Brescia House School

30 July 2014

08h00 – 11h00

GRADE 12

Mathematics Paper 2

Time: 3 hours 150 marks

Name: _____ Maths Teacher: _____

Please read the following instructions carefully

1. This question paper consists of 15 questions on 22 pages and a separate formula sheet. Please check that your question paper is complete.
2. All questions must be answered in black or blue pen on the question paper.
3. Read the questions carefully.
4. A non-programmable calculator may be used.
5. All answers must be rounded off correct to one decimal place, unless otherwise stated.
6. It is in your own interest to write legibly and to present your work neatly.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Total
Out of	12	14	24	7	14	10	4	7	4	6	6	12	9	11	10	150
Attained																

Examiner: Mr B. Dannatt
Moderators: Mrs A. Bux, Mrs K. Raeburn External: Mrs C. Kennedy

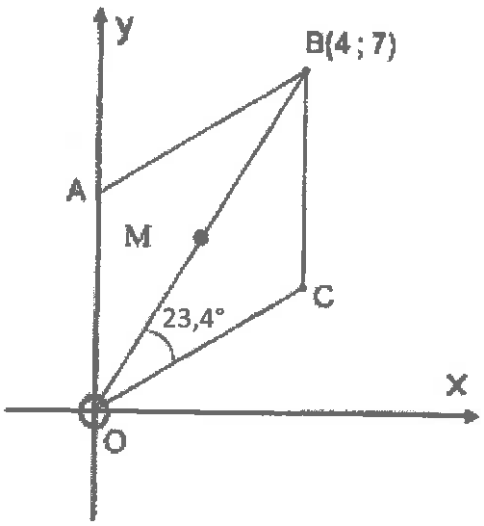
SECTION A

QUESTION 1

[12]

OABC is a parallelogram where O is the origin and A lies on the y-axis.

$B\hat{O}C = 23{,}4°$.



1.1 Determine the co-ordinates of M, the point where the diagonals of the parallelogram intersect. (2)

$M(2; \frac{7}{2})$

1.2 Calculate $B\hat{O}X$. (3)

$B\hat{O}X = \tan^{-1}(\frac{7}{4})$

$= 60{,}3°$

1.3 Hence, calculate $C\hat{O}X$. (1)

$C\hat{O}X = 36{,}9°$

1.4 Determine the gradient of AB. (2)

$m_{OC} = \tan 36{,}9°$

$= 0{,}8$

$\therefore m_{AB} = 0{,}8$

1.5 Determine the co-ordinates of C. (2)

eqn OC: $y = 0,8x$

at $x = 4$, $y = 3,2$

$\therefore C(4; 3,2)$

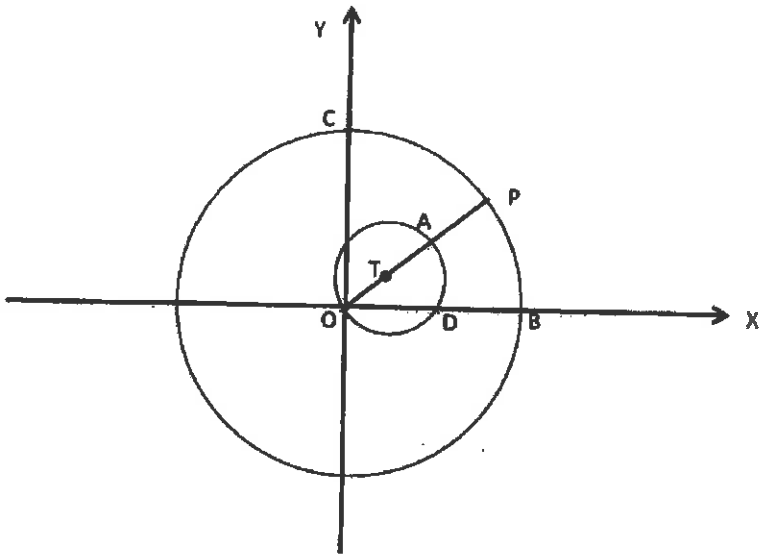
1.6 Determine the co-ordinates of A. (2)

$\Delta y = 7 - 3,2 = 3,8$

$\therefore A(0; 3,8)$

QUESTION 2 **[14]**

A is the midpoint of OP with P(4; 4) and lies on the smaller circle with diameter OA and centre T. B is the x-intercept of the circle centre O.



