CORNWALL HILL COLLEGE

Mathematics Department

EXAMINATIONS – 28 JULY 2014

Mathematics Paper 2

Grade: 12 **Time:** 3 Hours

Marks: 150

Examiner: Mrs T. Knoetze

Moderator(s): Mrs. M. van Niekerk, Mrs. S. Hickling and Mr. P. van Schalkwyk

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Instructions:

1. This question paper consists of 24 pages. Please ensure that your paper is complete.
2. **Answer ALL the questions on the question paper and hand this in at the end of the examination.**
3. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
4. Round off your answers to **two decimal** digits where necessary.
5. All the necessary working details must be clearly shown.
6. It is in your own interest to write legibly and to present your work neatly.

Q 1 (8)	Q 2 (26)	Q 3 (23)	Q 4 (18)	Q 5 (10)	Q 6 (17)	Q 7 (24)	Q 8 (6)	Q 9 (6)	Q 10 (12)	TOTAL

SECTION A [75]

QUESTION 1 [8]

In preparation for the Soccer World cup the Brazilian coaching team looked at the amount of goals scored by Brazil in the last nine tournaments.

(a) Fill in the missing amounts in the cumulative frequency table below. (2)

Tournament year	Goals scored	Cumulative frequency
1978	10	10
1982	15	25
1986	10	35
1990	4	39
1994	11	50
1998	14	64
2002	18	82
2006	10	92
2010	9	101

✓A ✓A

(b) Determine the average number of goals scored in the last nine tournaments by Brazil. (2)

$\bar{x} = \frac{101}{9}$ ✓m

$= 11,22$ ✓A

(c) Determine the standard deviation correct to two decimal places. (2)

$\sigma = 3,79$ ✓✓A

(d) In how many tournaments do the goals scored, lie within one standard deviation of the mean? (2)

$[11,22 - 3,79 \ ; \ 11,22 + 3,79]$

$[7,43 \ ; \ 15,01]$ ✓m

$\therefore$ 7 tournaments. ✓A

QUESTION 2 **[26]**

(a) If $\cos 65^0 = m$, determine the following expressions in terms of m.

(1) $\cos 115^0$ (1)

$= \cos(180° - 65°)$

$= -\cos 65^0$ ✓A

$= -m$

(2) $\sin 25^0$ (1)

$= \sin(90° - 65°)$

$= \cos 65°$

$= m$ ✓A

(3) $\sin 130^0$ (4)

$= \sin 2(65°)$ ✓A

$= 2\sin 65° \cos 65°$ ✓A

$= 2\ m\ \sqrt{1-m^2}$

✓A ✓A

(b) Given: $psin\beta - q = 0$, and $90° \leq \beta \leq 270°$, with $p > 0$ and $q > 0$.
Use a sketch to evaluate, in terms of p and q:

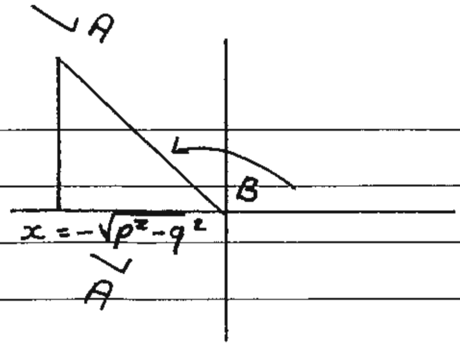
✓m

$\sin \beta = \frac{q}{p} = \frac{y}{r}$

$x^2 = p^2 - q^2$

$x = \pm\sqrt{p^2 - q^2}$

✓A



(1) $\cos\beta$ (4)

$= \frac{-\sqrt{p^2-q^2}}{p}$ ✓CA

(2) $\cos 2\beta$ (2)

$= 1 - 2\sin^2 B$ ✓A $\quad 2\cos^2 B - 1$

$= 1 - 2\left(\frac{q}{p}\right)^2 \quad = 2\left(\frac{-\sqrt{p^2-q^2}}{p}\right)^2 - 1$

