BRIDGE HOUSE
PRE-PRIMARY • PREPARATORY • COLLEGE

GRADE 12

SEPTEMBER EXAMINATIONS 2014

MATHEMATICS PAPER 2

Time: 3 hours **Total: 150**

NAME: Memo.

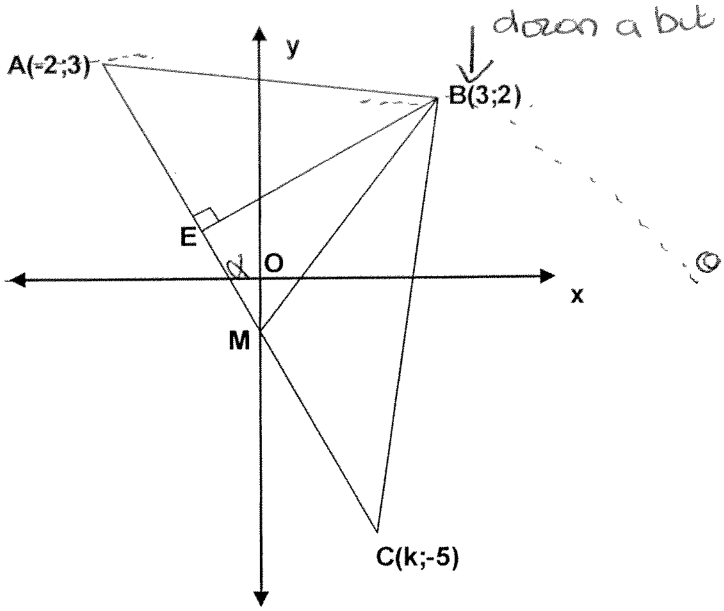
Read the following instructions carefully:

1. **ALL QUESTIONS TO BE ANSWERED ON THIS PAPER.** There are additional pages at the end, if needed.
2. This question paper consists of 17 pages and 2 separate Formula sheets. Please check that your question paper is complete.
3. Read the questions carefully.
4. Number your answers exactly as the questions are numbered.
5. All the necessary working details must be clearly shown.
6. Approved non-programmable calculators may be used unless otherwise stated.
7. Answers should be rounded off to **two decimal** digits where necessary, unless otherwise stated.
8. It is in your own interest to write legibly and to present your work neatly.
9. ~~Detach the Answer Sheet and staple it to your answer script.~~ Make sure your name is on this page.

SECTION A:

Question 1:

In the diagram alongside, $A(-2;3), B(3;2)$ and $C(k;-5)$ are three points in a Cartesian plane. M, the midpoint of AC, lies on the y-axis. $BE \perp AC$, with E a point on AC.



a. Determine M, the midpoint of AC. (2)

$m\left(0; \frac{3-5}{2}\right)$

$m(0;-1)$

b. Show that $k = 2$. (2)

$\frac{-2+k}{2} = 0$

$-2+k = 0 \quad \therefore k = 2.$

c. Determine the gradient of AC. (2)

$m_{AC} = \frac{-5-3}{2+2} = \frac{-8}{4} = -2.$

d. Determine the equation of altitude BE. (3)

$m_{BE} = \frac{1}{2}$ (AC⊥BE)

$y = \frac{1}{2}x + c$

$2 = \frac{1}{2}(3) + c$

$\frac{1}{2} = c \quad \therefore y = \frac{1}{2}x + \frac{1}{2}$

e. If $E(-\frac{3}{5};\frac{1}{5})$, calculate the area of ΔABM. $A(-2;3)$; $m(0;-1)$; $B(3;2)$ (5)

$AM = \sqrt{(-2-0)^2+(3+1)^2}$ ✓

$= \sqrt{4+16}$

$= \sqrt{20}$ ✓ 4,47

$BE = \sqrt{(3+\frac{3}{5})^2+(2-\frac{1}{5})^2}$

$= \frac{9\sqrt{5}}{5}$ ✓ 4,02

$\therefore$ Area $\Delta ABm = \frac{1}{2}(\sqrt{20})(\frac{9\sqrt{5}}{5})$ ✓

$= 9$ units^2 ✓ 8,98

f. Calculate the size of $C\hat{A}B$ rounded off to ONE decimal digit. (7)

$m_{AC} = \frac{3+5}{-2-2} = \frac{8}{-4} = -2$ ✓ $m_{AB} = \frac{3-2}{-2-3} = -\frac{1}{5}$ ✓