2.1 Write down the co-ordinates of A. (2)

A(2; 2)

2.2 Calculate the size of $O\hat{P}B$ to 1 decimal place. (4)

$m_{OP} = 1$

$\therefore \theta = 45°$

$OP = OB$ radii

$\therefore O\hat{P}B = 67,5°$

2.3 Determine the equation of the smaller circle. (3)

$T(1;1)$

$\therefore (x-1)^2 + (y-1)^2 = 2$

2.4 Determine the gradient of the tangent to the circle centre T at D on the x-axis. (5)

at D, $y = 0$

$\Rightarrow (x-1)^2 = 1$

$\Rightarrow x = 2$ or $x = 0$

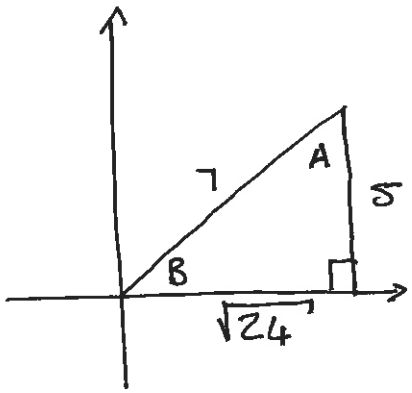
$\therefore D(2;0)$

$\therefore m_{TD} = \frac{1}{-1} = -1$

$\therefore m_t = 1$

QUESTION 3 **[24]**

3.1 If $\hat{A} + \hat{B} = 90°$ and $7 \sin \hat{B} = 5$, with the aid of a drawing, calculate the value of $\frac{\cos 2A}{\sin 2B}$. (7)



$$\frac{1 - 2\sin^2 A}{2\sin B \cos B}$$

$$= \frac{1 - 2\left(\frac{\sqrt{24}}{7}\right)^2}{2 \cdot \frac{5}{7} \cdot \frac{\sqrt{24}}{7}}$$

$$= \frac{1 - \frac{48}{49}}{\frac{10\sqrt{24}}{49}} = \frac{1}{10\sqrt{24}}$$

3.2.1 Prove that $\sin(45° + x) . \sin(45° - x) = \frac{1}{2} \cos 2x$ (5)

$$LHS = (\sin 45° \cos x + \cos 45° \sin x)(\sin 45° \cos x - \cos 45° \sin x)$$

$$= \frac{\sqrt{2}}{2}(\cos x + \sin x) \times \frac{\sqrt{2}}{2}(\cos x - \sin x)$$

$$= \frac{2}{4}(\cos^2 x - \sin^2 x)$$

$$= \frac{1}{2}\cos 2x$$

$$= RHS$$

3.2.2 Hence, determine the maximum value of $\sin(45° + x) . \sin(45° - x)$ (1)

$$max = \frac{1}{2}$$

3.3 Determine the value of $\frac{\cos 45°}{\sin 15°} - \frac{\sin 45°}{\cos 15°}$, without the use of a calculator. (5)

$$= \frac{\cos 45° \cos 15° - \sin 45° \sin 15°}{\sin 15° \cos 15°}$$

$$= \frac{\cos(45° + 15°)}{\frac{1}{2}\sin(2.15°)}$$

$$= \frac{\cos 60°}{\frac{1}{2}\sin 30°}$$

$$= 2$$

3.4 Determine the general solution for x if, $\cos 2x - \cos x = 2$ (6)

$$2\cos^2 x - 1 - \cos x - 2 = 0$$

$$2\cos^2 x - \cos x - 3 = 0$$

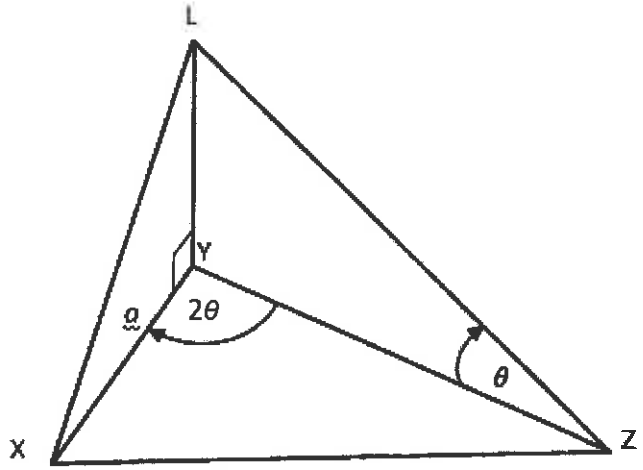
$$(2\cos x - 3)(\cos x + 1) = 0$$

$\cos x = \frac{3}{2}$ or $\cos x = -1$

no soln $x = 180° + k.360°; k \in Z$

QUESTION 4 [7]

In the diagram, a lamp post LY, is placed at a corner of a triangular field XYZ. The area of the field is $A\ m^2$ and $X\hat{Y}Z = 2\theta$. The angle of elevation to the top of the lamp post, measured from Z, is θ and $XY = a$ metres.