$= 1 - 2\frac{q^2}{p^2}$ ✓CA $\quad = -2\left(\frac{p^2-q^2}{p^2}\right) - 1$

(c) Prove that: $\frac{2\sin(180^0+A)\sin(90^0-A)}{\cos^4 A - \sin^4 A} = \tan(360^0 - 2A)$ (6)

$LHS = \frac{2(-\sin A)(\cos A)}{(\cos^2 A - \sin^2 A)(\cos^2 A + \sin^2 A)}$ ✓A ✓A ✓A

$= \frac{-2\sin A\cos A}{(\cos^2 A - \sin^2 A)(1)}$

$= \frac{-\sin 2A}{\cos 2A}$ ✓A ✓A

$= -\tan 2A$

$RHS = \tan(360^\circ - 2A)$

$= -\tan 2A.$ ✓A

(d) Given $f(x) = 2\cos(x + 45^0)$ and $g(x) = \tan x$
Determine:

(1) The amplitude of $f(x)$ (1)

2 ✓A

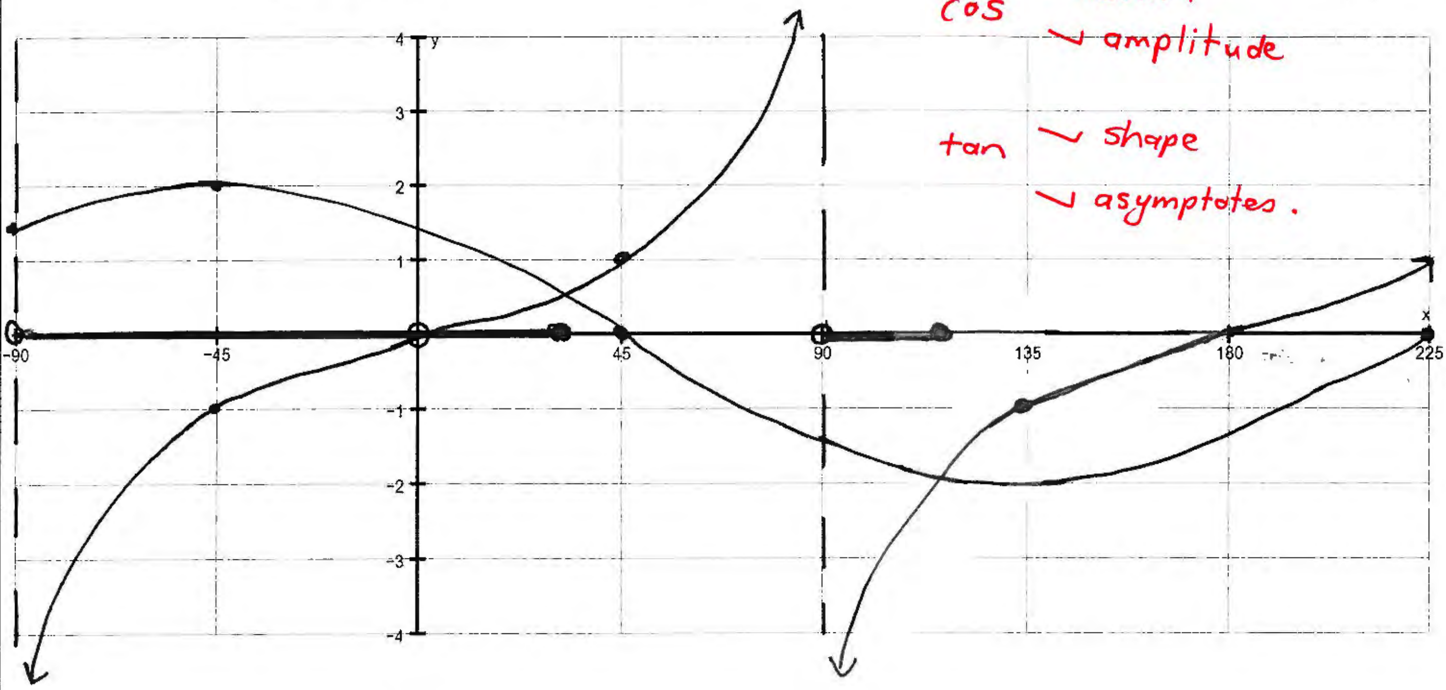
(2) The period of $g(x)$ (1)

180° ✓A

(3) Sketch the graphs of $f(x)$ and $g(x)$ for the interval $x \in [-90°;\ 225°]$ on the grid below. (4)

cos ✓shift
✓amplitude

tan ✓shape
✓asymptotes.

