CORNWALL HILL COLLEGE

Mathematics Department

EXAMINATIONS – 21 JULY 2014

Mathematics – Paper 1

Grade: 12 **Time: 3 Hours**

Marks: 150

Examiner: Mrs. S. Hickling

Moderator(s): Mrs. M. van Niekerk, Mrs. T. Knoetze and Mr. P. van Schalkwyk

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Instructions:

1. The question paper consists of 9 pages and 12 Questions. Please check that your paper is complete.
2. Read the questions carefully.
3. It is in your own interest to write legibly and to present your work neatly.
4. All the necessary working details must be clearly shown.
5. Approved calculators may be used except where otherwise stated.
6. Answers to be rounded off to **2 decimal digits** where necessary.
7. *GOOD LUCK!*

SECTION A **[77 MARKS]**

QUESTION 1 **[18]**

a) Solve for x:

(1) $(x^{\frac{2}{3}} - 4)(x^{\frac{1}{3}} + 2) = 0$ (2)

(2) $\frac{x^2 + 1}{x + 1} \leq 0$ (1)

(3) $2^{2x+1} - 3(2^{2x-1}) + 4^x = 12$ (4)

(4) $x^2 - 4x - 1 = \frac{2}{x^2 - 4x - 2}$ (5)

b) There are four solutions to the equation $(x^2 - 5)(2x^2 - 9x + 9) = 0$

Solve for x if:

1) $x \in Z$ (2)

2) $x \in Q^{\prime}$ (2)

c) The roots of a quadratic equation are $3 \pm \sqrt{12 - 3a^2}$. Determine the value(s) of a for which the roots will be equal. (2)

QUESTION 2 **[16]**

a) Consider the number sequence 2; 5; 2; 9; 2; 13; 2; 17; ...

(1) Write down the next two terms of the sequence, given that the pattern continues. (1)

(2) Calculate the sum of the first 100 terms of the sequence. (4)

b) The seventh term of a geometric sequence is $\frac{3\,645}{64}$ and the fourth term is $\frac{135}{8}$, determine the first term. (4)

c) The sum of 5 + 15 + 45 + ... to n terms is 605. Determine the value of n. (3)

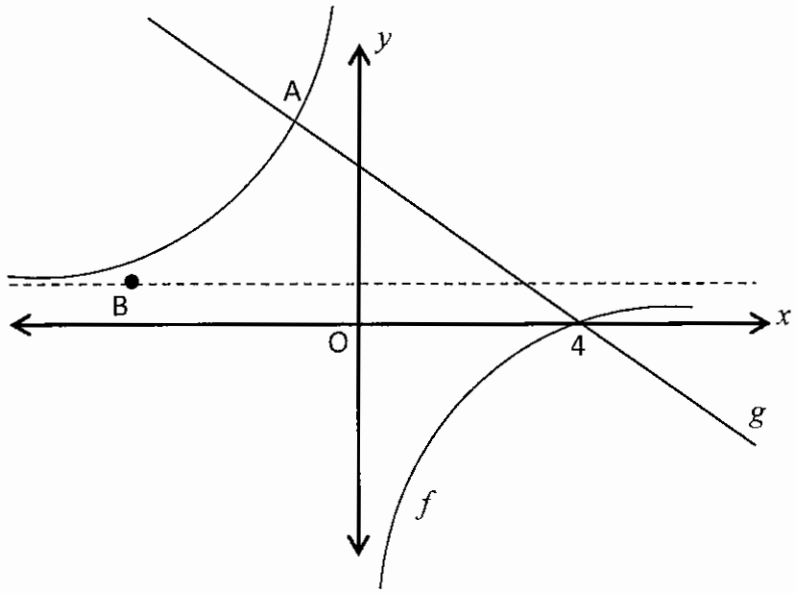
d) Show that:

$$\sum_{n=1}^{\infty}(k-\frac{3}{2})^n = \frac{2k-3}{5-2k}$$

(4)

QUESTION 3 **[27]**

a) Given: $f(x) = -\frac{4}{x} + 1$



(1) Give the equation of the asymptote through point B. (1)

