MATHEMATICS
GRADE 12
PRELIMINARY EXAM
PAPER 1 – 15 JULY 2014

SECTION A
QUESTION 1

(a) (1) $5.3^{2x+1} = 45$

$3^{2x+1} = 9$ ✓

$3^{2x+1} = 3^2$

$2x+1 = 2$ ✓

$x = \frac{1}{2}$ ✓ (3)

(2) $\log_5 x = 3$

$x = 5^3$ ✓

$x = 125$ ✓ (2)

(3) $(2x+3)(x-4) \geq 0$

$x \leq \frac{-3}{2}$ ✓ or $x \geq 4$ ✓ (2)

(4) $\frac{x^2+7}{x^2-2x-3} + \frac{2}{x+1} + \frac{1}{x-3} = 0$

$\frac{x^2+7}{(x-3)(x+1)}$ ✓ $+ \frac{2}{x+1} + \frac{1}{x-3} = 0$; $x \neq 3$, $x \neq -1$

$\times (x-3)(x+1)$: $x^2 + 7 + 2(x-3) + (x+1) = 0$ ✓

$x^2 + 3x + 2 = 0$ ✓

$(x+2)(x+1) = 0$

$x = -2$ ✓ or $x \neq -1$ ✓

N.A. (5)

(b) $kx^2 - 6x + 9 = 0$

$\Delta = (-6)^2 - 4(k)(9)$ ✓

$= 36 - 36k$ ✓

$36 - 36k < 0$ ✓

$k > 1$ ✓

(4)

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QUESTION 2

(a) (1) $T_4 = \frac{13}{27}$ ✓

(1)

(2) A.S. $-5 \; ; \; 1 \; ; \; 7$ (+6, +6) $\quad T_n = 6n - 11$

G.S. $1 \; ; \; 3 \; ; \; 9$ (×3, ×3) $\quad T_n = 1.3^{n-1}$

$= 3^{n-1}$

$\therefore T_n = \frac{6n-11}{3^{n-1}}$ ✓ ✓

(2)

(b) (1) $w+5 \; ; \; 2w+3 \; ; \; w-7 \; ; \; \ldots$

$(2w+3)-(w+5) \quad (w-7)-(2w+3)$

$(2w+3)-(w+5) = (w-7)-(2w+3)$ ✓ ✓

$w - 2 = -w - 10$

$2w = -8$

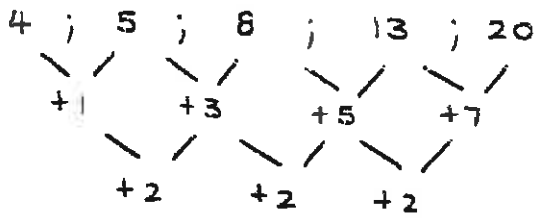
$w = -4$ ✓

(3)

(2) $d = -6$ ✓

(1)

(c) (1)



$2a = 2$ ✓ $\quad 3a + b = 1$ $\quad a + b + c = 4$

$a = 1$ ✓ $\quad 3(1) + b = 1$ $\quad 1 - 2 + c = 4$

$b = -2$ ✓ $\quad c = 5$ ✓

$T_n = n^2 - 2n + 5$ ✓

(4)

(2) 1 ; 3 ; 5 ; 7 ; ...

$a = 1 \quad d = 2$

$S_8 = \frac{8}{2}[2(1) + (8-1)(2)]$ ✓

$= 64$ ✓

(2)

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QUESTION 3

(a) $93\,000 = 145\,000(1 - i)^5$ ✓

$i = 1 - \sqrt[5]{\frac{93\,000}{145\,000}}$ ✓

$i = 0,084996...$

$i = 8,5\%$ ✓

(3)

(b) $4\,200\,000 = x\left[\frac{\left(1 + \frac{0,076}{12}\right)^{264} - 1}{\frac{0,076}{12}}\right]$ ✓

$4\,200\,000 = x\,(678,128302...)$ ✓

$x = R\,6\,193,52$ ✓

OR:

$4\,200\,000 = x\left[\frac{\left(1 + \frac{0,076}{12}\right)^{384} - 1}{\frac{0,076}{12}}\right]$ ✓ ✓

(4)

$x = R\,2\,583,86$

③

(c) (1) $32\,000 = 1274,27\left[\frac{1-\left(1+\frac{0,092}{12}\right)^{-n}}{\frac{0,092}{12}}\right]$ ✓