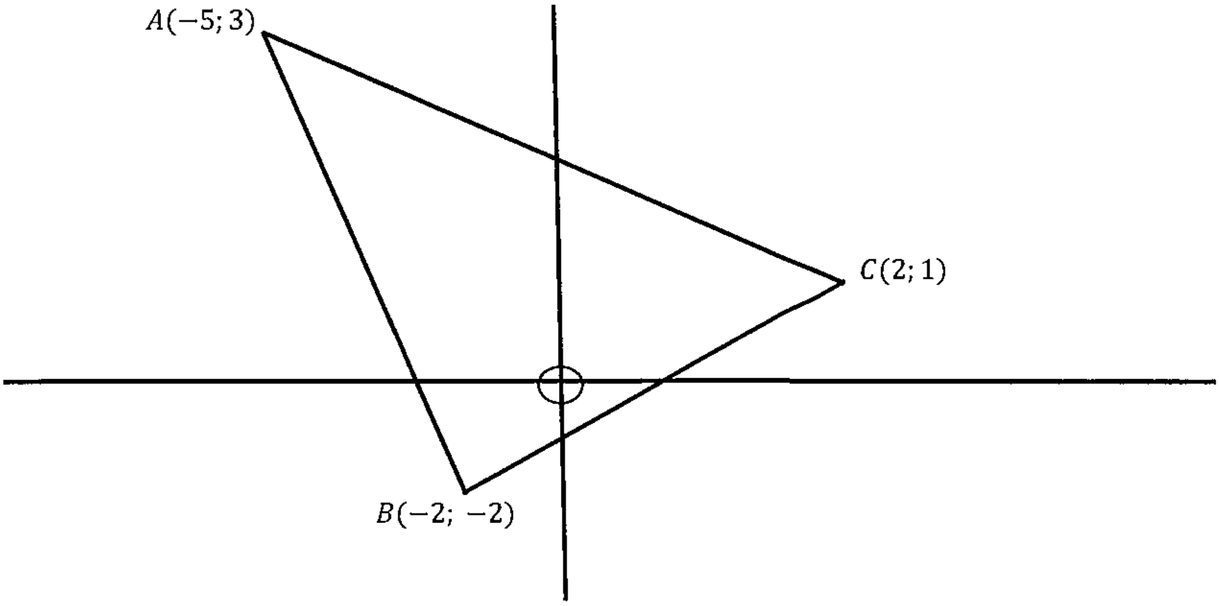


(4) Indicate, using thickened lines, the intervals on the horizontal (x)-axis where $f(x) \geq g(x)$ for the interval $x \in (-90°;\ 225°)$. (2)

✓ intervals
✓ dots.

QUESTION 3 **[23]**

(a) Given $\triangle ABC$ with vertices $A(-5;3)$, $B(-2;-2)$ and $C(2;1)$.



Determine:

(1) The length of AC, in simplest surd form. (2)

$AC = \sqrt{(2-(-5))^2 + (1-3)^2}$ m

$= \sqrt{53}$ A

(2) The midpoint of AC. (2)

$\left(\frac{-5+2}{2} ; \frac{3+1}{2}\right)$

$= \left(\frac{-3}{2} ; 2\right)$

A A

(3) The equation of the straight line passing through C, parallel to AB. (3)

$m_{AB} = \frac{-2-(3)}{-2-(-5)}$ $C(2;1)$

$= \frac{-5}{3}$

$y - y_1 = m(x - x_1)$ ✓A ✓m

$y - 1 = -\frac{5}{3}(x - 2)$

$y = -\frac{5}{3}x + 4\frac{1}{3}$ ✓A

$y = -\frac{5}{3}x + \frac{13}{3}$

(4) The size of $B\hat{A}C$. (5)

