Section A   **76 marks**

Question One   **Multiple Choice**   **8 marks**

Choose the most correct response, and circle the corresponding letter.

Handwritten: $f(2) = 0$ $\cancel{8} + 8 + 2a + b = 0$

1.1.   If $x^2 - 4$ is a factor of $f(x) = x^3 + 2x^2 + ax + b$ then   *Handwritten:* $f(-2) = 0$ $-\cancel{8} + 8 - 2a + b = 0$

Handwritten: $2b = -16$ $b = -8$ $2a = 8$

A. $a = -2; b = 8$ B. $a = 2; b = \dfrac{2}{3}$

C. $a = 4; b = -4$ (D.) $a = -4; b = -8$

1.2.   Which of the following relations define a function?

Handwritten over A: many –1 ✓   *Handwritten over B:* 1–many ✗

(A.)   {(2 ; 1);(3 ; 1);(4 ; 1);(5 ; 1)}   B.   {(2 ; 1);(1 ; 3);(4 ; 1);(1 ; 5)}

Handwritten over C: 1– many   *Handwritten over D:* 1–many ✗

C.   {(2 ; 3);(3 ; 4);(4 ; 3);(3 ; -2)}   D.   {(2 ; 1);(2 ; 2);(2 ; 3);(2 ; 4)}

1.3.   For which value of p will { (2 ; p) ; (2 ; 3p-1)} be a function?

Handwritten: $p = 3p - 1$
$1 = 2p$
$p = \frac{1}{2}$

A.   no value of p   B.   3

C.   4   (D.)   0,5

1.4.   The graph $y = (x - a)^2(x + b); a > 0; b > 0$ will look like   *Handwritten sketch with "a"*
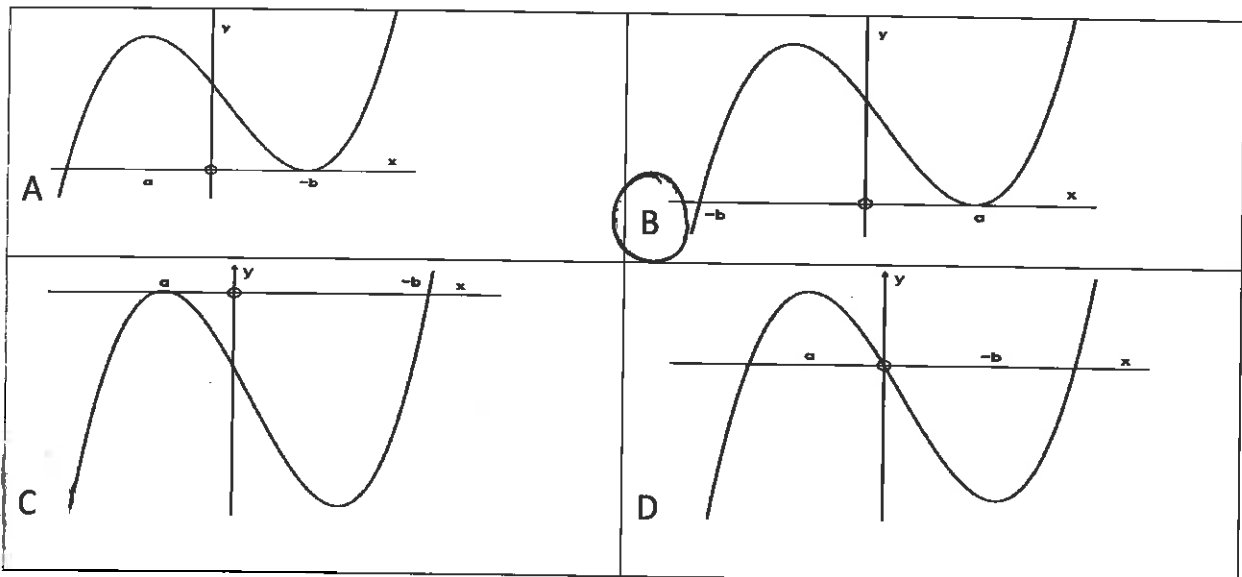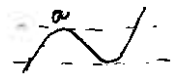
A, B, C, D — four graphs (B is circled).