(2) Determine the coordinates of point A, the point where the straight line $g(x) = -x + 4$ intercepts the hyperbola. (4)

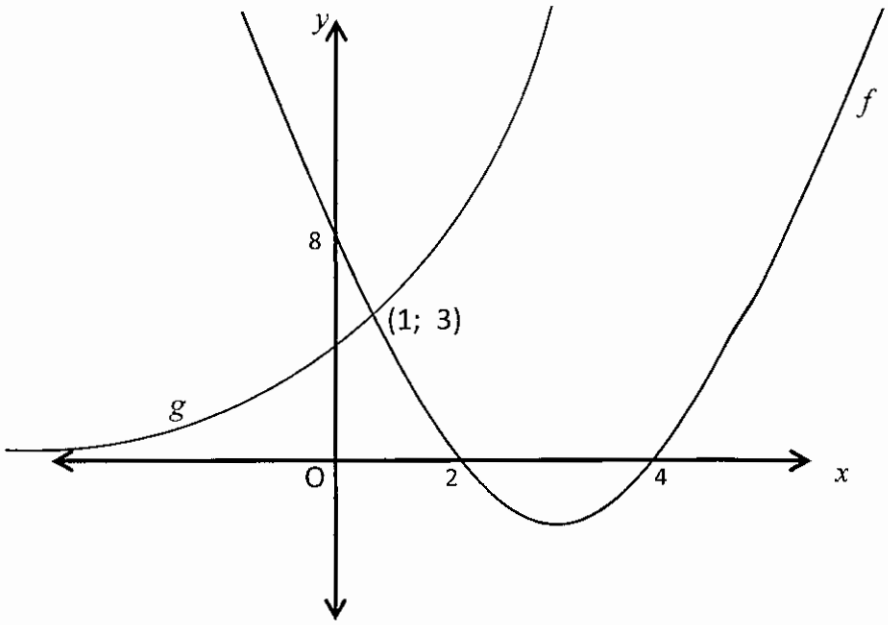
(3) Calculate the length of AB when B is the reflection of point A in the line $y = -x$. Leave the answer in surd form. (3)

(4) Give the equation of h if h is the translation of f by two units to the left. (1)

(5) Hence give an equation for any of the axis of symmetry for h. (1)

(6) Determine the values of x for which $f(x) \geq g(x)$ (2)

b) The sketch shows $f(x) = ax^2 + bx + c$ and $g(x) = k^x$. Both pass through the point (1; 3).



(1) Determine the values of a, b and c. (4)

(2) Determine the value of k. (1)

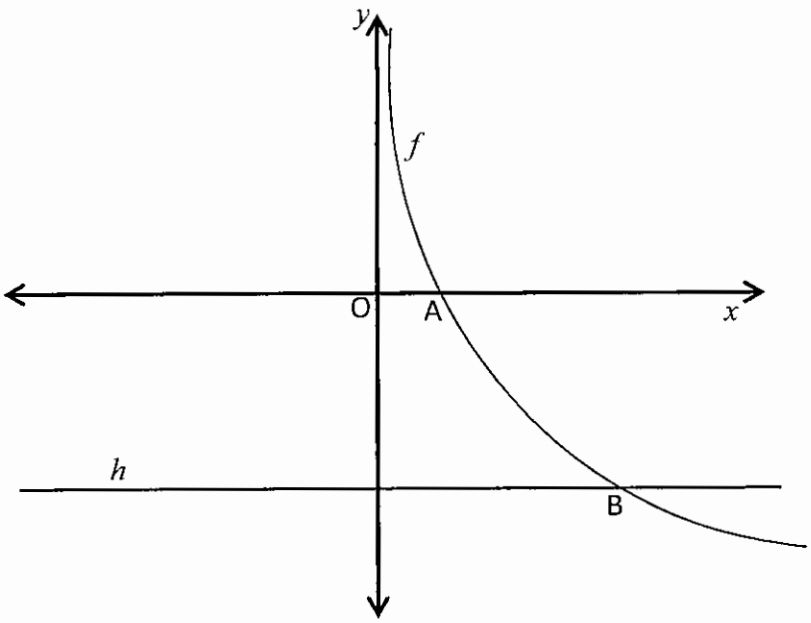
(3) Determine the range of $f(x)$. (2)

(4) Consider the graph of $f(x)$.