$\tan\theta = m_{AB}$

$= \frac{-5}{3}$ ✓A

$\therefore \theta = -59{,}03° + 180°$

$= 120{,}96°$ ✓m

$\tan\theta = m_{AC}$

$= \frac{1-3}{2-(-5)}$

$= \frac{-2}{7}$ ✓A

$\theta = -15{,}95° + 180°$

$= 164{,}05°$ ✓m

$\therefore B\hat{A}C = 164{,}05° - 120{,}96°$

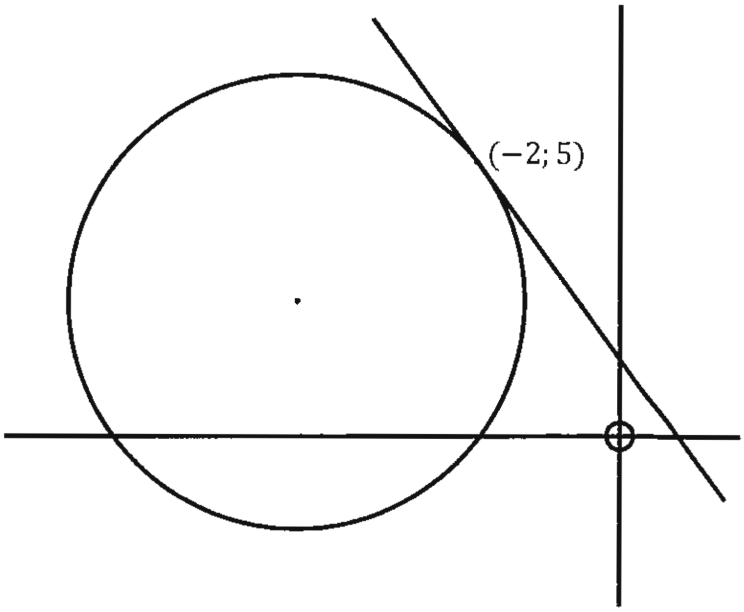
$= 43{,}09°$ ✓m

(5) The co-ordinate of D if ABCD is a parallelogram. (2)

✓A ✓A

$D(-1;6)$

(b) Sketched is the circle with the equation $x^2 + 12x + y^2 - 4y = -15$ passing through point $(-2; 5)$.



(1) Determine the centre and the length of the radius of the circle. (5)

$x^2 + 12x + (6)^2 + y^2 - 4y + (-2)^2 = -15 + (6)^2 + (-2)^2$

$(x + 6)^2 + (y - 2)^2 = 25$ ✓A ✓A ✓A

✓m

$\therefore (-6; 2)$ $r = 5$ ✓m

(2) Determine the equation of the tangent to the circle passing through point $(-2; 5)$. (4)

$m_{radius} = \frac{5-2}{-2-(-6)}$ $m_{tangent} = -\frac{4}{3}$ ✓m

$= \frac{3}{4}$ ✓A

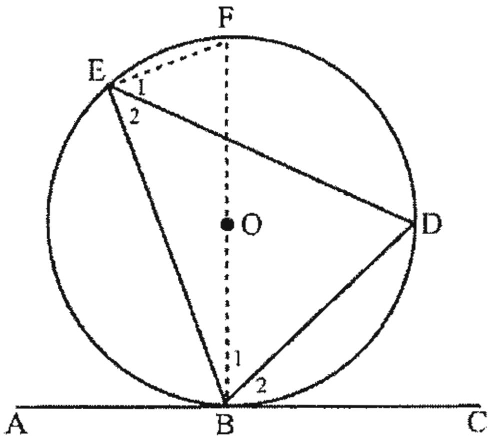
$y - 5 = -\frac{4}{3}(x - (-2))$ ✓m

$y = -\frac{4}{3}x + 2\frac{1}{3}$ ✓

$y = -\frac{4}{3}x + \frac{7}{3}$

QUESTION 4 [18]

(a) Prove the theorem that the angle between a tangent to a circle and a chord drawn from the point of contact is equal to an angle in the alternate segment. (5)