$-2 = \tan\alpha$ ✓ $-\frac{1}{5} = \tan\theta$

$116,57° = \alpha$ ✓ $168,69° = \theta$ ✓

$\therefore$ coint $\angle = 63,43°$ $\therefore$ coint $\alpha = 11,31°$

$\therefore C\hat{A}B = 63,43° - 11,31°$ ✓

$= 52,1°$ ✓

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Question 2:

Determine, without the use of a calculator, the numerical value of:

$$\frac{cos^2(-325°)-sin^2(145°)}{\cos 340° \times tan200°}$$ **[7]**

$\frac{\cos^2(360° - 35°) - \sin^2(180° - 35°)}{\cos(360° - 20°) \times \tan(180° + 20°)}$

$= \frac{\cos^2 35° - \sin^2 35°}{\cos 20° \times \tan 20°}$ ✓✓ ✓

$= \frac{\cos 70°}{\cancel{\cos} 20° \times \frac{\sin 20°}{\cancel{\cos 20}°}}$ ✓

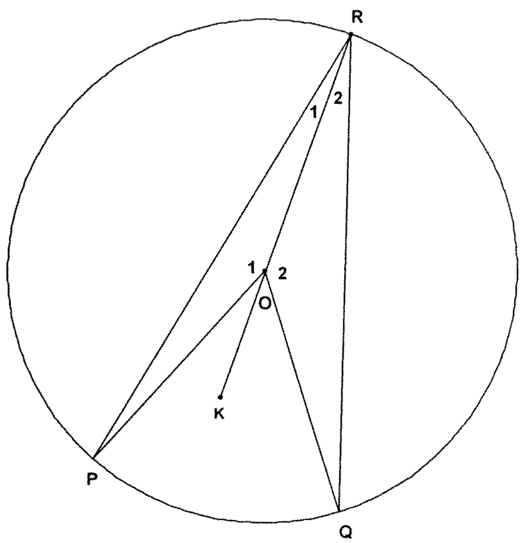
More lines

$= \frac{\sin 20°}{\sin 20°} = 1$ ✓ ✓

Question 3:

a. In the figure O is the centre of the circle. Using the sketch prove that the angle subtended by an arc at the centre of the circle is double the size of the angle subtended by the same arc at any point on the circumference of the circle.

(6)



Let $\hat{R}_1 = x$ and $\hat{R}_2 = y$ ✓

$\hat{P} = x$ and $\hat{Q} = y$ radii ✓

$\hat{O}_1 = 180° - 2x$ ∠ sum △ POR ✓

$\hat{O}_2 = 180° - 2y$ ∠ sum △ POR ✓

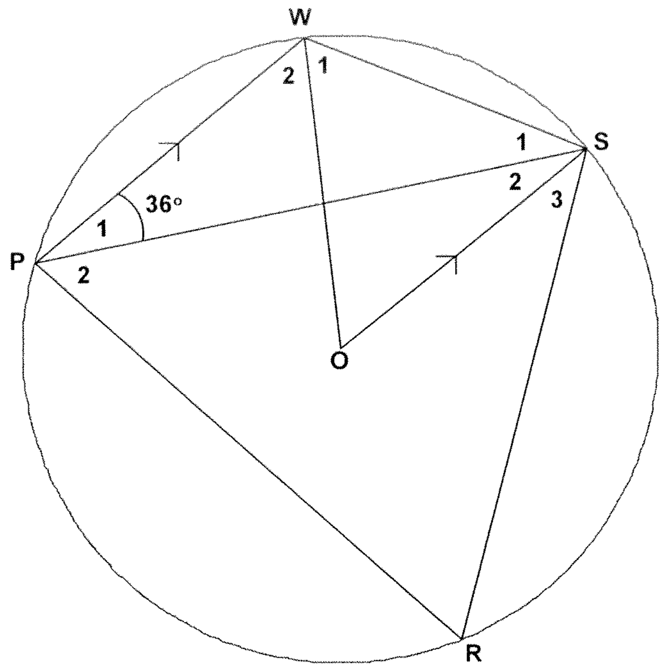
$P\hat{O}Q + \hat{O}_1 + \hat{O}_2 = 360°$ ∠'s rd pt. ✓

$\therefore P\hat{O}Q = 360° - (180° - 2x) - (180° - 2x)$

$= 2(x+y)$ ✓

$= 2P\hat{R}Q$

b. In the diagram, O is the centre of the circle. PWSR is a cyclic quadrilateral. PS, WO and OS are drawn. PW||OS and $\hat{P}_1 = 36°$.