(i) Explain why $f(x)$ does not have an inverse. (1)

(ii) What are the values of x for which $f(x)$ will have an inverse? (1)

c) The graph of $f(x) = \log_{\frac{1}{3}} x$ and $h(x) = -3$ are given below:



(1) Determine the coordinates of:

(i) A (1)

(ii) B (1)

(2) Use the graph to solve for x if $\log_{\frac{1}{3}} x \geq -3$ (2)

(3) Determine the equation of $f^{-1}(x)$. (1)

(4) Determine the new equation of $f(x)$, if $f(x)$ is reflected about the x-axis. (1)

QUESTION 4 **[10]**

a) Given: $f(x) = x^2 - 3x$, find $f'(x)$ from first principles. (4)

b) Find $\frac{dy}{dx}$ in each of the following:

(1) $y = 2\sqrt{x} - \frac{4}{9}x^3 + \pi$ (2)

(2) $8x^3 - 2xy + y - 1 = 0;\ x \neq \frac{1}{2}$ (4)

QUESTION 5 **[6]**

A survey was conducted among the 640 soccer players at the Soccer World Cup about their preferences for Adidas, Nike and Puma soccer boots.
The findings were:
196 prefer Adidas
256 prefer Nike
284 prefer Puma
36 prefer all three types
60 prefer Puma and Adidas
43 prefer Nike and Puma
64 do not prefer any of these three types
x prefer Nike and Adidas but not Puma.

Draw a Venn diagram to summarize the information and determine how many soccer players prefer Nike and Adidas but not Puma. (6)

SECTION B **[73 MARKS]**

QUESTION 6 **[8]**

a) (1) Factorise: $a^2 + 5a + 6$ (1)

(2) Hence, simplify the following: $\dfrac{a^{\frac{3}{2}} + 5a^{\frac{1}{2}} + 6a^{\frac{-1}{2}}}{a^2 - 4}$ (3)

b) Find the value(s) of k for which the nature of the roots of the following equations will be the same:

$x^2 - 5x + 7 = 0$ and $kx^2 + kx + 2 = 0$ (4)

QUESTION 7 **[14]**

A new car currently costs R250 000.

a) The value of the vehicle depreciates at 18 % p.a. on a reducing balance basis.
In which year will the car be valued at 30 % of its original value? (5)

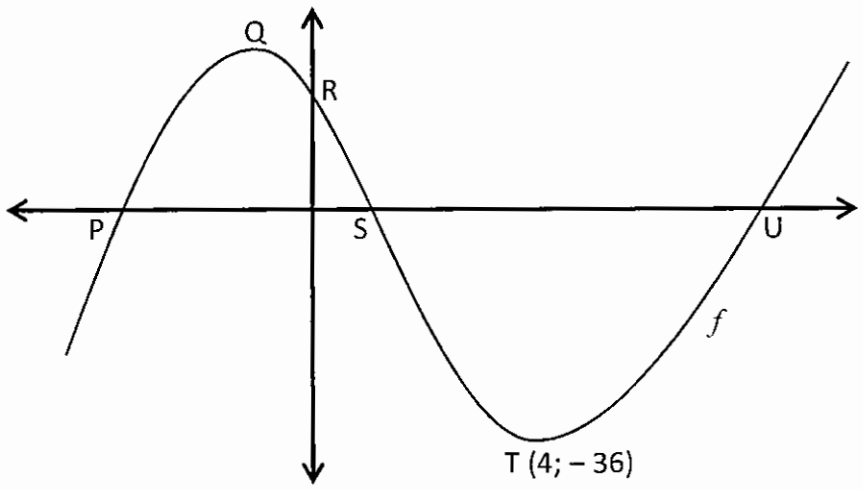
b) What will a new car cost in 5 years time if inflation is calculated at 8 % p.a.? (2)

c) A man wants to start saving so that he can afford to buy a car in 5 years' time.
He opens a savings account with an interest rate of 9,5 % p.a. compounded monthly.
What must his monthly payments into the savings account be, so that he can afford to buy the car in 5 years time? (4)

d) Calculate the effective annual interest rate as a percentage and correct to 2 decimal places for the interest rate in 7 c). (3)