Given: Tangent ABC ✓A

Required to prove: $\hat{B}_2 = \hat{E}_2$

Proof:

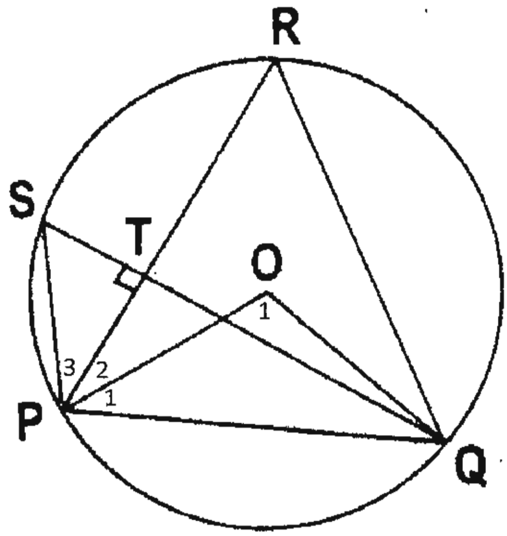
$\hat{B}_1 + \hat{B}_2 = 90°$ ✓A tan ⊥ radius

$\hat{E}_1 + \hat{E}_2 = 90°$ ✓A ∠ in semi circle

$\hat{E}_1 = \hat{B}_1$ ✓A ∠'s in same segment

$\therefore \hat{B}_2 = \hat{E}_2$. ✓A

(b) In the diagram below O is the centre of the circle and $S\hat{T}P = 90^o$. If $\hat{R} = 43°$ calculate the sizes of each of the following angles. No reasons required.



(1) $\hat{O}_1$ (1)

$\hat{O}_1 = 2\hat{R}$

$= 86°$ ✓ A

(2) $\hat{S}$ (1)

$\hat{S} = \hat{R} = 43°$ ✓ A

(3) $\hat{P}_1$ (1)

$\hat{P}_1 = \dfrac{180° - 86°}{2}$

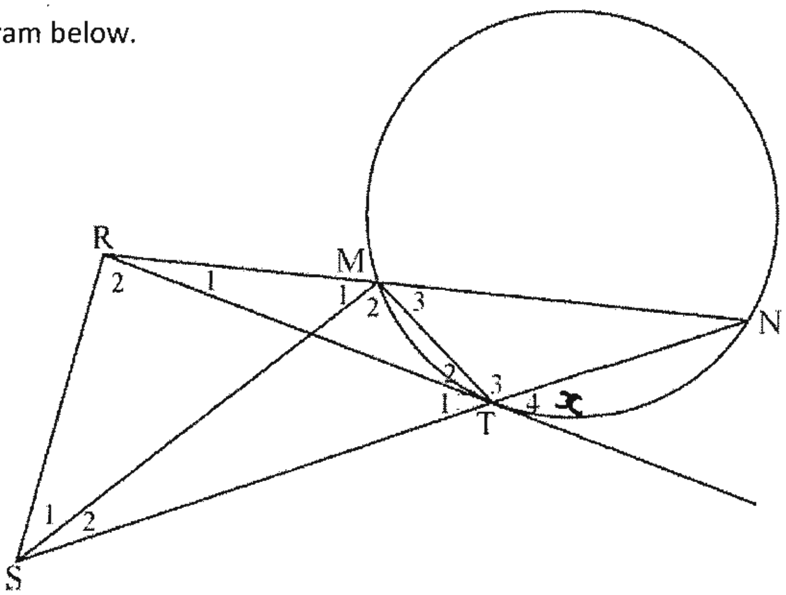
$= 47°$ ✓ A

(4) $\hat{P}_3$ (1)

$\hat{P}_3 = 180° - 90° - \hat{S}$

$= 47°$ ✓ A

(c) Refer to the diagram below.



RT is a tangent to the circle at T. Lines MN and NT are produced to R and S respectively, so that RS = RT. S and M are joined. Let $\hat{T}_4 = x$. Supply proper reasons.