$0,19252853\ldots = 1-\left(1+\frac{0,092}{12}\right)^{-n}$

$\left(1+\frac{0,092}{12}\right)^{-n} = 0,80747147\ldots$ ✓

$-n = \log_{\left(1+\frac{0,092}{12}\right)} 0,80747147\ldots$ ✓

$-n = -27,999\ldots$

$n \approx 28$ months ✓

(2) $OB = 1274,27\left[\frac{1-\left(1+\frac{0,092}{12}\right)^{-10}}{\frac{0,092}{12}}\right]$ ✓ (4)

$= R12\,221,46$ ✓✓

OR

$OB = 32\,000\left(1+\frac{0,092}{12}\right)^{18} - 1274,27\left[\frac{\left(1+\frac{0,092}{12}\right)^{18}-1}{\frac{0,092}{12}}\right]$ ✓ ✓ ✓

$= R12\,221,40$ ✓

(4)

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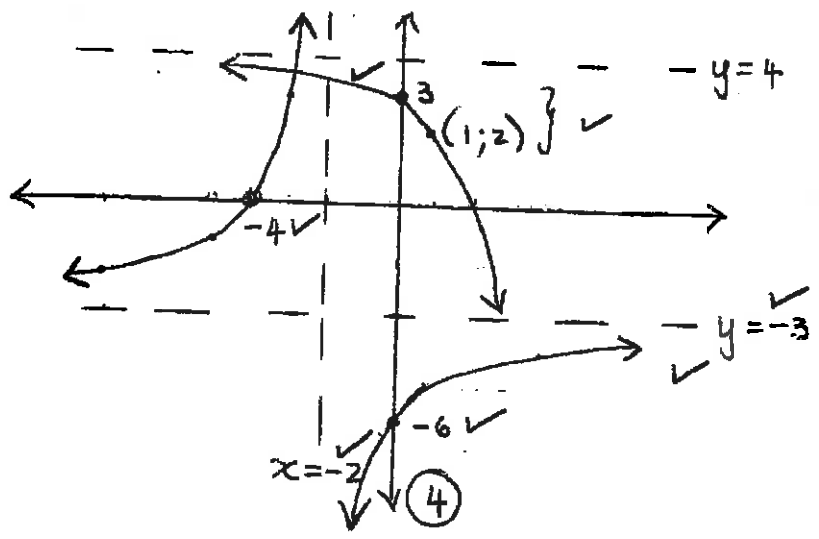
QUESTION 4

(a)



(5)

(b) $y = (x+2) - 3$ ✓

$y = x - 1$ ✓

(2)

(c) (1) * See previous page

(2)

(2) $g(x) = a.b^x + q$

$q = 4$ ✓

$g(x) = a.b^x + 4$

Subst. $(0;3)$ into equation

$3 = a.b^0 + 4$ ✓

$a = -1$ ✓

Subst. $(1;2)$ into equation

$2 = -1.b^1 + 4$ ✓

$b = 2$ ✓

$g(x) = -2^x + 4$

(5)

[14]

QUESTION 5

(a) $f(x) = -x^3$ $\quad f(x+h) = -(x+h)^3$ ✓

$= -(x^3 + 3x^2h + 3xh^2 + h^3)$

$= -x^3 - 3x^2h - 3xh^2 - h^3$ ✓

$$f'(x) = \lim_{h\to 0} \frac{-x^3 - 3x^2h - 3xh^2 + h^3 - (-x^3)}{h}$$ ✓

$$= \lim_{h\to 0} \frac{-3x^2h - 3xh^2 + h^3}{h}$$

$$= \lim_{h\to 0} \frac{h(-3x^2 - 3xh + h^2)}{h}$$ ✓

$$= \lim_{h\to 0} (-3x^2 - 3xh + h^2)$$ ✓

$= -3x^2$ ✓

(6)

(b) (1) $y = 2\sqrt{x} - \frac{3}{x^2}$

$= 2x^{1/2} - 3x^{-2}$ ✓

$\frac{dy}{dx} = x^{-1/2}$ ✓ $+ 6x^{-3}$ ✓

(3)

(2) $y = \frac{(x-7)(x+4)}{x+4}$ ✓

$= x - 7$ ✓

$\frac{dy}{dx} = 1$ ✓

(3)