QUESTION 8 **[16]**

The shown below is defined by the equation: $f(x) = x^3 + ax^2 + bx + 12$. The point S (1; 0) is an x- intercept and T (4; –36) and Q are stationary points.



a) Show that $a = -5$ and $b = -8$. (7)

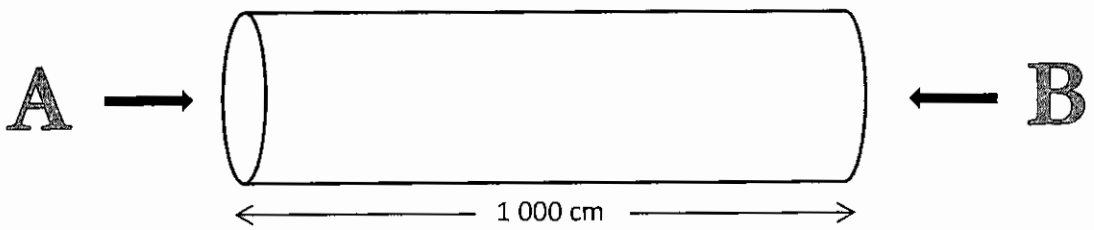
b) Find the co-ordinates of Q. (4)

c) Find the co-ordinates of P and U. (2)

d) Determine the equation of a tangent to $f(x)$ at point P. (3)

QUESTION 9 **[8]**

Two particles are projected simultaneously towards each other from the opposite ends of a straight tube, 1 000 cm long. Particle A travels 51 cm in the first second, 49 cm in the second second, 47 cm in the third second, etc. Particle B travels 25 cm in the first second, 24 cm in the second second, 23 cm in the third second, etc.



a) How far does particle A travel in t seconds? (3)

b) Determine how long it takes the particles to meet. (5)

QUESTION 10 **[10]**

a) If the letters of the word **SOCCER** are arranged randomly into a "word",

(1) How many 6 letter words can be formed? (2)

(2) What is the probability that the letters **CC** are next to each other? (3)

b) In the Soccer World Cup the players are being tested for the use of illegal substances. Of the 640 soccer players participating in the World Cup 50 are using an illegal substance.

A test for an illegal substance is 96 % accurate if the substance is present in the bloodstream and 99 % accurate when it is not in the bloodstream. What % of the soccer players will test positive for an illegal substance?

(Make use of a tree diagram to help you). (5)

QUESTION 11 **[10]**

A soccer ball is kicked in the air. Its height h in metres, t seconds after it has been kicked, is given by the formula $h = 20(t^2 - \frac{1}{3}t^3)$.

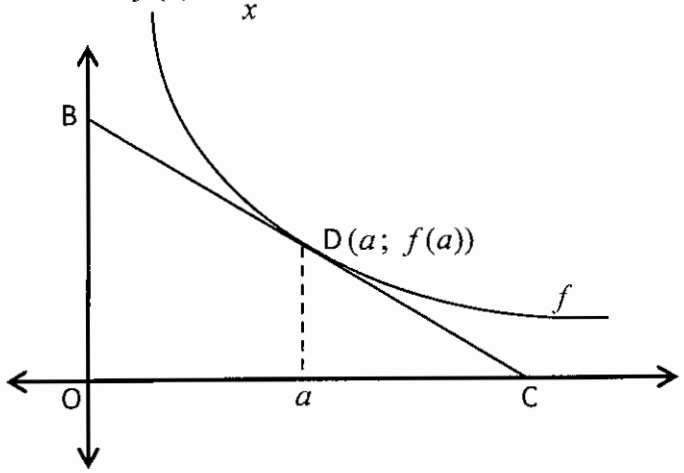
a) What is the velocity of the ball after 1,5 seconds? (3)

b) For what value of t is the **velocity** a maximum? (3)

c) For what value of t is the height of the ball a maximum? Calculate the greatest height from the ground. (4)

QUESTION 12 [7]