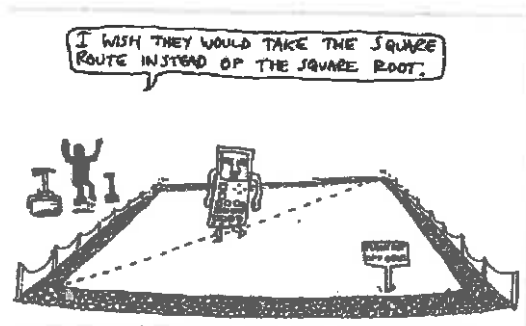Question Two   Match the Columns   10 marks

Below the table, match column A with the respective letter in column B.

If $f(x) = \frac{4x+16}{2}$, then $f(x) = 2x + 8$

Column A	Column B
2.1. $f^{-1}(x)$ $x = 2y + 8$ $\therefore = \frac{x-8}{2}$	A. $y = \frac{1}{2x+8}$
2.2. $f\left(\frac{1}{x}\right) = \frac{2}{x} + 8$	B. $y = \frac{4x^0}{2}$
2.3. $\frac{1}{f(x)}$ $\frac{1}{2x+8}$	C. $y = \frac{2}{x} + 8$
2.4. $f(-x)$ $-2x + 8$	D. $y = -2x + 8$
2.5. $f'(x) = 2$	E. $y = \frac{1}{2}x - 4$

2.1. E   2.2. C   2.3. A   2.4. D   2.5. B

Question Three   19 marks

Solve for x, stating any necessary restrictions.

3.1. $(2x + 3)(3 - x) = 4$ (3)

$$-2x^2 + 3x + 9 - 4 = 0$$
$$2x^2 - 3x - 5 = 0$$
$$(2x - 5)(x + 1) = 0$$
$$x = \frac{5}{2} \; ; \; -1.$$

3.2. $x + \sqrt{2x - 5} = 2$ (4)

$$\sqrt{2x - 5} = 2 - x$$
$$2x - 5 = 4 - 4x + x^2$$
$$x^2 - 6x + 9 = 0$$
$$(x - 3)^2 = 0$$
$$x = 3. \quad \text{check: no soln.}$$

Remember to isolate $\sqrt{}$ before $(\;)^2 = (\;)^2$ & then to check solution.

3.3. $7^x = 14$ (2)

$$x \log 7 = \log 14$$
$$\therefore x = 1,4.$$

P 1D.
rounding off correctly

3.4. a. (Prove) that $\left(x + \frac{1}{x}\right)^2 = \left(x - \frac{1}{x}\right)^2 + 4$ (2)

L: $x^2 + 2 + \frac{1}{x^2}$ R: $x^2 - 2 + \frac{1}{x^2} + 4$

Prove means keep
LHS / RHS
separate.

$$x^2 + 2 + \frac{1}{x^2}$$
$$= L.$$

b.   Hence solve for x: $2\left(x+\frac{1}{x}\right)^2 - 5\left(x-\frac{1}{x}\right) - 5 = 0;$

leaving answer in surd form if necessary.   (8)

$\therefore 2\left[\left(x-\frac{1}{x}\right)^2 + 4\right] - 5\left(x-\frac{1}{x}\right) - 5 = 0$

let $k = x - \frac{1}{x}$

$\therefore 2k^2 + 8 - 5k - 5 = 0$

$2k^2 - 5k + 3 = 0$

$(2k - 3)(k - 1) = 0$

$\therefore x - \frac{1}{x} = \frac{3}{2}$ $x - \frac{1}{x} = 1$

$2x^2 - 3x - 2 = 0$ $x^2 - x - 1 = 0$

$(2x + 1)(x - 2) = 0$ $x = \frac{1 \pm \sqrt{1 - 4(-1)}}{2}$

$x = -\frac{1}{2}$ or 2 $= \frac{1 \pm \sqrt{5}}{2}$

Hence means use
what you have
just done.

Always look for k method if
a question gets complicated.

P 1 N.
leave as a
surd.

Question Four

4.1.   Complete the following table

type of sequence	T_1	T_2	T_3	T_4	T_5	T_n
arithmetic	-16	-4	8	20	32	$-16 + (n-1)12$ $= 12n - 28$
geometric	2	-4	8	-16	32	$2 \cdot (-2)^{n-1}.$

For arithmetic T_2 to T_3: $+12$

For geometric T_2 to T_3: $\times -2$

(6)

T_n means give the general formula!