4.1 Determine YZ in terms of A, a and 2θ (3)

$$A = \frac{1}{2}a\,YZ\sin 2\theta$$

$$\therefore YZ = \frac{2A}{a\sin 2\theta}$$

4.2 Hence, show that the height of the lamp post is given by $LY = \frac{A}{a\cos^2\theta}$ (4)

$$\frac{LY}{YZ} = \tan\theta$$

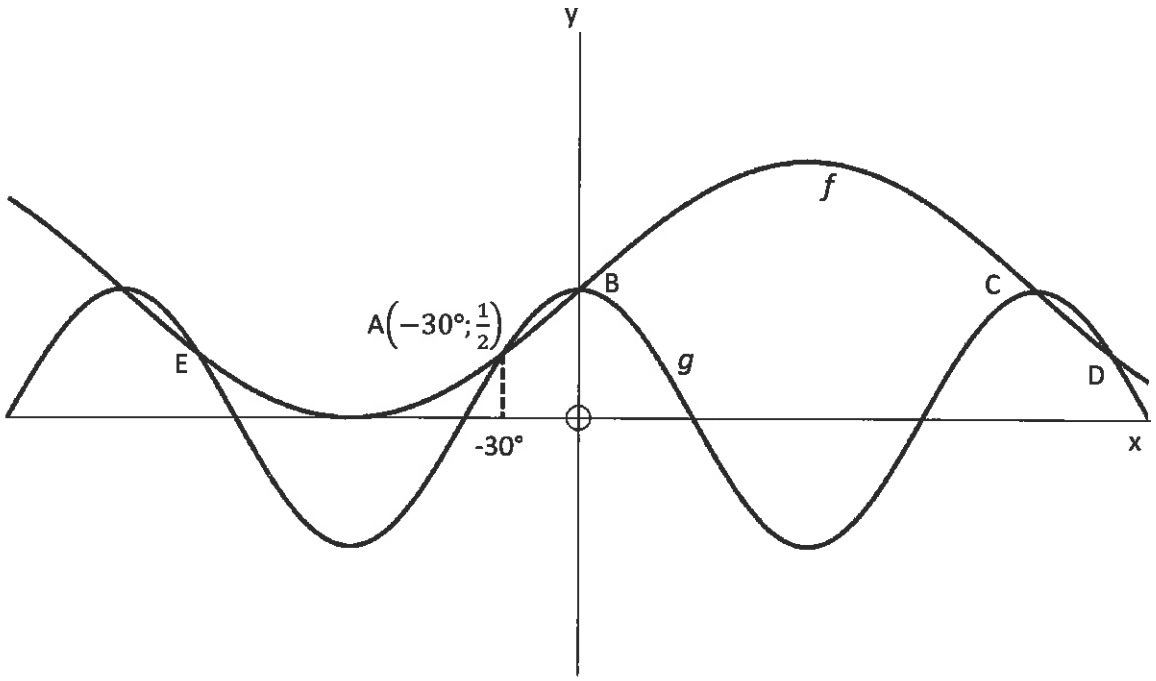
$$LY = \frac{2A}{a\sin 2\theta}\cdot\tan\theta$$

$$= \frac{2A}{2a\sin\theta\cdot\cos\theta}\cdot\frac{\sin\theta}{\cos\theta}$$

$$= \frac{A}{a\cos^2\theta}$$

QUESTION 5 **[14]**

5.1 In the diagram, the graphs of $f(x) = 1 + \sin x$ and $g(x) = \cos 2x$ are shown below for $x \in [-225°; 225°]$.



5.1.1 Write down the amplitude of f. (1)

1

5.1.2 Write down the period of g. (1)

$180°$

5.1.3 Determine the value of $f(90°) - g(90°)$. (2)

$2 - (-1)$

$= 3$

5.1.4 If $x = -30°$ at A, write down the x-coordinates of B, C, D and E. (4)

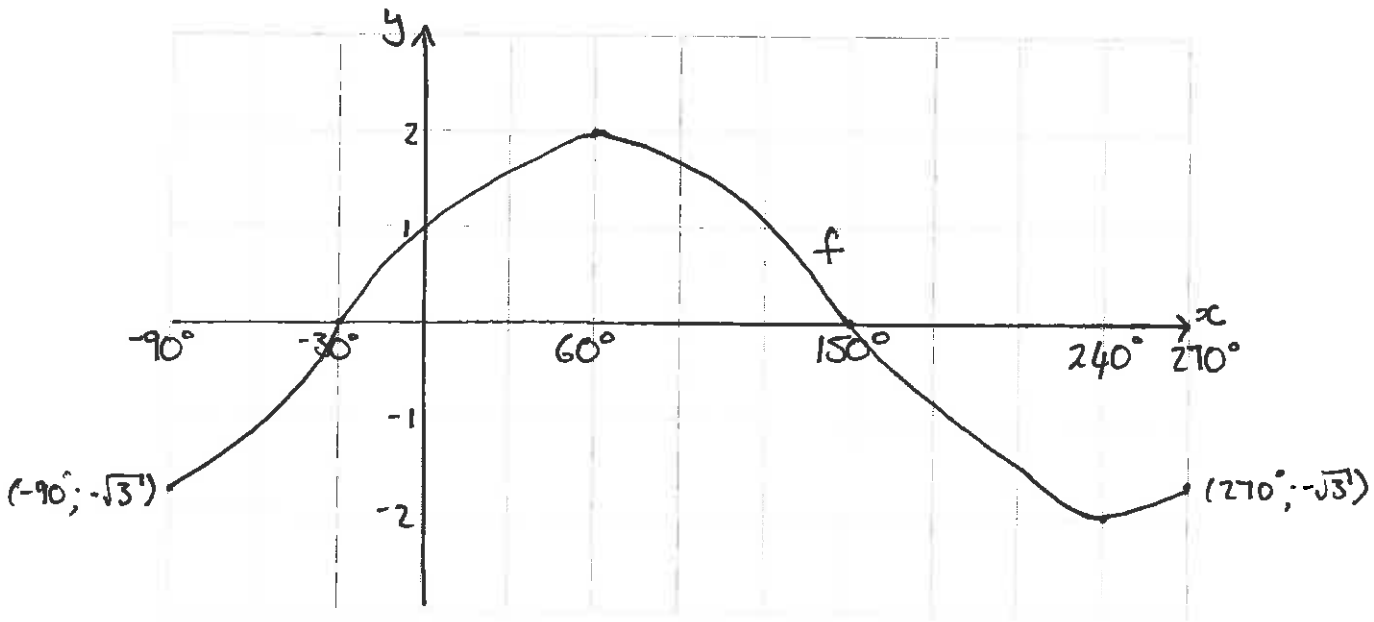
B $(0; 1)$ C $(180°; 1)$ D $(210°; \frac{1}{2})$