Given: $f(x) = \frac{1}{x}$. The tangent at D where $x = a$ is drawn.



a) Show that the equation of the tangent at D is $x + a^2y = 2a$. (4)

b) Calculate the area of Δ OBC. (3)

Gr 12 - July 2014 Memo

Question 1 [18]

a) (1) $(x^{2/3} - 4)(x^{1/3} + 2) = 0$

$x^{2/3} = 4$ or $x^{1/3} = -2$

$x = \pm 8$ ✓ $x = -8$ ✓

(2)

(2) $\frac{x^2 + 1}{x + 1} \leq 0$

$\therefore x + 1 < 0$

$x < -1$ ✓

(1)

(3) $2^{2x+1} - 3(2^{2x-1}) + 4^x = 12$

$2^{2x+1} - 3 \cdot 2^{2x-1} + 2^{2x} = 12$ ✓ ✓

$2^{2x}(2^1 - 3 \times 2^{-1} + 1) = 12$

$2^{2x}(1\frac{1}{2}) = 12$

$2^{2x} = 8$ ✓

$\therefore 2x = 3$

$x = 3/2$ ✓

(4)

(4) $x^2 - 4x - 1 = \frac{2}{x^2 - 4x - 2}$

Let $k = x^2 - 4x$ ✓

$k - 1 = \frac{2}{k - 2}$ ✓

$k^2 - 3k + 2 = 2$

$k(k - 3) = 0$

$x^2 - 4x = 0$ ✓ or $x^2 - 4x - 3 = 0$

$x(x - 4) = 0$ ✓ $x = \frac{4 \pm \sqrt{28}}{2}$

$x = 0$ or $x = 4$ ✓

$= 4{,}65$ or $-0{,}65$

(5)

b) $(x^2 - 5)(2x^2 - 9x + 9) = 0$

$(x^2 - 5)(2x - 3)(x - 3) = 0$

$x = \pm\sqrt{5}$ or $x = 3/2$ or $x = 3$

(1) $x = 3$ ✓ (2)

(2) $x = \pm\sqrt{5}$

$= \pm 2{,}24$. ✓ (2)

c) $\Delta = 0$

$12 - 3a^2 = 0$ ✓

$4 = a^2$

$\pm 2 = a$ ✓ (2)

Question 2 [16]

a) (1) 2 ; 21 ✓ (1)

(2) $S_{100} = 2 \times 50 + \frac{50}{2}[2 \times 5 + 49(4)]$ ✓

$= 100 + 5150$ ✓

$= 5250$ ✓ (4)

b) $ar^6 = \frac{3645}{64}$ ——— ① $ar^3 = \frac{135}{8}$ ——— ②

$\frac{ar^6}{ar^3} = \frac{\frac{3645}{64}}{\frac{135}{8}}$ ✓

$r^3 = \frac{27}{8}$

$\therefore r = \frac{3}{2}$ ✓

Subs into ①: $a\left(\frac{3}{2}\right)^3 = \frac{135}{8}$

$a = 5$ ✓ (4)

c) $5 + 15 + 45 + \ldots \qquad S_n = 605$

$$605 = \frac{5(3^n - 1)}{3 - 1}$$

$$242 = 3^n - 1$$

$$243 = 3^n$$

$$3^5 = 3^n \qquad (3)$$

$$\therefore \quad n = 5$$

d) $$\sum_{n=1}^{\infty} (k - 3/2)^n = (k - 3/2) + (k - 3/2)^2 + (k - 3/2)^3 + \ldots$$

$$S_\infty = \frac{(k - 3/2)}{1 - (k - 3/2)}$$

$$= \frac{2k - 3}{2} \div \frac{2 - 2k + 3}{2}$$

$$= \frac{2k - 3}{5 - 2k} \qquad (4)$$

Question 3 [27]

a) (1) $y = 1$ (1)

(2) $$\frac{-4}{x} + 1 = -x + 4$$

$$-4 + x = -x^2 + 4x$$

$$x^2 - 3x - 4 = 0$$

$$(x - 4)(x + 1) = 0$$

$x = 4$ or $x = -1$

$y = -(-1) + 4$

$= 5$

$\therefore$ A $(-1; 5)$ (4)