(1) Give 3 other angles that are equal to x. (6)

$\hat{T}_4 = \hat{T}_1 = x$ ✓ ✓vert. opp ∠'s
$\hat{T}_4 = \hat{M}_3 = x$ ✓ ✓tan-chord
$\hat{T}_1 = \hat{S}_1 + \hat{S}_2 = x$ ✓ ✓ isos △; RS = RT

(2) Prove that RSTM is a cyclic quadrilateral (3)

$\hat{M}_3 = \hat{S}_1 + \hat{S}_2 = x$ ✓ proved above

∴ RSTM ✓a cyclic quad. ext ✓∠ of quad = int opp ∠.

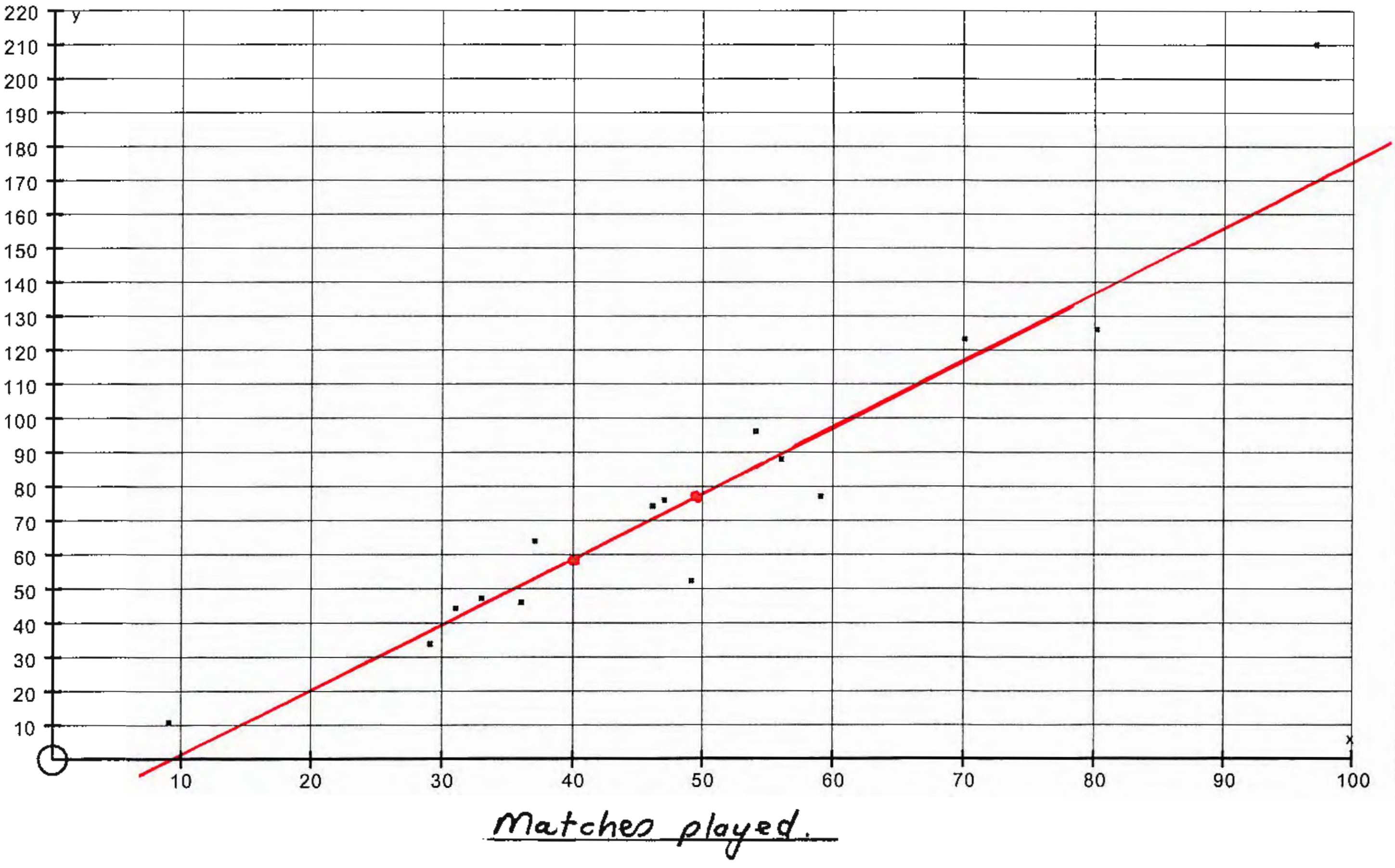
SECTION B [75]

QUESTION 5 [10]

The all-time rankings in World Cup tournaments of fifteen countries are depicted below. It indicates the matches played (MP) by that country and the amount of goals scored (GS).