How do you keep warm in a cold room?

You go to the corner, because it's always 90 degrees.

4.2. Recently Mrs Hultzer's son had a miracle return to health after being diagnosed with malaria. After doing some research on the topic, Mrs Raeburn found the following information:

> Since 2003, the deaths per 100 000 people at risk due to malaria in Africa have roughly followed the following pattern:

T_1	T_2	T_3	T_4	T_5
2003	2004	2005	2006	2007
126,2	123,2	119	113,6	107

$-3 \quad -4,2 \quad -5,4 \quad -6,6 \quad \therefore$ not linear.

$-1,2 \quad -1,2 \quad -1,2 \quad \therefore$ quadratic

a. State whether the pattern is arithmetic, geometric or quadratic. (3)

Quadratic — second difference $-1,2$ (as shown above)

b. Hence determine the nth term. (3)

$2a = -1,2 \qquad 3a+b = -3 \qquad a+b+c = 126,2$

$a = -0,6 \qquad b = -1,2 \qquad c = 128$

$T_n = -0,6n^2 - 1,2n + 128$

or $\quad -\frac{3}{5}n^2 - \frac{6n}{5} + 128$

c. What percentage reduction has taken place between 2003 and 2014? (3)

$\%$ reduction $= \dfrac{V_{2003} - V_{2014}}{V_{2003}} \qquad V = $ Value.

NB $V_{2014} = T_{12} = -\frac{3}{5}(12^2) - \frac{6}{5}(12) + 128 = 27,2$
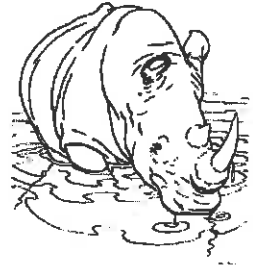
$\therefore \% = \dfrac{126,2 - 27,2}{126,2} = 78,4\%$

4.3. The Eco committee keep us informed about the rhino population in South Africa. It has been mentioned that the population is depleting at a rate of 12% p.a. on a reducing balance. If there are now only 3200 rhino left, how many will there be in five years' time? (2)

$$A = 3200 \, (1 - 12\%)^5$$
$$= 1688$$

P 1 D.
rounding

Question Five   12 marks

Determine the derivatives of the following functions, using the correct notation. Leave your answers with positive exponents.

5.1 $D_x \left[4x^{\frac{3}{2}} + \sqrt[3]{x} + (b+1)^3 \right]$ (3)

$$= Dx \left[4 \, x^{3/2} + x^{1/3} + (b+1)^3 \right]$$

$$= 4 \cdot \frac{3}{2} x^{1/2} + \frac{1}{3} x^{-2/3}$$

$$= 6x^{1/2} + \frac{1}{3x^{2/3}}$$

P 1 N.
→ differentiation or function
→ +ve exponents.

5.2 $f(x) = \left(2x + \frac{1}{x} \right)^2$ (3)

$$f(x) = 4x^2 + 4 + x^{-2}$$

$$f'(x) = 8x - 2x^{-3}$$

$$= 8x - \frac{2}{x^3}$$

5.3. $\frac{dy}{dx}$ if $x^3 y = \frac{5}{3}$ (2)

$$y = \frac{5}{3} x^{-3}$$

$$\frac{dy}{dx} = -5x^{-4} = \frac{-5}{x^4}$$

This tested all the rules.
If you got these wrong — revise your differentiation.

5.4. Determine the equation of the tangent to $f(x) = x - x^2$ which passes through the point $(a ; 6)$; $a > 0$. (4)

$f'(x) = 1 - 2x$

$\therefore (2 ; 6)$

$m_T = 1 - 2(2)$

$= -3$

$y - 6 = -3(x - 2)$

$y = -3x + 12$

$a + a^2 = 6$

$a^2 + a - 6 = 0$

$(a+3)(a-2) = 0$

$a = -3$ or 2

$a > 0$

Question Six   3 marks

Seven cheerleaders each have **one** of the letters of the word BRESCIA on their backs. Typically, when the cheerleaders get excited, they might not stay in the correct order.

a.   How many arrangements could the girls possibly stand in, if order does not matter? (1)

$7!$

b.   Cheerleaders B and R are good friends, and they are keeping together all the time. How many ways can the cheerleaders arrange themselves if B and R stay together but the others do not? (2)