Calculate the sizes of the following angles:

i. $S\hat{O}W$ = 72° (2)

∠ at centre = 2 × ∠ at circumference

ii. $\hat{W}_2$ = 72° (2)

alt ∠'s PW ∥ SO

iii. $\hat{S}_2$ = 36° (2)

alt ∠'s PW ∥ SO

iv. $O\hat{S}W$ (3)

$\hat{W}_1 = O\hat{S}W$ isos △ OWS OW = OS radii

$= \frac{180° - 72°}{2}$ = 54°

v. $\hat{R}$ (3)

$\hat{W}_1 + \hat{W}_2$ = 72° + 54°

= 126°

∴ $\hat{R}$ = 54° opp ∠'s of cyclic quad suppl.

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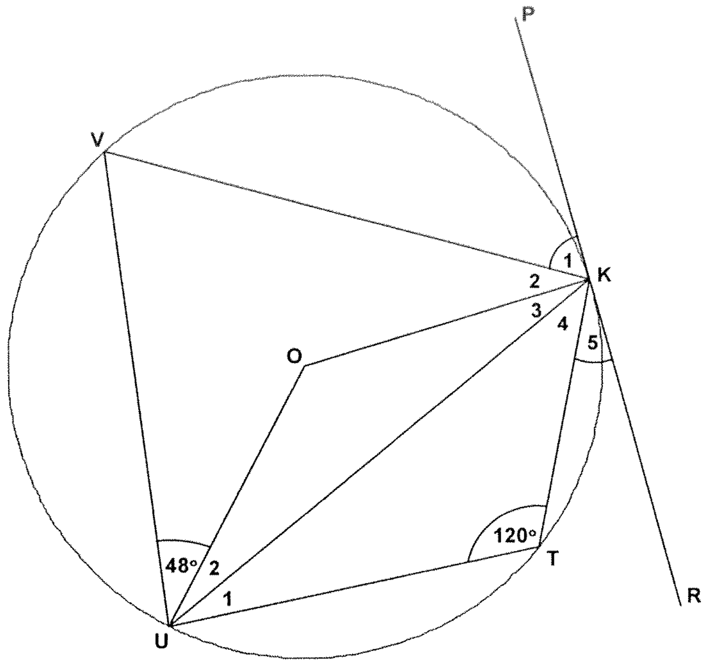
Question4:

In the diagram below, O is the centre of the circle KTUV. PKR is a tangent to the circle at K.

$O\hat{U}V = 48°$ and $K\hat{T}U = 120°$.



Calculate, with reasons, the sizes of the following angles:

a. $\hat{V}$ = 60° ✓ (2)

opp ∠'s of cyclic quad are suppl ✓

b. $K\hat{O}U$ = 120° ✓ (2)

∠ at centre = 2x ∠ at circumference ✓

c. $\hat{U}_2$ = $\hat{K}_3$ Isos △ KOU KO = OU radii ✓ (2)

$= \frac{180° - 120°}{2} = 30°$ ✓

d. $\hat{K}_1$ = 48° + 30° (2)

= 78° ✓ tan- chord ✓

e. $\hat{K}_2$ = 180° - (30 + 78° + 60°) (2)