E $(-120°; \frac{1}{2})$

5.1.5 State the equation of $h(x)$, if h is obtained by shifting the graph of g 30° to the right. (1)

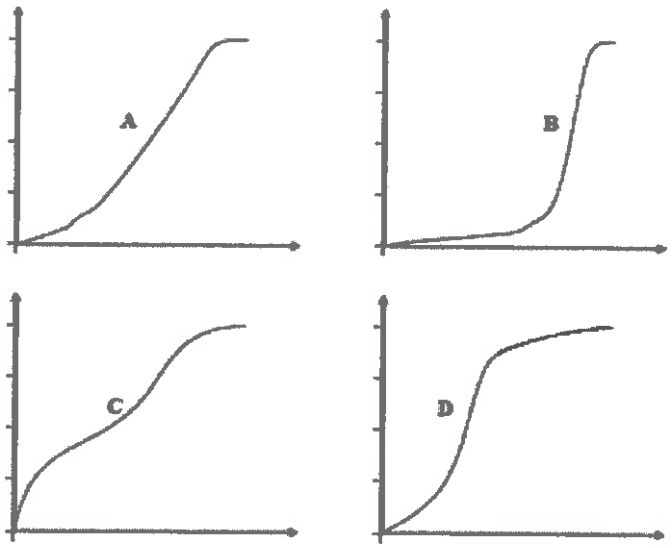
$h(x) = \cos(2x - 60°)$

5.2 Draw a neat graph of the function $f(x) = 2\cos(x - 60°)$ for $x \in [-90°; 270°]$ on the grid below. (5)

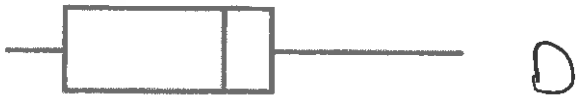


QUESTION 6 **[10]**

6.1 Four cumulative frequency diagrams are given below:



Which one of the diagrams best fits the box-and-whisker diagram given below? Write down only the letter. (2)



6.2 The following information summarises the year marks for a class of 20 students:

$$\sum_{i=1}^{20}(x_i - \bar{x})^2 = 1560 \qquad \sum_{i=1}^{20} x_i = 1220$$

6.2.1 Determine the mean mark for the class. (2)

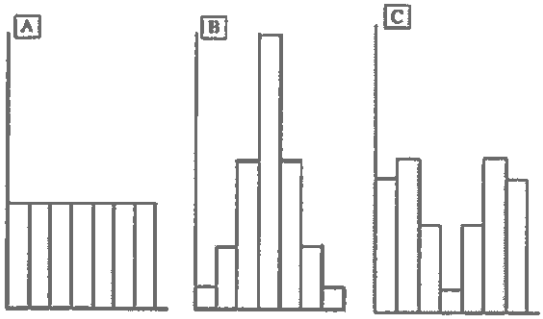
$\frac{1220}{20} = 61$

6.2.2 Determine the standard deviation for the class. (3)

$s^2 = \frac{1560}{20}$ $\therefore s = \sqrt{78}$

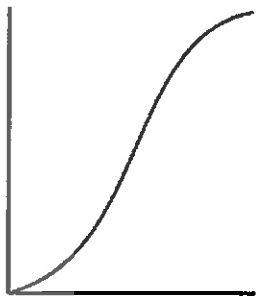
$= 8,8$

6.3 Three populations A, B and C are the same size and have the same range. Frequency histograms for the three populations are given below:

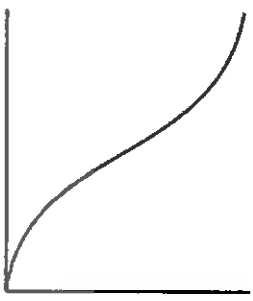


6.3.1 Which population (A, B or C or none of them) has a standard deviation of zero? A (1)

6.3.2 Both of the cumulative frequency curves below correspond to one of the three populations. Write the letter of the population that corresponds with the given cumulative frequency curve. (2)



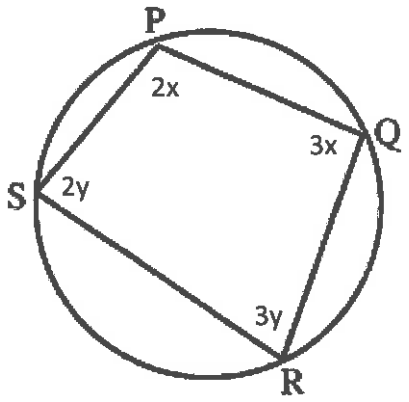
Graph 1 B



Graph 2 C

QUESTION 7 **[4]**

Calculate, with reasons, x and y.