	Argentina	Belgium	Brazil	Chile	Czech Republic	England	France	Italy	Mexico	Poland	Russia	South Africa	Spain	Sweden	Uruguay
MP	70	36	97	29	33	59	54	80	49	31	37	9	56	46	47
GS	123	46	210	34	47	77	96	126	52	44	64	11	88	74	76

The scatter plot representing the relationship between these two sets of data is show below.



(a) Determine the equation of the regression line, give your answer correct to four decimal places. (2)

A A

$y = -24,2974 + 2,0907x$

(b) Discuss with reasons the correlation of this data. (2)

A A

$r = 0,95$ Strong positive correlation

(c) Germany has played in 99 world cup matches, predict the number of goals they should have scored. (1)

$y = -24,2974 + 2,0907(99)$

$= 182,68$

$\therefore$ 183 goals m

(d) Is your prediction in (c) considered interpolation or extrapolation? Explain. (2)

Interpolation A

99 lies within the data range. A

(e) Sketch the line of best fit on the given scatterplot showing at least 2 significant points. (3)

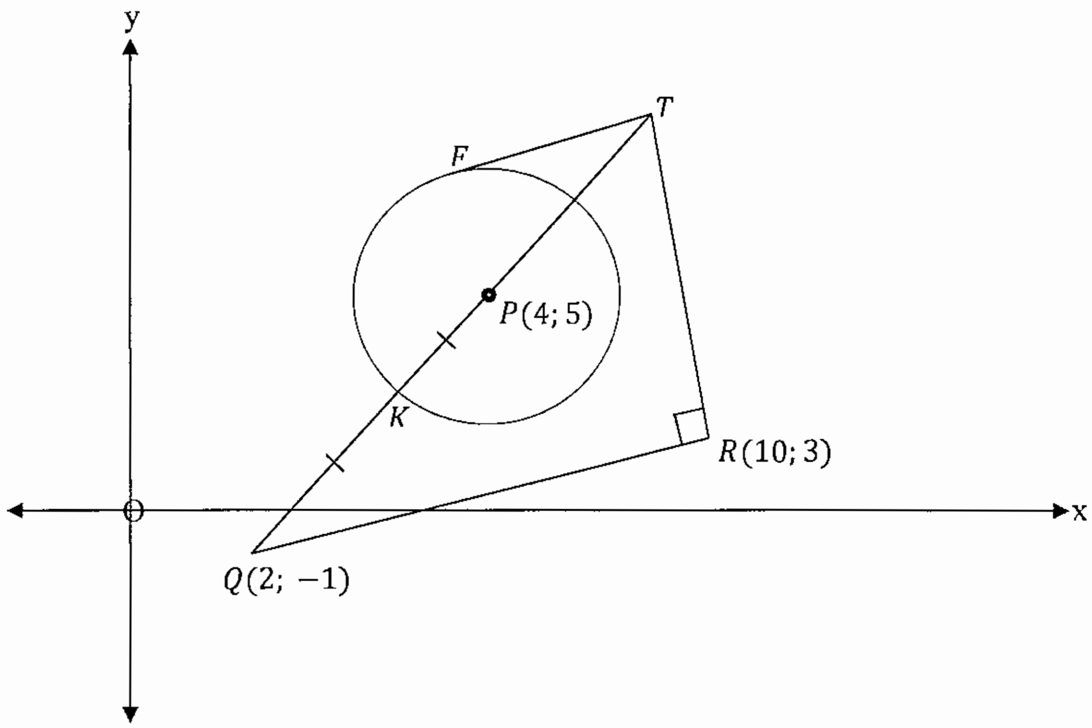
A m line

$(\bar{x}\,;\,\bar{y}) = (49\,;\,78)$

$(40\,;\,59)$ m

QUESTION 6 [17]

P(4; 5), Q(2; −1) and $R(10;3)$ are given points in the figure alongside. QP is produced to T and $TR \perp QR$. The circle with centre P passes through K, the midpoint of PQ.