(c) $y = -7(-1) + 3$

$= 10$

$(-1; 10)$ ✓

$g(-1) = 10$

$a(-1)^2 + b(-1) + 5 = 10$ ✓

$a - b + 5 = 10$

$a - b = 5$ — ①

$g(x) = ax^2 + bx + 5$

$g'(x) = 2ax + b$ ✓

$g'(-1) = -7$

$2a(-1) + b = -7$ ✓

$-2a + b = -7$ — ②

① + ②: $-a = -2$

$a = 2$ ✓

$2 - b = 5$

$b = -3$ ✓

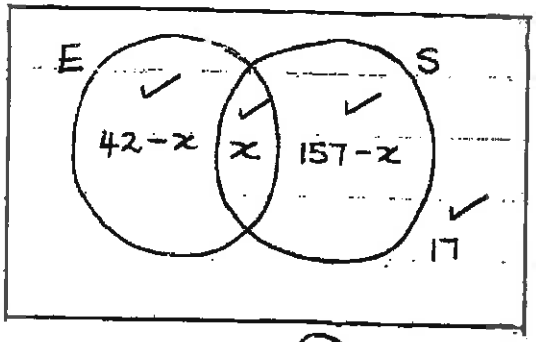
(6)

[18]

SECTION B

QUESTION 6

(a) (1)



(6)

(4)

(2) $42 - x + x + 157 - x + 17 = 180$ ✓

$216 - x = 180$

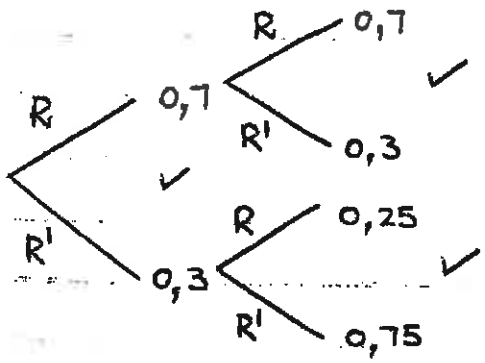
$x = 36$ ✓

(3)

(3) $\frac{42-36}{180} = \frac{6}{180} = \frac{1}{30}$ ✓

(2)

(b)



$(0,7 \times 0,7) + (0,3 \times 0,25)$ ✓

$= 0,565$ ✓

(5)

(c) $P(A \cap B) = \frac{1}{5} \times \frac{2}{3}$

$= \frac{2}{15}$ ✓

$P(A \cup B) = \frac{1}{5} + \frac{2}{3} - \frac{2}{15}$ ✓

$= \frac{11}{15}$ ✓

$P(A \cup B)' = 1 - \frac{11}{15}$

$= \frac{4}{15}$ ✓

(4)

[18]

QUESTION 7

(a) $4^m = p(2^{2m-1}) + p$

$2^{2m} = p \cdot 2^m \cdot 2^{-1} + p$ ✓

$2^{2m} - \frac{p(2^m)}{2} = p$ ✓

$2^{2m}\left(1 - \frac{p}{2}\right) = p$ ✓

$2^{2m}\left(\frac{2-p}{2}\right) = p$ ✓

$2^{2m} = \frac{2p}{2-p}$

(4)

(b) $x^2 - 3x + 2 = (x-2)(x-1)$ ✓

$h(2) = 0$ and $h(1) = 0$

$2(2)^3 + p(2)^2 + 7(2) + q = 0$ ✓ $\quad 2(1)^3 + p(1)^2 + 7(1) + q = 0$ ✓

$16 + 4p + 14 + q = 0 \quad -(1) \quad 2 + p + 7 + q = 0 \quad -(2)$

(1) − (2): $14 + 3p + 7 = 0$ ✓

$3p = -21$

$p = -7$ ✓

$2 - 7 + 7 + q = 0$

$q = -2$ ✓

(6)

[10]

QUESTION 8

(a) $\sum_{n=2}^{k} (3n - 11) = 994$

$-5 - 2 + 1 + 4 + \ldots + 3k - 11 = 994$

$a = -5$ and $d = 3$ ✓

$S_n = \frac{n}{2}[2(-5) + (n-1)(3)] = 994$ ✓ or $\frac{k-1}{2}(-5 + 3k - 11) = 994$ ✓

$\frac{n}{2}[3n - 13] = 994$ $\quad (k-1)(3k-16) = 1988$

$\times 2$: $3n^2 - 13n - 1988 = 0$ ✓ $\quad 3k^2 - 19k - 1972 = 0$ ✓

$(3n - 71)(n - 28) = 0$ ✓ $\quad (3k - 68)(k - 29) = 0$ ✓

$n = 28$ ✓ $\quad k = 29$ ✓

$\therefore k = 29$ ✓

(6)