= 12° ✓ ∠'s of △ ✓

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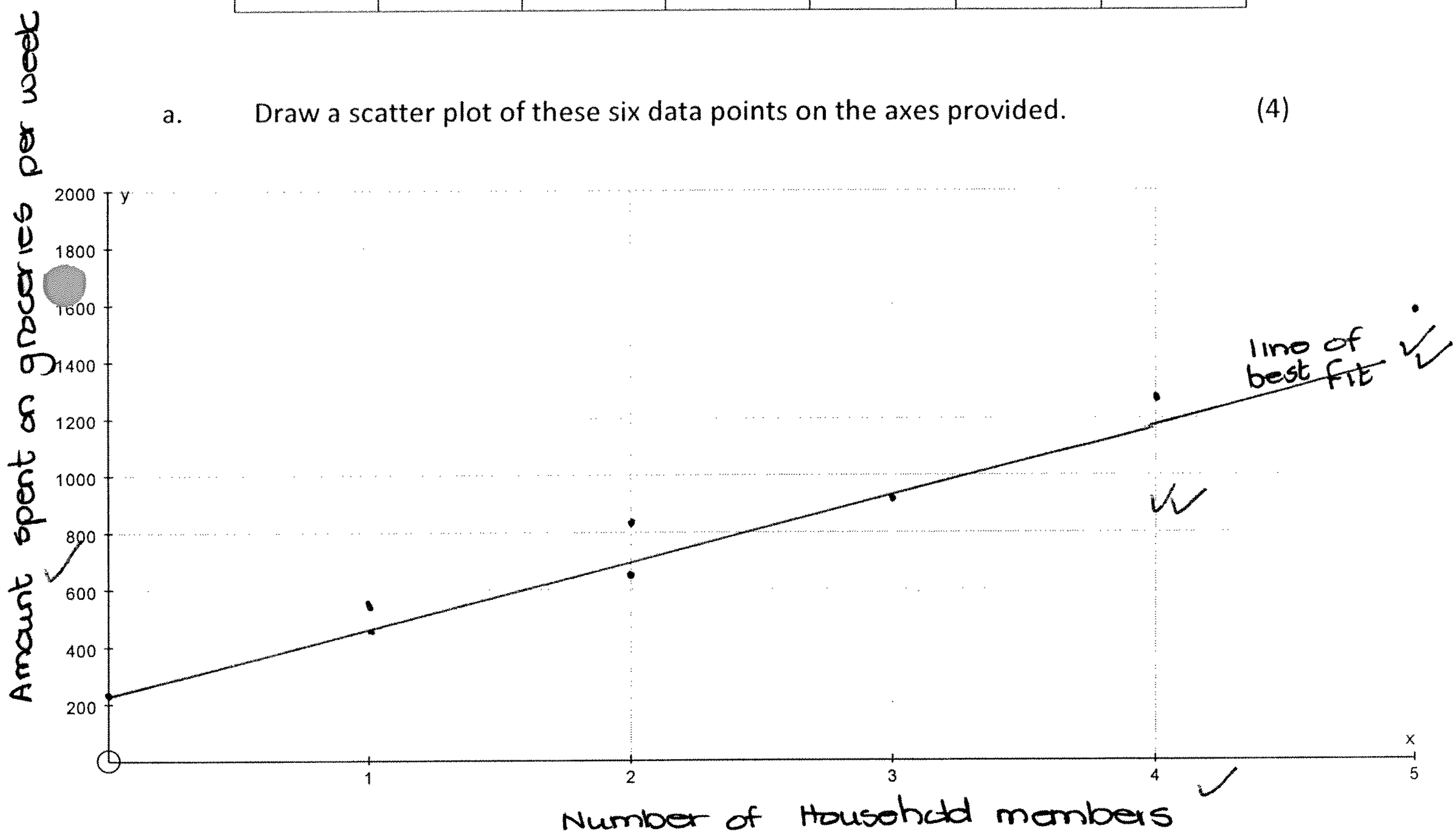
Question 5:

The number of household members, x, and the amount spent on groceries per week, y, are measured for six households in a local area. They appear in the table below:

x	2	2	3	4	1	5
y	R653,25	R812,09	R901,63	R1260,12	R544,46	R1586,82

a. Draw a scatter plot of these six data points on the axes provided. (4)



b. Describe the relationship between the number of household members and the amount spent on groceries per week. (2)

The more household members there are, the more money that is spent on groceries per week.

c. Using your calculator, determine the best-fitting line for these data points and plot it on the same graph. (4)

$a = 217,16$ $b = 262,08$

$\therefore y = 217,16 + 262,08x$

d. What would you estimate a household of six spend on groceries per week? (2)

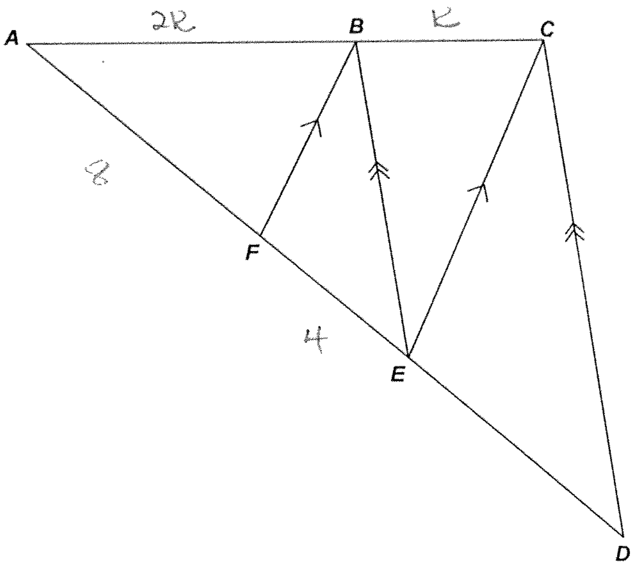
$y = 217,16 + 262,08(6) = R1789,64$

[12]

