot of the tower.



$$\widehat{MJL} = x$$

$$\widehat{KML} = 90^\circ + x$$

$$\widehat{MKL} = 2x$$

$$JL = 2 \text{ units}$$



(i) Show that  $KL = 1$ .

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(6)

(ii) Show that  $MK = 2 \cos 2x - 1$ .

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(4)

(3) Find the values of  $x$  for which MK exists.

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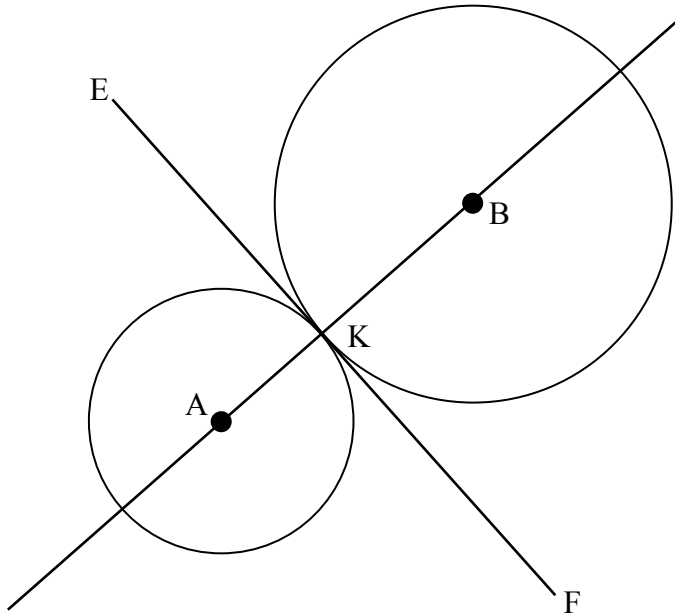
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(3)  
[23]

**QUESTION 7**

- (a) In the diagram circle A and circle B touch each other at a common point K. EF is the common tangent to the circles at K.



- (1) Complete the following table:

Circle	Centre	Radius	Equation
A	(-3; -2)	2	
B			$x^2 + y^2 - 2x - 2y - 7 = 0$

(1)

(2)

(2) Determine the coordinates of the point K.

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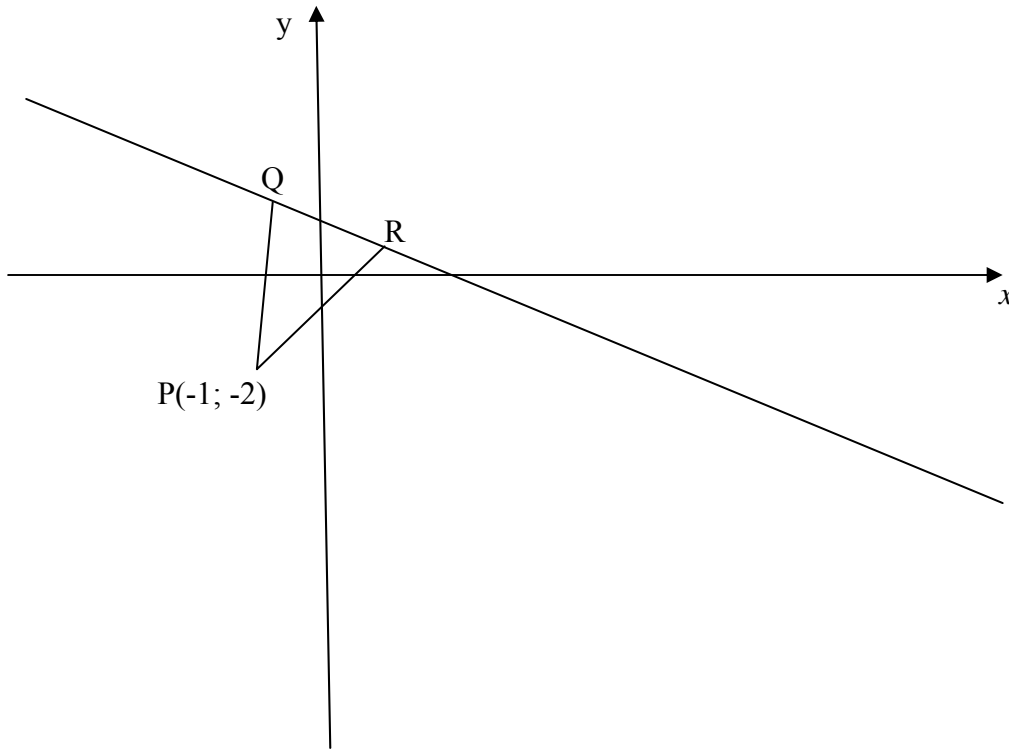
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(7)



- (b) The point  $P(-1; -2)$  is one vertex of an equilateral triangle  $\Delta PQR$ , having one of its sides on the line  $x + 2y - 5 = 0$ .  $T$  is a point on  $QR$  so that  $PT \perp QR$ .



- (1) Determine the equation of  $PT$ .

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(4)

- (2) Determine the length of  $PT$ .

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(5)

(3) Determine the length of PR, correct to one decimal digit.