$2x + 3y = 180°$ } opp $\angle$s cyclic quad

$2y + 3x = 180°$ }

$6x + 9y = 180.3$

$6x + 4y = 180.2$

$5y = 180°$

$y = 36°$

$x = 36°$

SECTION B

QUESTION 8 **[7]**

State whether the following statements are true or false. If the statement is false, correct it so that it is true.

8.1 If the mean is greater than the median then the data is skewed right.

True.

8.2 The 50th percentile is another term to describe the mean.

False. Another term for median.

8.3 The median is influenced by outliers.

False. The mean is influenced by outliers.

8.4 The modal value need not necessarily lie in the interval with the highest frequency.

True.

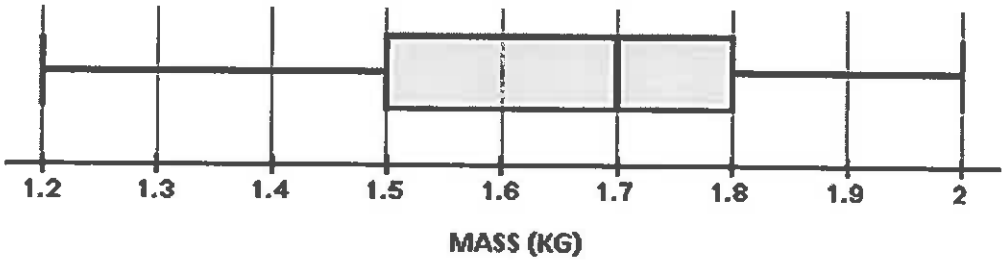
8.5 The mean may be a misleading measure of central tendency if the data is skewed.

True.

QUESTION 9

[4]

The riverine rabbit is one of the most endangered animals in Southern Africa. It is estimated that there are less than 250 riverine rabbits left in the wild. The mass of 60 riverine rabbits is summarized by the box and whisker plot below:



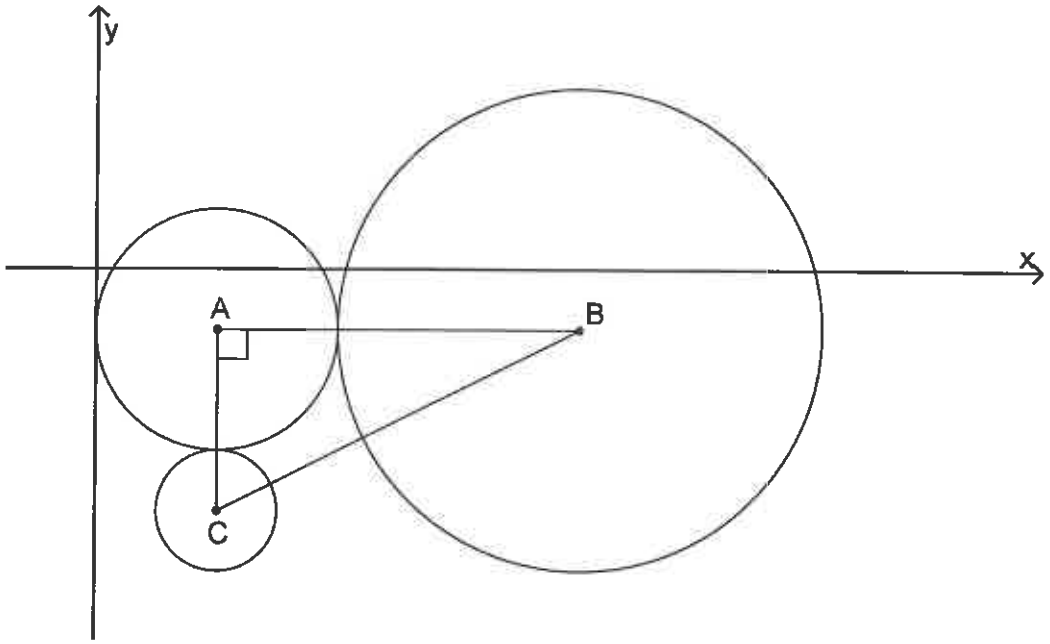
Complete the frequency table below by giving the values for A, B, C and D.