(3) $A(-1;5)$ $B(-5;1)$

$$AB = \sqrt{(-1+5)^2 + (5-1)^2}$$
$$= 4\sqrt{2}$$ (3)

(4) $h(x) = \frac{-4}{x+2} + 1$ (1)

(5) $y = (x+2) + 1$ or $y = -(x+2) + 1$
$= x + 3$ (1) $= -x - 1$

(6) $[-1;0)$ $[4;\infty)$ (2)

b) (1) $y = a(x-2)(x-4)$
Subs $(0;8)$
$8 = a(0-2)(0-4)$
$1 = a$

$\therefore y = 1(x-2)(x-4)$
$= x^2 - 6x + 8$

$\therefore a = 1$ $b = -6$ $c = 8$ (4)

(2) Subs $(1;3)$ into: $y = k^x$
$3 = k^1$ (1)

(3) $x = 3$ subs into $y = x^2 - 6x + 8$
$= (3)^2 - 6(3) + 8$
$= -1$

$\therefore y \geq -1$ (2)

(4) (1) $f^{-1}(x)$ – one to many / vertical line, no functio (1)

(2) $x \geq 3$ or $x \leq 3$ (1)

c) (1) (i) $A(1;0)$ ✓ (1)

(ii) $B(27;-3)$ ✓

$y = \log_{1/3} x$

(1) $-3 = \log_{1/3} x$

$\therefore x = (1/3)^{-3}$

$= 27$

(2) $(0;27]$ ✓ ✓ (2)

(3) $f^{-1}(x) = (1/3)^x$ ✓ (1)

(4) $f(x) = -\log_{1/3} x$

$= \log_3 x$ ✓ (1)

Question 4 [10]

a) $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ ✓

$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - 3x - 3h - (x^2 - 3x)}{h}$

$= \lim_{h\to 0} \frac{2xh + h^2 - 3h}{h}$ ✓

$= \lim_{h\to 0} 2x + h - 3$ ✓

$= 2x - 3$ ✓ (4)

b) (1) $y = 2\sqrt{x} - \frac{4}{9}x^3 + \pi$

$= 2x^{1/2} - \frac{4}{9}x^3 + \pi$

$\frac{dy}{dx} = x^{-1/2} - \frac{4}{3}x^2$ ✓ ✓ (2)

(c) $8x^3 - 2xy + y - 1 = 0$

$y - 2xy = 1 - 8x^3$

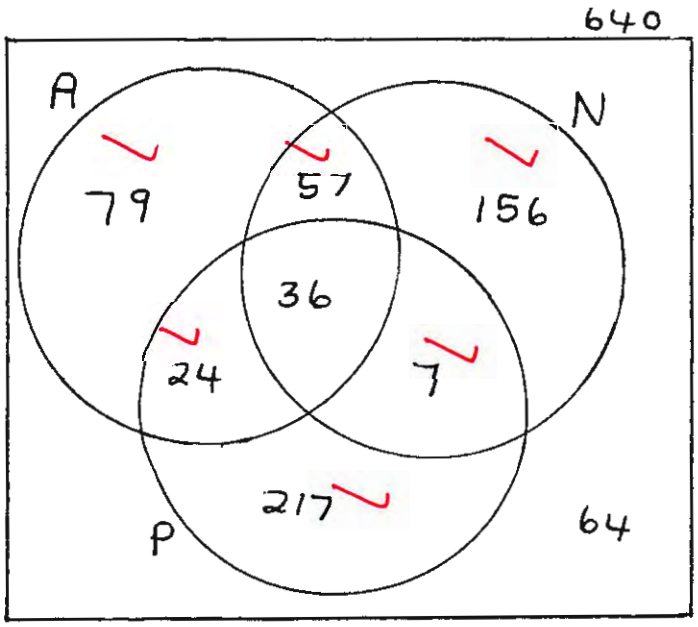
$y(1 - 2x) = 1 - 8x^3$

$y = \frac{(1 - 2x)(1 + 2x + 4x^2)}{(1 - 2x)}$

$y = 1 + 2x + 4x^2$

$\frac{dy}{dx} = 2 + 8x$ (4)