[12]

Question 6:

In ΔADC, E is a point on AD and B is a point on AC such that EB||DC. F is a point on AD such that FB||EC. It is also given that $AB = 2BC = 2k$.



a. Determine the value of $AF: FE$. (2)

AF : FE = AB : BC line ‖ to 3rd side of Δ

= 2 : 1

b. Calculate the length of ED if $AF = 8\ cm$. (5)

AF : FE = 2 : 1 Proven

∴ FE = 4 cm

AE : ED = AB : BC line ‖ to 3rd side of Δ

= 2 : 1

$\therefore \frac{12}{ED} = \frac{2k}{1k}$

[7]

6cm = ED

more lines

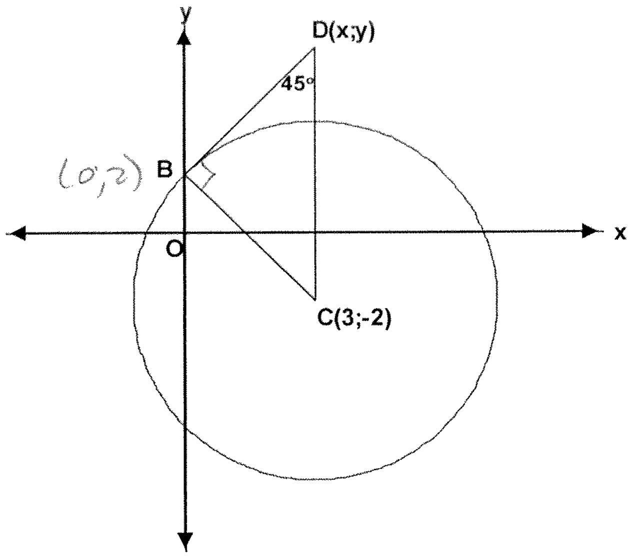
SECTION B:

Question 7:

In the diagram alongside, BD is a tangent to the circle at point B, which lies on the y-axis. The centre of the circle is $C(3;-2)$. The equation of the tangent BD is given by

$3x - 4y + 8 = 0.$

$B\hat{D}C = 45°$



a. Determine the co-ordinates of B. (2)

$3x - 4y + 8 = 0$

$3(0) - 4y + 8 = 0$ ✓

$8 = 4y$

$2 = y$ ✓ $B(0;2)$

b. Show fully that $x^2 - 6x + y^2 + 4y - 12 = 0$ is the equation of the circle. (4)

$(x-3)^2 + (y+2)^2 = r^2$ ✓

$r^2 = (2+2)^2 + (0-3)^2$ ✓

$= 16 + 9$

$= 25$

$\therefore (x-3)^2 + (y+2)^2 = 25$

$x^2 - 6x + 9 + y^2 + 4y + 4 - 25 = 0$ ✓

$x^2 - 6x + y^2 + 4y - 12 = 0$ ✓

more lines

c. Determine the value(s) of q if $x + q = 0$ is the equation of a tangent to the circle. ~~(4)~~ 3

$r = 5$ ✓

$\therefore q = 2$ ✓ or $q = -8$ ✓

d. i) Why is the length of BD = 5? $\hat{B} = 90°$ ✓ tan ⊥ chord ✓ ~~(2)~~ 3

$\therefore \triangle BDC$ is isosceles ✓ $\hat{D} = \hat{C} = 45°$

$\therefore BD = 5$

ii) Hence, determine the co-ordinates of D. (6)

$CD^2 = 5^2 + 5^2$ Pythagoras

$= 50$

$\therefore (x-3)^2 + (y+2)^2 = 50$

$x^2 - 6x + 9 + y^2 + 4y + 4 = 50$ ① ✓

$3x - 4y + 8 = 0$

$3x + 8 = 4y$

$\frac{3x+8}{4} = y$ ② ✓

② in ①. $x^2 - 6x + 9 + \left(\frac{3x+8}{4}\right)^2 + 4\left(\frac{3x+8}{4}\right) + 4 = 50$ ✓