Mass interval	Frequency
$1{,}2 \leq x < 1{,}3$	***A***
$1{,}3 \leq x < 1{,}4$	6
$1{,}4 \leq x < 1{,}5$	7
$1{,}5 \leq x < 1{,}6$	7
$1{,}6 \leq x < 1{,}7$	***B***
$1{,}7 \leq x < 1{,}8$	***C***
$1{,}8 \leq x < 1{,}9$	***D***
$1{,}9 \leq x < 2{,}0$	2

A = 7
B = 8
C = 15
D = 13

QUESTION 10 **[6]**

In the diagram, circle centre C touches circle centre A with C on the same vertical line as A. Circle centre B touches circle centre A (with B on the same horizontal as A). AC = 3 units and AB = 6 units. The equation of the circle centre A is given by $(x-2)^2+(y+1)^2=4$. (Diagram not drawn to scale)



10.1 Determine the equation of the circle centre C, if AC is parallel to the y-axis. (3)

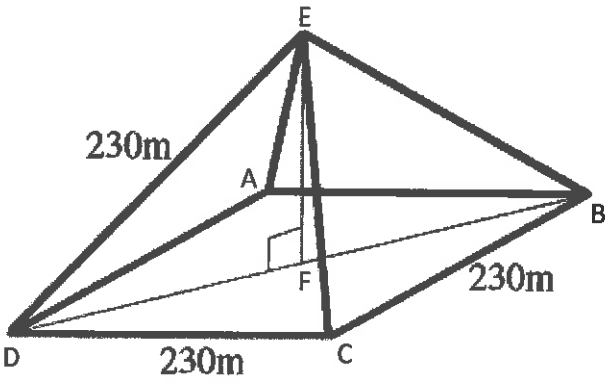
$(x-2)^2+(y+4)^2=1$

10.2 Write down the equation of circle B. (3)

$(x-8)^2+(y+1)^2=16$

QUESTION 11 **[6]**

Below is the pyramid of King Khafra at Giza. It consists of a square base and four equilateral triangles.



11.1 Show that the height of the pyramid is 162,6m. (4)

$BD = \sqrt{2.230^2}$

$h = \sqrt{230^2 - \left(\frac{\sqrt{2.230^2}}{2}\right)^2}$

$= 162,6m$

11.2 Determine the volume of the pyramid, if

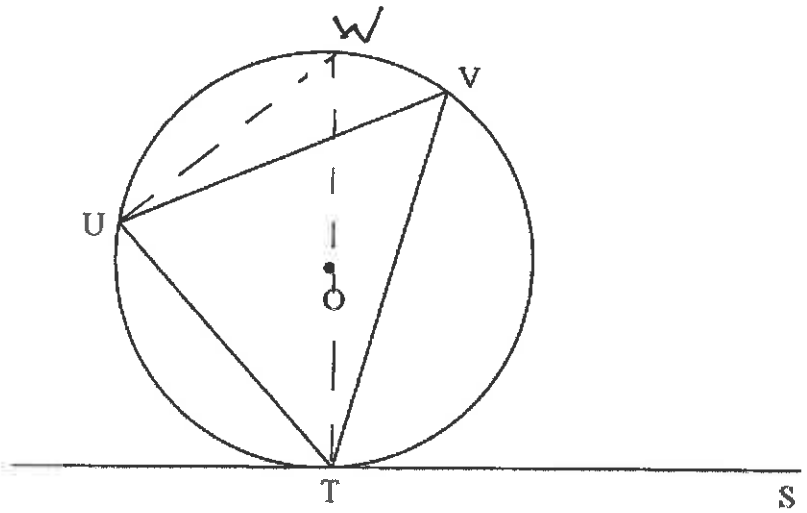
$Volume = \frac{1}{3}(Area\ of\ base) \times height$ (2)

$= \frac{1}{3} \times 230^2 \times 162,6$

$= 2867180\ m^3$

QUESTION 12 **[12]**

12.1 In the figure, **ST** is a tangent to the circle with centre O. Points U and V lie on the circumference of the circle. Prove the theorem that states $V\hat{T}S = U$.



Hint: Draw diameter TOW and join W to U. (6)

Let $W\hat{T}V = x$ and $V\hat{T}S = y$

then $W\hat{T}S = 90°$ (rad $\perp$ tan)

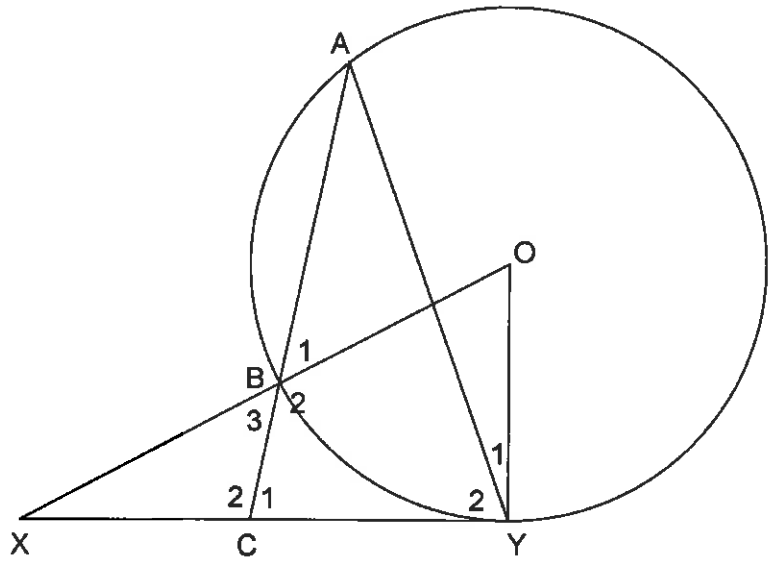
$\therefore x + y = 90°$

but $W\hat{U}T = 90°$ ($\angle^s$ in semi-O)

and $W\hat{T}V = W\hat{U}V = x$ ($\angle^s$ in same segment)