Question 5 [6]



$136 - x + x + 213 - x + 36 + 7 + 24 + 217 + 64 = 640$

$-x + 697 = 640$

$57 = x$

Question 6 [8]

a) (1) $a^2 + 5a + 6$

$= (a + 3)(a + 2)$ ✓ (1)

(2) $\dfrac{a^{3/2} + 5a^{1/2} + 6a^{-1/2}}{a^2 - 4}$

$= \dfrac{a^{-1/2}(a^2 + 5a + 6)}{(a+2)(a-2)}$ ✓ ✓

$= \dfrac{a^{-1/2}(a+3)(a+2)}{(a+2)(a-2)}$ (3)

$= \dfrac{a^{-1/2}(a+3)}{(a-2)}$ ✓

b) $x^2 - 5x + 7 = 0$

$\Delta = b^2 - 4ac$

$= (-5)^2 - 4(1)(7)$

$= -3$ ✓

Roots non-real

$\therefore k^2 - 8k$ ✓ < 0 (4)

$k(k - 8) < 0$

$0 < k < 8$ ✓

$kx^2 + kx + 2 = 0$

$\Delta = k^2 - (4)(k)2$

$= k^2 - 8k$

✓

Question 7 [14]

a) $A = P(1 - i)^n$

$75\,000 = 250\,000(1 - 0{,}18)^n$

$\frac{75000}{250000} = (0{,}82)^n$

$n = \log_{0{,}82}\frac{75}{250}$

$n = 6{,}0668....$

$\therefore$ In the 7th year (5)

b) $A = P(1 + i)^n$

$= 250000(1 + 0{,}08)^5$

$= R\,367\,332{,}02$ (2)

c) $367\,332{,}02 = \frac{x\left[\left(1 + \frac{0{,}095}{12}\right)^{60} - 1\right]}{\frac{0{,}095}{12}}$

$x = R\,4\,806{,}61$ (4)

d) $1 + i = \left(1 + \frac{i}{m}\right)^m$

$1 + i = \left(1 + \frac{0{,}095}{12}\right)^{12}$

$i = \left(1 + \frac{0{,}095}{12}\right)^{12} - 1$

$i = 0{,}09924....$

$\therefore$ 9,92 % p.a. (3)

Question 8 [16]

a) $m = 3x^2 + 2ax + b$

$= 3(4)^2 + 2a(4) + b$

$0 = 48 + 8a + b$ ——— ①

Subs $(1; 0)$: $0 = 1 + a + b + 12$

$-13 = a + b$ ——— ②

$\therefore -48 - 8a = -13 - a$

$-7a = 35$

$a = -5$

Subs into ②:

$-13 = -5 + b$

$-8 = b$ (7)

$y = x^3 - 5x^2 - 8x + 12.$

b) $3x^2 - 10x - 8 = 0$

$(3x + 2)(x - 4) = 0$

$x = -2/3$ or $x = 4$

$y = 14\frac{22}{27}$

$Q\left(-\frac{2}{3}; 14\frac{22}{27}\right)$ (4)

c) $x^3 - 5x^2 - 8x + 12 = 0$

$(x - 1)(x^2 - 4x - 12) = 0$

$(x - 1)(x - 6)(x + 2) = 0$

$x = 1$ or $x = 6$ or $x = -2$

$\therefore P(-2; 0)$ $U(6; 0)$ (2)

d) $m_{tan} = 3x^2 - 10x - 8$

$= 3(-2)^2 - 10(-2) - 8$

$= 24$ ✓

$y - 0 = 24(x + 2)$ ✓

$y = 24x + 48.$ (3)

Question 9 [8]

A: 51 ; 49 ; 47 ; ...