$\boxed{2!}$ _ _ _ _ _

$6! \, 2!$

Question Seven
7 marks

A converging Geometric Series is shown: $1 + 3p + 9p^2 + 27p^3 + \cdots$

7.1. For which values of p will the series have a sum to infinity? (4)

$$r = 3p \qquad -1 < 3p < 1 \quad \text{for convergence}$$
$$-\tfrac{1}{3} < p < \tfrac{1}{3}.$$

7.2. If $p = \frac{1}{4}$, find the sum to infinity. (3)

$$\therefore a = 1 \qquad r = \frac{3}{4} \qquad S_\infty = \frac{1}{1-3/4}$$
$$= 4.$$

Section B
74 marks

Question Eight
9 marks

Mrs G is trying to explain cell growth of babies being formed in the womb. Refer to the information in the table and answer the questions that follow:

Day	5	6	7	8
Number of cells	48	96	192	384

8.1. How many cells will there be on day 9? (1)

768

8.2. How many cells were there on day 4? (1)

24.

8.3. If the number of cells on day 1 represents the number of babies in the womb, then how many babies are in the womb? (1)

1   2   3   4

3   6   12   24
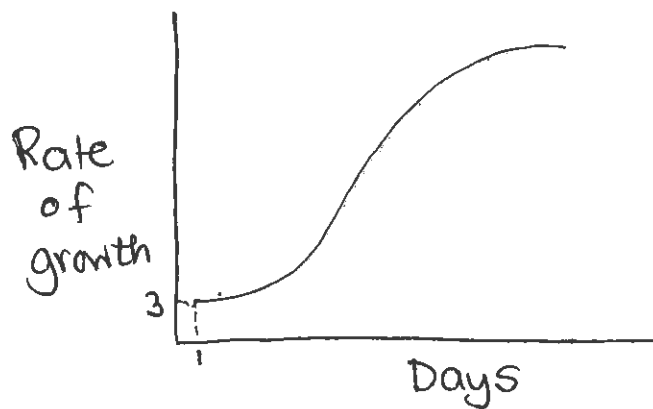
3

8.4.	How many cells will there be in total on day n?	(2)

$$S_n = \frac{3(2^n - 1)}{2 - 1} = 3 \cdot (2^n - 1)$$

8.5.	Why do you think the rate of growth of cells slows down in the last two weeks of pregnancy?	(1)

Baby fully formed.

8.6.	Draw a rough sketch of what you think the graph would look like that shows the rate of growth from beginning to end of pregnancy. Do NOT label points, the shape of the graph is important.	(3)

problem solving question
seeing if you understand
real life situations.

Fetal Growth From 8 to 40 Weeks

| Embryo at 8 Weeks | Fetus at 12 Weeks | 16 | 20 | 24 | 28 | 32 | 36 | 40 |
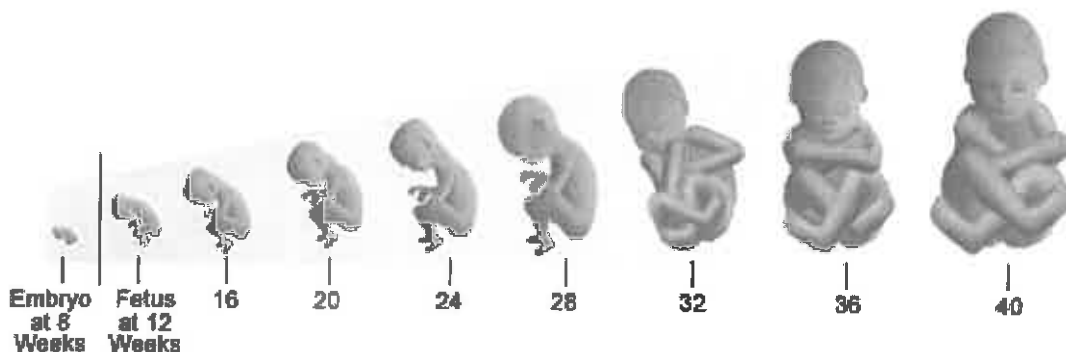
Question Nine   13 marks

9.1. Nicole finds two investments, in the first investment(at Bank for the People) she gets a nominal interest rate of 15% p.a. compounded monthly while in the second (called Bank for Woman) she receives a 17% p.a. compounded semi-annually.

a. Which one of the investments gives a better effective interest rate?