$x^2 - 6x + 9 + \frac{9x^2 + 48x + 64}{16} + 3x + 8 + 4 - 50 = 0$

$16x^2 - 96x - 464 + 9x^2 + 48x + 64 + 48x = 0$

$25x^2 - 400 = 0$ ✓

$25(x+4)(x-4) = 0$

$x = -4$ or $x = 4$ ✓ $\therefore y = \frac{3(4)+8}{4} = 5$ more lines

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$D(4;5)$ ✓

Question 8:

a. Determine the general solution of $cos4\theta \times cos40° + sin4\theta \times sin40° = -1$ if $\theta \in [-90°; 180°]$. ✓ (6)

$\cos(4\theta - 40°) = -1$

$4\theta - 40° = \pm 180° + k \cdot 360°;\ k \in \mathbb{Z}$ ✓

$4\theta = 220° + k \cdot 360°;\ k \in \mathbb{Z}$ or $4\theta = -140° + k \cdot 360°;\ k \in \mathbb{Z}$

$\theta = 55° + k \cdot 90°;\ k \in \mathbb{Z}$ ✓ $\theta = -35° + k \cdot 90°;\ k \in \mathbb{Z}$ ✓

$\theta = 55°;\ -35°;\ 145°;$ ✓✓

less lines

b. Show that $2cos\theta \times cos2\theta + \frac{sin^2 2\theta}{cos\theta} = 2cos\theta$ (7)

LHS: $2\cos\theta(\cos^2\theta - \sin^2\theta) + \frac{(2\sin\theta\cos\theta)^2}{\cos\theta}$ RHS: $2\cos\theta$

$= 2\cos^3\theta - 2\cos\theta\sin^2\theta + \frac{4\sin^2\theta\cos^2\theta}{\cos\theta}$

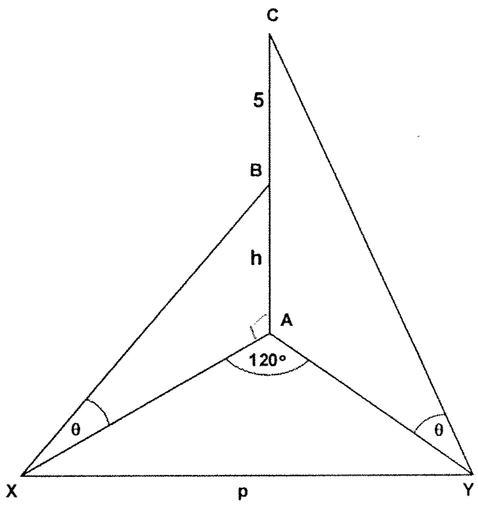
$= 2\cos^3\theta + 2\cos\theta\sin^2\theta$

$= 2\cos\theta(\cos^2\theta + \sin^2\theta)$

$= 2\cos\theta$

$=$ RHS.

c. In the diagram alongside AC is a vertical tower on the horizontal plane AXY. B is a point on CA. The angles of elevation of both B and C are θ. $X\hat{A}Y = 120°$ and $BC = 5\ metres$.



i. Express XY^2 in terms of AX and AY. (3)

$XY^2 = AX^2 + AY^2 - 2(AX)(AY)\cos 120°$

$= AX^2 + AY^2 + 2(AX)(AY)\cos 60°$

$= AX^2 + AY^2 + 2(AX)(AY)\frac{1}{2}$

$= AX^2 + AY^2 + (AX)(AY)$

ii. If $AB = h$ and $XY = p$, prove that $p^2 = \frac{3h^2+15h+25}{tan^2\theta}$ (8)