B: 25 ; 24 ; 23 ; ...

a) $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_t = \frac{t}{2}[2\times 51 + (t-1)(-2)]$

$= \frac{t}{2}[102 - 2t + 2]$

$= 52t - t^2$ (3)

b) $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_t = \frac{t}{2}[2\times 25 + (t-1)(-1)]$

$= \frac{t}{2}[50 - t + 1]$

$= \frac{51t}{2} - \frac{t^2}{2}$

$52t - t^2 + \frac{51t}{2} - \frac{t^2}{2} = 1000$

$104t - 2t^2 + 51t - t^2 = 2000$

$0 = 3t^2 - 155t + 2000$

$0 = (3t - 80)(t - 25)$

$t = \frac{80}{3}$ or $t = 25$ s

$= 26,6$

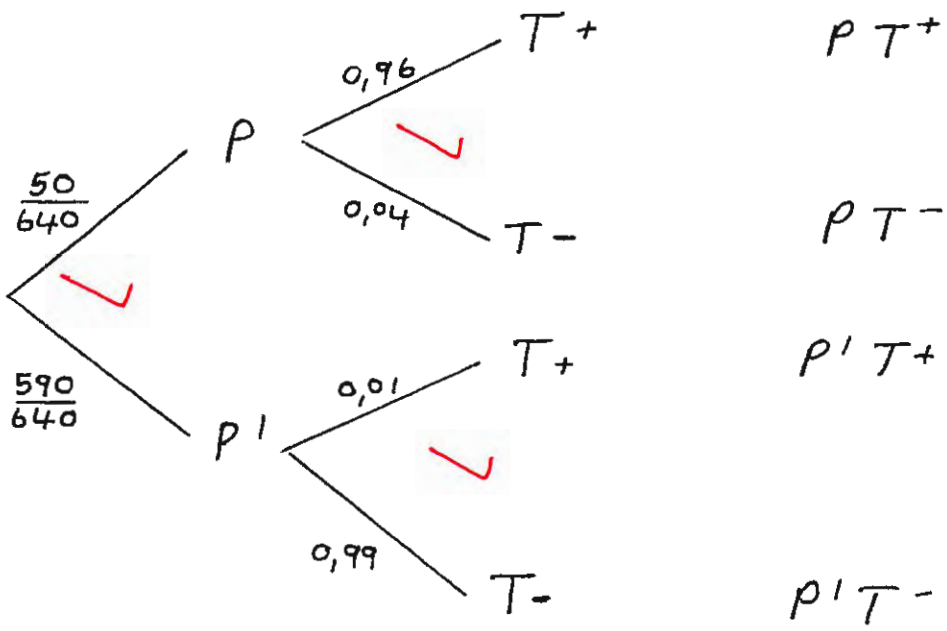
NA. (5)

Question 10 [10]

a) (1) $\frac{6!}{2!} = 360$ (2)

(2) $\frac{2!\ 5!}{360} = \frac{2}{3}$ (3)

b)



$\left(\frac{50}{640} \times 0,96\right) + \left(\frac{590}{640} \times 0,01\right)$

$= 8,42\ \%.$ (5)

Question 11 [10]

a) $h = 20\left(t^2 - \frac{1}{3}t^3\right)$

$= 20t^2 - \frac{20}{3}t^3$

$\frac{dh}{dt} = 40t - 20t^2$ (3)

$= 40(1,5) - 20(1,5)^2$

$= 15\ m/s$

b) $40 - 40t = 0$

$40 = 40t$

$1 = t$ (3)

c) $40t - 20t^2 = 0$

$20t(2 - t) = 0$

$t = 0$ or $t = 2$

NA

$\therefore h = 20(2^2 - \frac{1}{3}(2)^3)$

$= 26\frac{2}{3}$ m. (4)

Question 12 [7]

a) Subs $x = a$ into $y = \frac{1}{x}$

$= \frac{1}{a}$

$\therefore D(a; \frac{1}{a})$

$m = -1x^{-2}$

$= \frac{-1}{a^2}$

$y - \frac{1}{a} = -\frac{1}{a^2}(x - a)$

$a^2y - a = -1(x - a)$

$a^2y - a = -x + a$

$a^2y + x = 2a.$ (4)

b) $a^2y + 0 = 2a$

$y = \frac{2a}{a^2}$

$= \frac{2}{a}$

$B(0; \frac{2}{a})$

$a^2(0) + x = 2a$

$x = 2a$

$C(2a; 0)$

Area $= \frac{1}{2}(2a)(\frac{2}{a})$

$= 2.$ (3)