(Show all your calculations)   (5)

People

$$1 + i_{eff} = \left(1 + \frac{15\%}{12}\right)^{12}$$

$$i_{eff} = 16,1\%$$

Bank for Woman

Woman

$$1 + i_{eff} = \left(1 + \frac{17\%}{2}\right)^{2}$$

$$i_{eff} = 17,7\%$$

This formula will always be tested.

b. If Nicole invests R 50 800 in at the Bank for the People what will her investment be worth at the end of three years?   (2)

$$A = 50\,800\left(1 + \frac{15\%}{12}\right)^{36}$$

$$= R\,79\,448,35.$$

9.2. Mrs Raeburn wants to buy a nice little holiday flat on the coast, and takes out a loan of R 740 000. The interest on the loan is 8,7% p.a. compounded monthly. Calculate how long, in years correct to 1 decimal place, it will take her to repay the loan if she starts her first payment one month after taking out the loan, and pays R 9 000 per month.

(6)

740 000

$$(A) = 1 + \frac{8,7\%}{12}$$
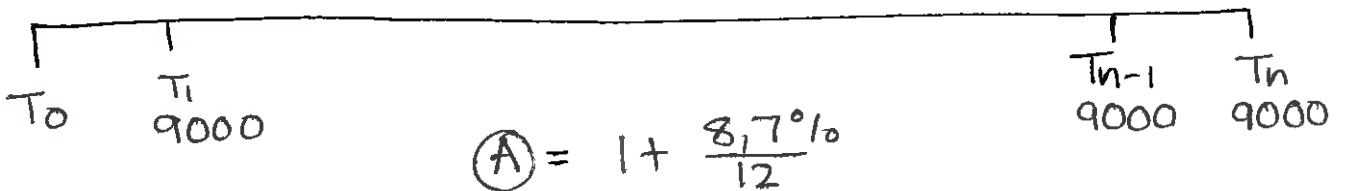
$$\therefore 740\,000\,(A)^{12t} = \frac{9000\left((A)^{12t} - 1\right)}{(A) - 1}$$

$$740(A)^{12t} \cdot (A) - 740(A)^{12t} = 9(A)^{12t} - 9$$

$$(A)^{12t}\left(740(A) - 749\right) = -9$$

$$(A)^{12t} = \frac{-9}{740(A) - 749}$$

$$12t = \frac{\log\left(\frac{-9}{740(A) - 749}\right)}{\log(A)}$$

$$t = 10,458\ldots$$

$$= 10 \text{ years} \quad 6 \text{ months}$$

$$\text{or} \quad 10,5 \text{ yrs.} \quad \text{or} \quad 125,5 \text{ months}$$
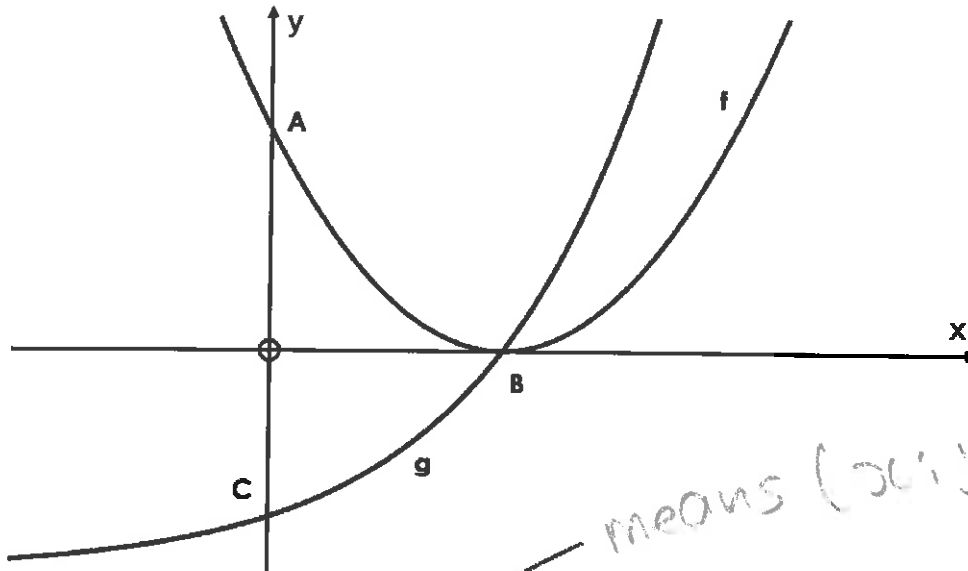
Question 10   18 marks