$\therefore V\hat{U}T = V\hat{T}S = y$ (complementary $\angle^s$)

12.2 XY is a tangent to the circle centre O. $\hat{X} = 20°$.



Determine, with reasons, the size of $\hat{A}$. (6)

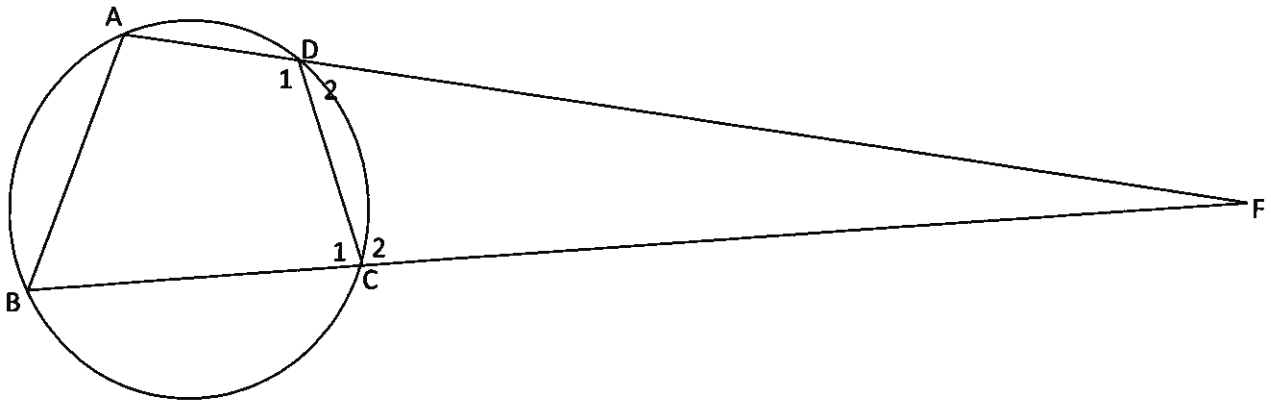
$\hat{Y}_1 + \hat{Y}_2 = 90°$ (rad ⊥ tan)

$\hat{O} = 70°$ ($\angle^s$ of $\triangle$)

$\hat{A} = 35°$ ($\angle$ at centre = 2$\angle$ at circumf)

QUESTION 13 **[9]**

In the diagram below, ABCD is a cyclic quadrilateral. AD and BC are extended and intersect at F.



Prove that:

13.1 $\frac{DC}{CF} = \frac{BA}{AF}$. (4)

In $\triangle DCF$ and $\triangle BAF$:

$\hat{D} = \hat{B}$ (ext $\angle$ cyclic quad = int opp $\angle$)

$\hat{C} = \hat{A}$ (ext $\angle$ cyclic quad = int opp $\angle$)

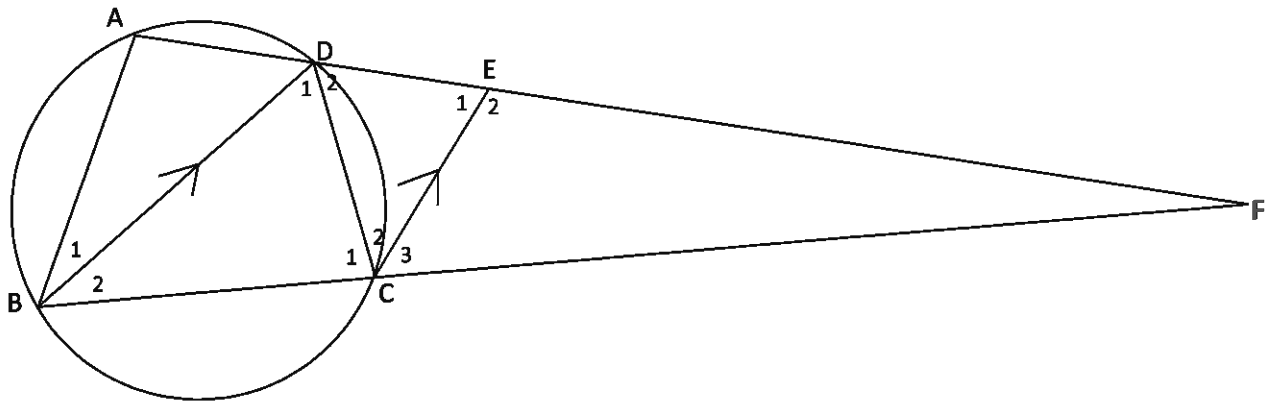
$\hat{F} = \hat{F}$ (common)

$\therefore \triangle DCF \;|||\; \triangle BAF$ (AAA)

$\therefore \frac{DC}{BA} = \frac{CF}{AF}$ (similar $\triangle$s)