(b) $1,5 + 2 \times 1,5 \times \frac{4}{5} + 2 \times 1,5 \times \left(\frac{4}{5}\right)^2 + \ldots$ ✓

G.S. $a = 2,4$ ✓ and $r = \frac{4}{5}$ ✓

$S_\infty = \frac{2,4}{1 - \frac{4}{5}}$ ✓

$= 12$ ✓ $\quad \therefore$ Total distance $= 12 + 1,5$

(8) $= 13,5\text{ m}$ ✓ (6)

[12]

QUESTION 9

(a) $V = l.b.h$

$9 = (3x)(x)(h)$ ✓

$h = \frac{3}{x^2}$ ✓

(2)

(b) $S.A. = (\text{Perimeter of base} \times H) + 2(\text{Area of base})$

$= (2l + 2b) \times H + 2(l \times b)$

$= (2(3x) + 2x)\left(\frac{3}{x^2}\right)$ ✓ $+ 2(3x)(x)$ ✓

$= \frac{24}{x} + 6x^2$ ✓

$C = \left(\frac{24}{x}\right)(50) + (6x^2)(100)$ ✓

$= \frac{1200}{x} + 600x^2$

(4)

(c) $C = 1200x^{-1} + 600x^2$

$C' = -1200x^{-2} + 1200x$ ✓

$\frac{-1200}{x^2} + 1200x = 0$ ✓

$\times x^2 \quad -1200 + 1200x^3 = 0$ ✓

$x^3 = 1$

$x = 1 \text{ m}$ ✓

(4)

[10]

QUESTION 10

(a) $f(x) = \log_m x$

Subst. $(4; 2)$ into equation

$2 = \log_m 4$ ✓

$m^2 = 4$ ✓

$m = 2$

(2)

(b) $x = \log_2 y$ ✓

$\therefore f^{-1}(x) = 2^x$ ✓

(2)

(c) $2 \overset{\checkmark}{<} x \leqslant \overset{\checkmark}{3}$

(2)

[6]

QUESTION 11

(a) $A(2;0)$ and $D(0;2)$ ✓

$f(x) = a(x+1)(x+1)(x-2)$ ✓

$= a(x^2+2x+1)(x-2)$

$= a(x^3-3x-2)$ ✓

Subst. $(0;2)$ into equation

$2 = a(0^3-3(0)-2)$ ✓

$2 = -2a$

$a = -1$

$\therefore f(x) = -1(x^3-3x-2)$

$= -x^3+3x+2$ ✓

$\therefore a = -1$ and $b = 3$

OR: $f'(x) = 3ax^2+b$ ✓

$0 = 3a(-1)^2+b$

$b = -3a$ ✓

$f(2) = 0$

$a(2)^3+b(2)+2 = 0$ ✓

$8a+2b+2 = 0$

÷2: $4a+b+1 = 0$

$4a-3a+1 = 0$ ✓

$a = -1$

$b = -3(-1)$ ✓

$= 3$

(6)

(b) $f(x) = -x^3 + 3x + 2$

$f'(x) = -3x^2 + 3$ ✓

$-3x^2 + 3 = 0$ ✓

$x^2 - 1 = 0$

$(x+1)(x-1) = 0$

$x = -1$ or $x = 1$ ✓

$f(1) = -(1)^3 + 3(1) + 2$

$= 4$ ✓

$B(1 ; 4)$

(4)

(c) $-1 < x < 1$ ✓ ✓

(2)

[12]

QUESTION 12

Let the path of the ball be represented by

$y = a(x-p)^2 + q$ ✓

T.P. at $(2,5 ; 4,3)$

$\therefore y = a(x-2,5)^2 + 4,3$ ✓

Subst $(0 ; 1,8)$ into equation

$1,8 = a(0-2,5)^2 + 4,3$ ✓

$a = \frac{-2}{5}$ / $-0,4$ ✓

$\therefore y = -0,4(x-2,5)^2 + 4,3$

$3,05 = -0,4(x-2,5)^2 + 4,3$ ✓

$3,125 = (x-2,5)^2$

$x - 2,5 = \pm\sqrt{3,125}$

$x = 2,5 \pm \sqrt{3,125}$

$x = 4,3\text{ m}$ ✓

[6]