The diagram represents the graphs of $f(x) = (x-2)^2$ and $g(x) = 2^x - 4$

Points A and C are the y intercepts of f and g respectively, and B is their common x intercept.

means (x, y)

10.1. Calculate the coordinates of A, B and C.   (3)

A: $(0; 4)$ B $(2; 0)$ C $(0; -3)$

$f(0) = 4$ $(x-2)^2 = 0$ $g(0) = -3$

10.2. Write down the equation of the asymptote of g.   (1)

$y = -4$.

10.3. a.   Determine the equation of $h(x) = f(2x) + 4$ (2)

$h(x) = (2x - 2)^2 + 4$.
It did not say
simplify.

b.   Determine p if $h(p) = 6$ (4)

$$(2p - 2)^2 + 4 = 6$$
$$(2p-2)^2 = 2$$
$$2p - 2 = \pm\sqrt{2}$$
$$2p = 2 \pm \sqrt{2}$$
$$p = \frac{2 \pm \sqrt{2}}{2}$$

10.4. a.   Write down the equation of $h^{-1}(x) =$ (3)

$$x = (2y - 2)^2 + 4$$
$$x - 4 = (2y-2)^2$$
$$\pm\sqrt{x-4} = 2y - 2$$
$$\frac{2 \pm \sqrt{x-4}}{2} = h^{-1}(x)$$

inverse
swop $x \ne y$
+
rewrite as
$y = \ldots$

b.   Evaluate $h^{-1}\left(\frac{1}{16}\right)$ same as   (2)

$$\frac{2 \pm \sqrt{\frac{1}{16} - 4}}{2}$$

$= $ undefined.

OR

$h(x) = \frac{1}{16}$.
$(2x-2)^2 + 4 = \frac{1}{16}$.
$(2x-2)^2 = \frac{-63}{16}$
∴ no solution

c.   Calculate the value of p correct to 2 decimal places if $h^{-1}(p) = 6$
                                                                (3)

$$\frac{2 \pm \sqrt{x-4}}{2} = 6$$
$$\pm\sqrt{x-4} = 10$$
$$x - 4 = 100$$
$$x = 104.$$

same as
$h(6) = (12-2)^2 + 4$
$= 104.$

Question Eleven
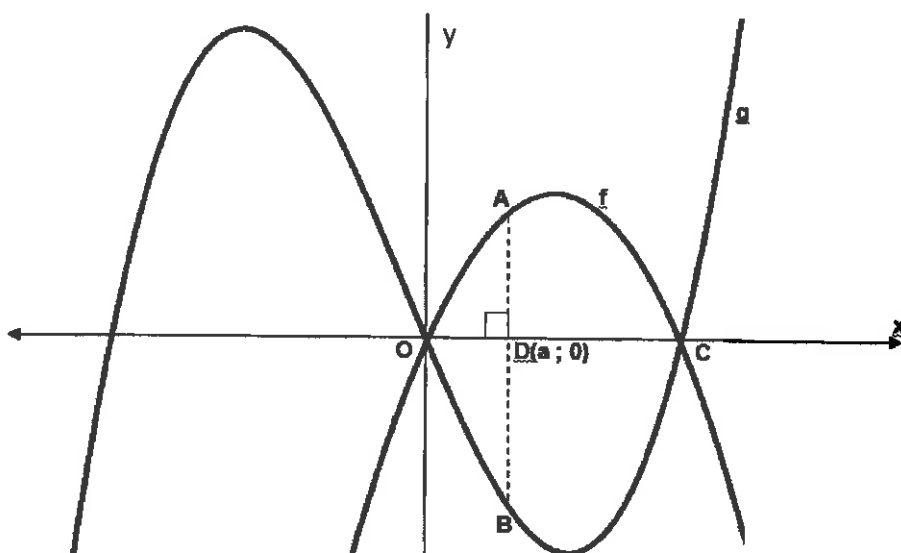
The sketch shows the graphs of

$$g(x) = x^3 + x^2 - 20x \qquad \text{and} \qquad f(x) = -2x^2 + 4x$$

ADB is perpendicular to the x axis, with A on f and B on g. D lies between O and C.