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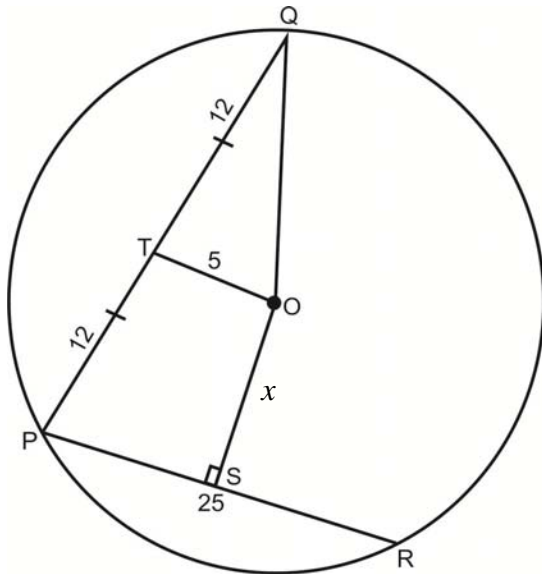
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(2)  
[23]

**QUESTION 8**

- (a) In the diagram, O is the centre of the circle.  
 $OS \perp PR$  and  $PT = TQ$



If  $PT = TQ = 12$ ,  $PR = 25$  and  $OT = 5$ , determine, giving reasons, the value of  $x$  correct to one decimal digit.

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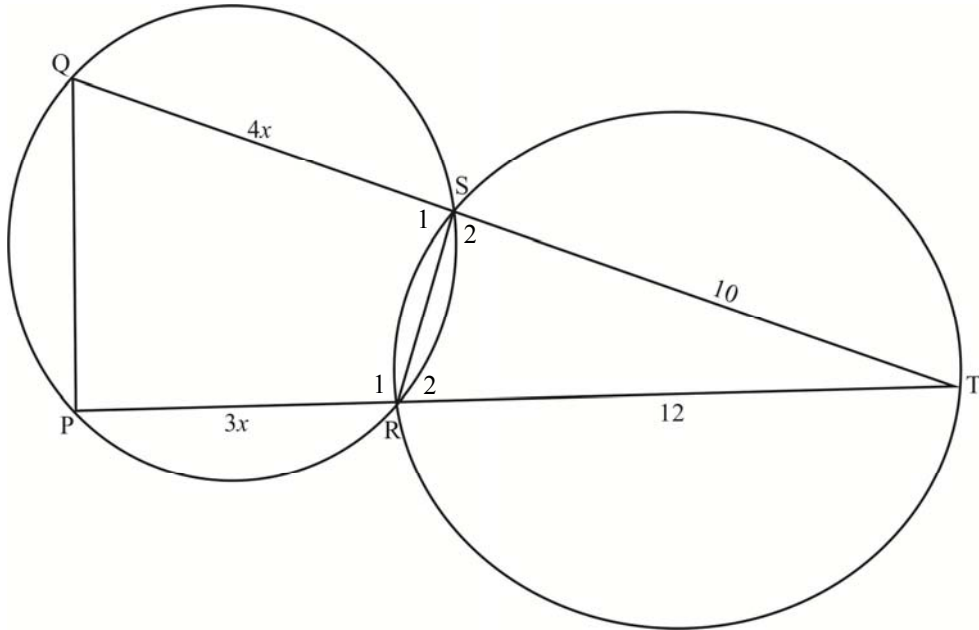
(7)



- (b) (1) Complete the statement:  
The corresponding sides of equiangular triangles are:

\_\_\_\_\_ (1)

- (2) Chords PR and QS are produced to meet outside the circle at T, on the edge of another circle.  
RT = 12 units, ST = 10 units, PR = 3x units and SQ = 4x units.



Determine the length of PR.

**Hint:** Use similar  $\Delta$ 's.

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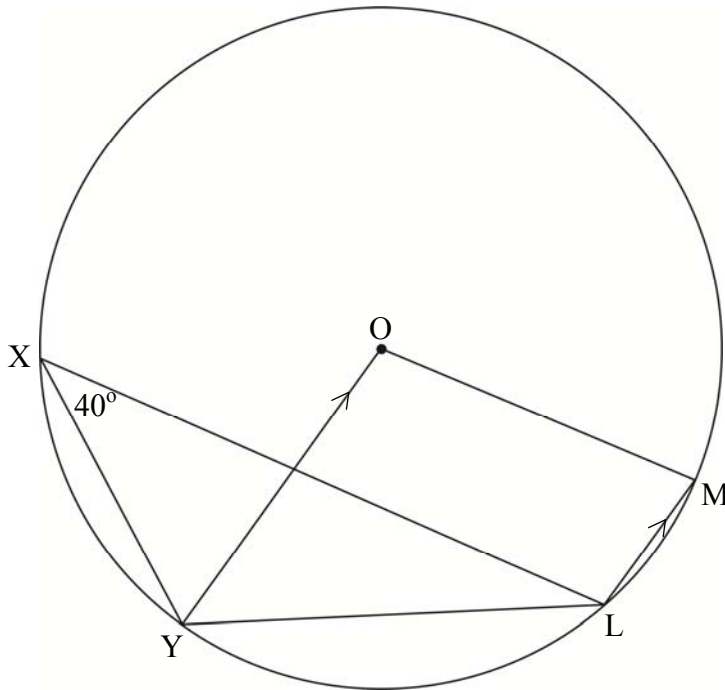
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\_\_\_\_\_

(8)

- (c) In the diagram O is the centre of the circle.  
 X, Y, L and M are points on the circumference.  
 $OY \parallel ML$ .  
 $\hat{YXL} = 40^\circ$ .



Calculate, giving reasons, the size of  $\hat{YOM}$ .

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(8)  
 [24]

<b>78 marks</b>
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**Total: 150 marks**