$\tan\theta = \frac{h}{AX}$ $\tan\theta = \frac{h+5}{AY}$

$AX = \frac{h}{\tan\theta}$ $AY = \frac{h+5}{\tan\theta}$

$\therefore XY^2 = AX^2 + AY^2 + (AX)(AY)$ more lines

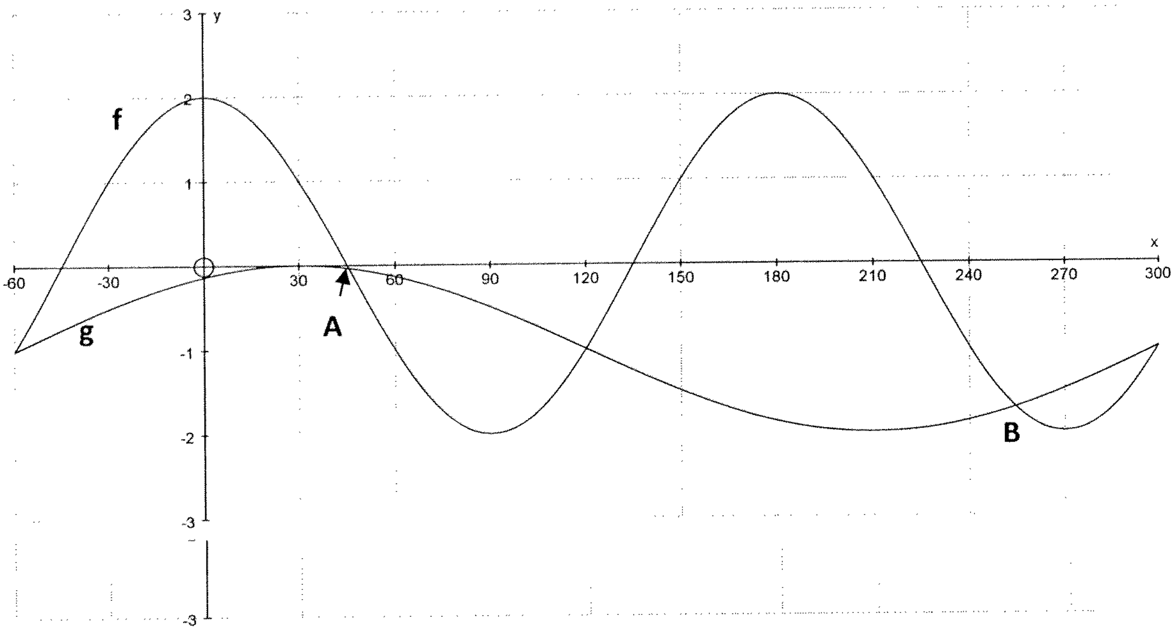
$p^2 = \left(\frac{h}{\tan\theta}\right)^2 + \left(\frac{h+5}{\tan\theta}\right)^2 + \left(\frac{h}{\tan\theta}\right)\cdot\left(\frac{h+5}{\tan\theta}\right)$

$$p^2 = \frac{h^2}{\tan^2\theta} + \frac{h^2+10h+25}{\tan^2\theta} + \frac{h^2+5h}{\tan^2\theta}$$

$$p^2 = \frac{3h^2+15h+25}{\tan^2\theta}$$

d. The graphs of $f(x) = a\cos bx$ and $g(x) = \sin(x+c) + d$ are sketched below.

A is the point $(46°; -0{,}05)$.



From the graphs and the values of point A, answer the following questions:

i. Determine the values of a, b, c and d. (4)

$a = 2$; $b = 2$

$c = 60°$; $d = -1$

ii. Give two x values (other than A) for which $f(x) = g(x)$ in the interval $[-\overset{6}{\cancel{8}}0°; 120°]$. (2)

$x = -60°$; $x = 120°$

iii. For which values of x is $f(x) < g(x)$ in the interval $[-30°; 120°]$? (2)

$x \in (46°; 120°)$

iv. For which values of x is $f(x).g(x) \leq 0$ in the interval $[-30°; 120°]$? (2)

$x \in [-30°; 45°]$

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Question 9:

The weights (in grams) of 27 packages of mince in a supermarket display are listed in the table below, in order from smallest to largest.

341	377	395	405	405	405	418
423	436	436	441	445	450	482
491	491	509	509	518	518	532
536	536	564	582	627	641	

a. Calculate the mean and the standard deviation of the above data. (2)

Mean $\bar{x} = 478,26$ ✓

Std Dev $s_x = 75,28$ ✓ OR 73,87

b. Determine if there are any outliers, using the formula $Q_1 - 1,5 \times IQR$ and $Q_3 + 1,5 \times IQR$. ~~(5)~~ 6

Q_1 pos: $\frac{27+1}{4} = 7$ pos $Q_3 = \frac{3}{4}(27+1) = 21$

$Q_1 = 418$ ✓ $Q_3 = 532$ ✓

$IQR = 532 - 418 = 114$ ✓

$Q_1 - 1,5 \times IQR$ $Q_3 + 1,5 \times IQR$

$= 418 - 1,5 \times 114$ $= 532 + 1,5 \times 114$ more lines

$= 247$ ✓ $= 703$ ✓ ∴ No outliers. ✓

c. Draw a box and whisker plot to show the distribution of the data in the table. (2)