$\therefore \frac{DC}{CF} = \frac{BA}{AF}$

13.2 It is further given that EC is a tangent to the circle and $BD//CE$.



Show that $\frac{DE}{EF} = \frac{DC}{CF}$. (5)

$\frac{DE}{EF} = \frac{BC}{CF}$ (EC||DB in $\triangle DFB$)

also $\hat{C}_2 = \hat{D}_1$ (alt $\angle$s; EC||DB)

and $\hat{C}_2 = \hat{B}_2$ (tan-chord)

$\therefore \hat{D}_1 = \hat{B}_2$

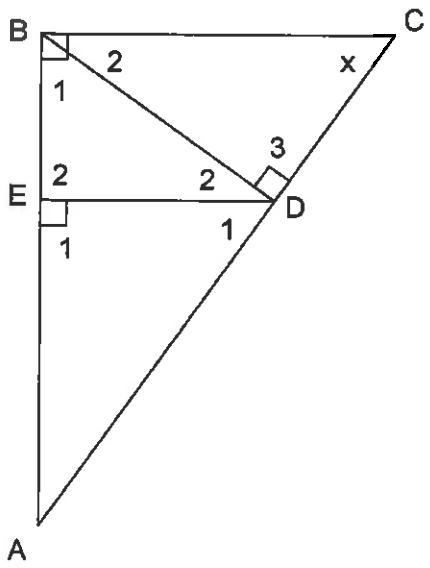
$\therefore BC = CD$ (isos $\triangle$)

$\therefore \frac{DE}{EF} = \frac{BC}{CF}$

QUESTION 14 **[11]**

ΔABC is a right-angled triangle with $\hat{B} = 90°$. D is a point on AC such that BD $\perp$ AC and E is a point on AB such that DE $\perp$ AB. E and D are joined.

AD : DC = 3 : 2 and AD = 15 cm.



14.1 Prove that $\Delta BDA \,|||\, \Delta CDB$. (3)

In ΔBDA and ΔCDB:

$\hat{A} = \hat{B}_2 = 90° - x$ ($\angle^s$ of Δ)

$B\hat{D}A = \hat{D}_3 = 90°$ ($\angle^s$ on str line)

$\hat{B}_1 = \hat{C} = x$ (rem $\angle$ of Δ)

$\therefore \Delta BDA \,|||\, \Delta CDB$ (equiangular Δs)

14.2 Calculate BD (Leave your answer in surd form). (3)

$$\frac{BD}{CD} = \frac{AD}{BD}$$

$$\therefore BD^2 = AD \cdot CD$$

$$= 15 \cdot 10$$

$$BD = \sqrt{150}$$

$$= 5\sqrt{6}$$

14.3 Calculate AE (Leave your answer in surd form). (5)

$\Delta ABD \text{ III } \Delta ADE$ (perp from right vertex)

$$\frac{AE}{AD} = \frac{AD}{AB}$$ (similar Δs)

$$\therefore AE = \frac{AD^2}{AB}$$

also $AB^2 = (5\sqrt{6})^2 + 15^2$ (pythag)

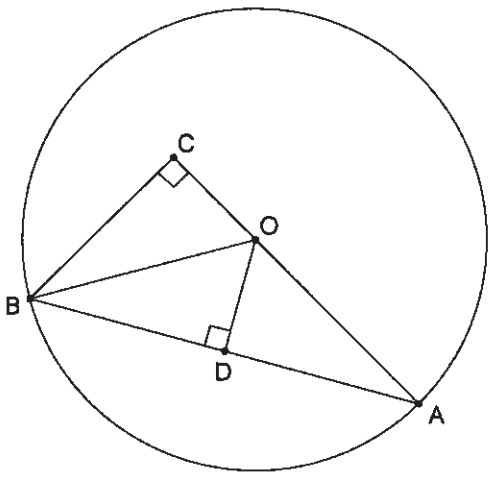
$$\therefore AB = 5\sqrt{15}$$

$$\Rightarrow AE = \frac{15^2}{5\sqrt{15}}$$

$$= 3\sqrt{15}$$

QUESTION 15 **[10]**

In the figure below, AB is a chord of a circle with centre O. AO is produced to C such that AC ⊥ BC. OD ⊥ AB.



15.1 Prove that ΔABC /// ΔAOD (3)

In ΔABC and ΔAOD:

$\hat{C} = \hat{D} = 90°$ (given)

$\hat{A} = \hat{A}$ (common)

$\hat{B} = \hat{O}$ (rem $\angle$ of Δ)

$\therefore \Delta ABC \,///\, \Delta AOD$ (LLL)

15.2 Hence prove that $2BD^2 = OA^2 + OA.OC$ (7)

$\frac{AD}{AC} = \frac{OA}{AB}$ (similar Δs)

$\therefore \frac{AD}{AC} = \frac{OA}{2AD}$ (AB = 2AD; line from centre $\perp$ chord)

$\therefore 2AD^2 = OA.AC$

but $AC = OA + OC$

$\therefore 2AD^2 = OA(OA + OC)$

$= OA^2 + OA.OC$