11.1. Write down the coordinates of A and B in terms of a.   (2)

$A(a; f(a))$ $B(a; g(a))$

$A(a; -2a^2 + 4a)$ $B(a; a^3 + a^2 - 20a)$

11.2. Hence, show that the length of AB is $AB = -a^3 - 3a^2 + 24a$ (1)

$AB = f(a) - g(a) \longrightarrow$ This step is important.

$= -2a^2 + 4a - (a^3 + a^2 - 20a)$

$= -a^3 - 3a^2 + 24a$.

11.3. Determine the value of a that ⟨MAXIMISES⟩ the length of AB.   (5)

max length $AB' = 0$.

$-3a^2 - 6a + 24 = 0$

$a^2 + 2a - 8 = 0$

$(a+4)(a-2) = 0$

$a = -4$ or 2 $a > 0$

$\therefore a = 2$.
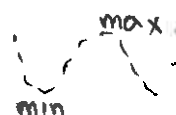
When you see Max/min in a calculus question means $\frac{d}{dx} = 0$

Given f(x) and g(x) are both parabolas, such that:

f(x) = – g(x);   f(0) = –3;   f(2) = f(6) = 0; $f'(a) = 0$;   f(a) = 5

12.1. Draw the sketch graph of g(x).   (4)
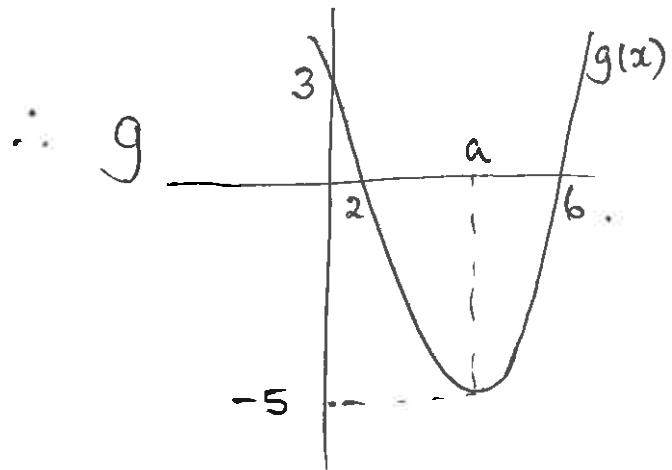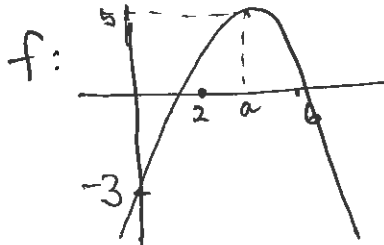
12.2. If h'(x) = g (x), state the value(s) of x for which h(x) is increasing.

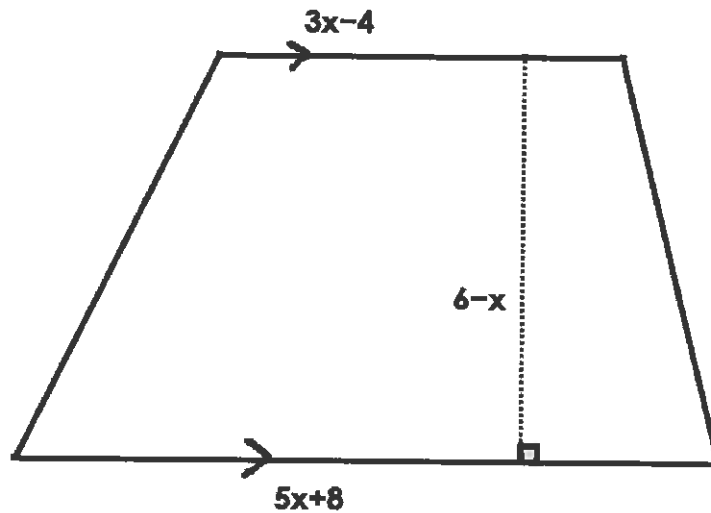h(x) increases   (2)

when g(x) > 0

x < 2   or   x > 6.

continue onto next page

The cross section of a trapezium based prism is drawn. The height of the prism is $2x + 5$