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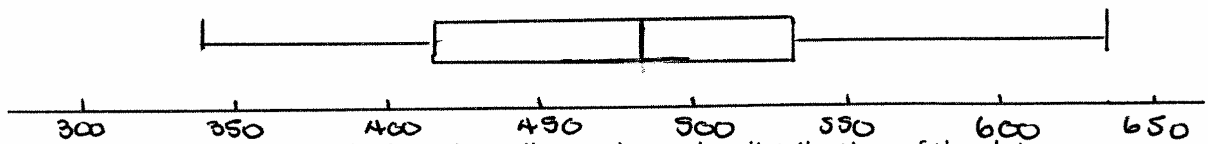


d. What does this box and whisker plot tell you about the distribution of the data ie. about the skewness of the data? ~~(2)~~ (1)

Data is symmetrical ✗

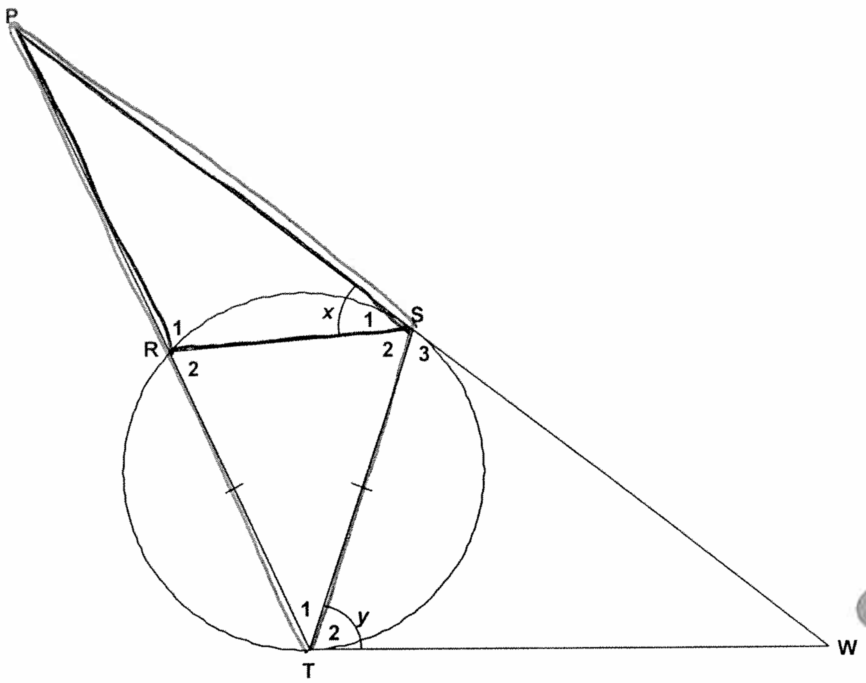
slightly left ✗

[11]

Question 10:

In the diagram, PSW and WT are tangents to circle RST at S and T respectively. PT is drawn and intersects the circle at R. RS and ST are joined. RT=TS. retype

Let $\hat{S}_1 = x$ and $\hat{T}_2 = y$.



a. Name, with reasons, THREE angles each equal to y. (6)

$\hat{R}_2 = y$ ✓ tan - chord theorem ✓
$\hat{S}_2 = y$ ✓ ∠'s opp = sides ✓
$\hat{S}_3 = y$ ✓ tan-chord theorem. ✓

b. Prove that $\Delta PRS|||\Delta PST$. (3)

$\hat{P}$ is common ✓
$\hat{S}_1 = \hat{T}_1 = x$ tan-chord th ✓
$\therefore P\hat{R}S = P\hat{S}T$ rem ∠'s of Δ ✓
$\therefore \Delta PRS ||| \Delta PST$

c. Prove that $PS \times RT = RS \times PT$. (3)

$\frac{PS}{PT} = \frac{RS}{ST}$ ✓ ΔPRS ||| ΔPST

but RT = ST ✓ given
$\therefore \frac{PS}{PT} = \frac{RS}{RT}$ $\therefore RT \times PS = RS \times PT$ ✓

[12]

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TOTAL FOR PAPER: 150 MARKS