3x−4

6−x

5x+8

Not a difficult question at all

13.1. Show that the area of the trapezium base is $12 + 22x - 4x^2$ (2)

$$A = h\left(\frac{a+b}{2}\right) \qquad a \| b$$

$$= \frac{(6-x)(3x-4+5x+8)}{2}$$

$$= \frac{(6-x)(8x+4)}{2}$$

$$= (6-x)(4x+2) = -4x^2 + 22x + 12.$$

if you could not get this

13.2. Determine the volume of the prism in terms of x. (3)

$$V = (-4x^2 + 22x + 12)(2x + 5)$$

$$= -8x^3 - 20x^2 + 44x^2 + 110x + 24x + 60$$

$$= -8x^3 + 24x^2 + 134x + 60$$

you should get this

13.3. Hence determine the value of x, for which the volume is a maximum. (4)

max $dV/dx = 0$: $-24x^2 + 48x + 134 = 0$

$$x = \frac{6 \pm \sqrt{237}}{6}$$

$$\therefore \text{ max} \quad x = \frac{6 + \sqrt{237}}{6}$$

In the FET phase at Brescia House, there are 240 girls. The following information was collected by Mrs McLoughlin regarding their participation in sport in Term 2.

122 girls play hockey (H)

58 girls do dancersize (D)

96 girls play netball   (N)

16 girls do all three activities

22 girls play hockey and do dancersize
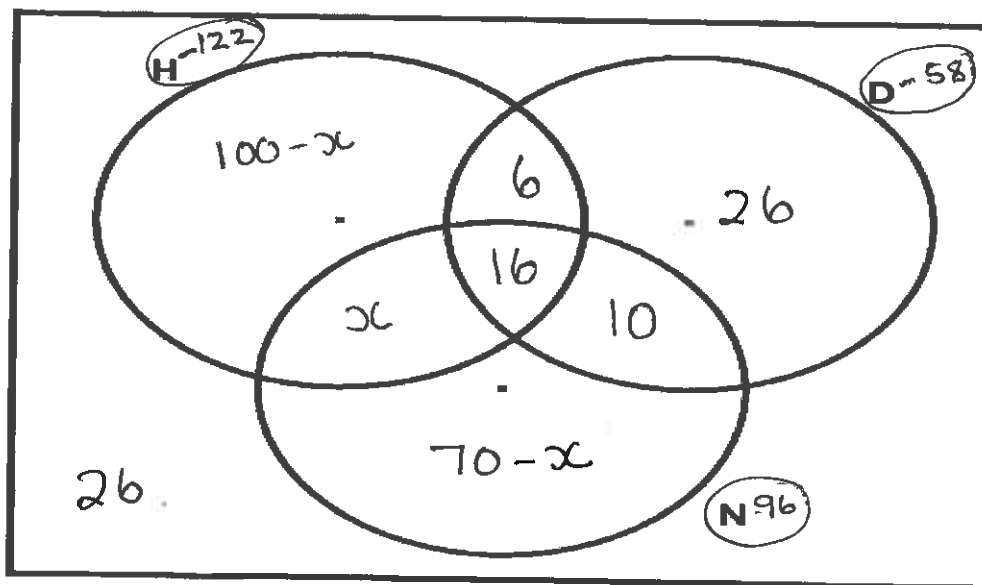
26 girls play netball and dancersize

26 girls do no sport at all

x girls play ONLY hockey and netball.

Not work it out but represent info as it is given

14.1. Complete the Venn Diagram below to represent the above information.

(4)

14.2. Determine the number of girls who play hockey AND netball. (3)

$$122 + 36 + 70 - x + 26 = 240$$

$$254 - x = 240$$

$$\therefore x = 14$$

Hockey and Netball   14 + 16
(not only)   = 30

14.3. Determine the probability (leaving your answer as a simplified fraction) that a girl in the FET phase who is selected at random:

i. only does dancersize (1)

$$\frac{26}{240} = \frac{13}{120}$$

ii. does not play netball (2)

$$1 - \frac{96}{240} = \frac{3}{5}$$

iii. participates in at least two of these activities. (1)

$$1 - \frac{86 + 26 + 56 + 26}{240} \qquad \text{or} \qquad \frac{46}{240}$$

$$= 1 - \frac{97}{120} \qquad\qquad = \frac{23}{120}$$

$$= \frac{23}{120}$$