(a) Determine:

(1) the co-ordinates of K. (2)

$K\left(\frac{4+2}{2} ; \frac{-1+5}{2}\right)$

✓A ✓A

$K(3 ; 2)$

(2) the equation of the circle. (3)

$(x-a)^2 + (y-b)^2 = r^2$ ✓A

$(x-4)^2 + (y-5)^2 = r^2$

Subs $(3;2)$

$(3-4)^2 + (2-5)^2 = r^2$ ✓m

$10 = r^2$ ✓A

$(x-4)^2 + (y-5)^2 = 10$

(3) the equation of the line QP. (3)

$m_{QP} = \frac{-1-5}{2-4}$

$= 3$ ✓A

$y - y_1 = m(x - x_1)$ ✓m

$y - 5 = 3(x - 4)$ ✓

$y = 3x - 7$ ✓A

(b) Show by calculation that T is the point $(6;11)$ if the equation of TR is $y = -2x + 23$. (4)

$y = -2x + 23$ $\qquad$ $y = 3x - 7$

✓m

$3x - 7 = -2x + 23$

$5x = 30$

$x = 6$ ✓CA

$y = 3(6) - 7$

$= 11$ ✓m

$\therefore T(6;11)$ ✓CA

(c) Determine (in surd form) the length of the tangent TF. (5)

$FT^2 = PT^2 - PF^2$ A ✓m

$= (\sqrt{40})^2 - (\sqrt{10})^2$ ✓

$= 40 - 10$

$= 30$ ✓

$FT = \sqrt{30}$ ✓m

$PT = \sqrt{(4-6)^2 + (5-11)^2}$

$= \sqrt{40}$ ✓A

$= 2\sqrt{10}$

QUESTION 7 [24]

(a) Simplify: $\dfrac{2+cos\beta-cos2\beta}{3sin\beta-sin2B}$ (4)

$= \dfrac{2 + \cos B - (2\cos^2 B - 1)}{3\sin B - 2\sin B\cos B}$ ✓A ✓A

$= \dfrac{3 + \cos B - 2\cos^2 B}{\sin B(3 - 2\cos B)}$

$= \dfrac{(1 + \cos B)(3 - 2\cos B)}{\sin B(3 - 2\cos B)}$ ✓A ✓A

$= \dfrac{(1 + \cos B)}{\sin B}$

(b) (1) Determine the general solution of:

$\cos 30^0 \cos 2x - \sin 30^0 \sin 2x = \cos 3x$ (5)

✓A

$\cos(30^\circ + 2x) = \cos 3x$

$30^\circ + 2x = \pm 3x + k \cdot 360^\circ;\ k \in \mathbb{Z}$

✓A ✓A

$30^\circ + 2x = 3x + k \cdot 360^\circ$ or $30 + 2x = -3x + k \cdot 360^\circ$

$-x = -30^\circ + k \cdot 360^\circ$ $\quad$ $5x = -30^\circ + k \cdot 360^\circ$

$x = 30^\circ - k \cdot 360^\circ$ ✓m $\quad$ $x = -6^\circ + k \cdot 72^\circ$ ✓m

(2) Hence determine x if $x \in [-90°;\ 90°]$ (2)

$x = -6°\ ;\ 30°\ ;\ 66°\ ;\ -78°$ ✓A ✓A

(c) Prove that: $\dfrac{\sin 4x}{\sin x} = 8cos^3x - 4cosx$ (5)

$LHS = \dfrac{\sin 2(2x)}{\sin x}$ ✓A

$= \dfrac{2\sin 2x\cos 2x}{\sin x}$ ✓A

$= \dfrac{4\sin x\cos x\cos 2x}{\sin x}$ ✓A

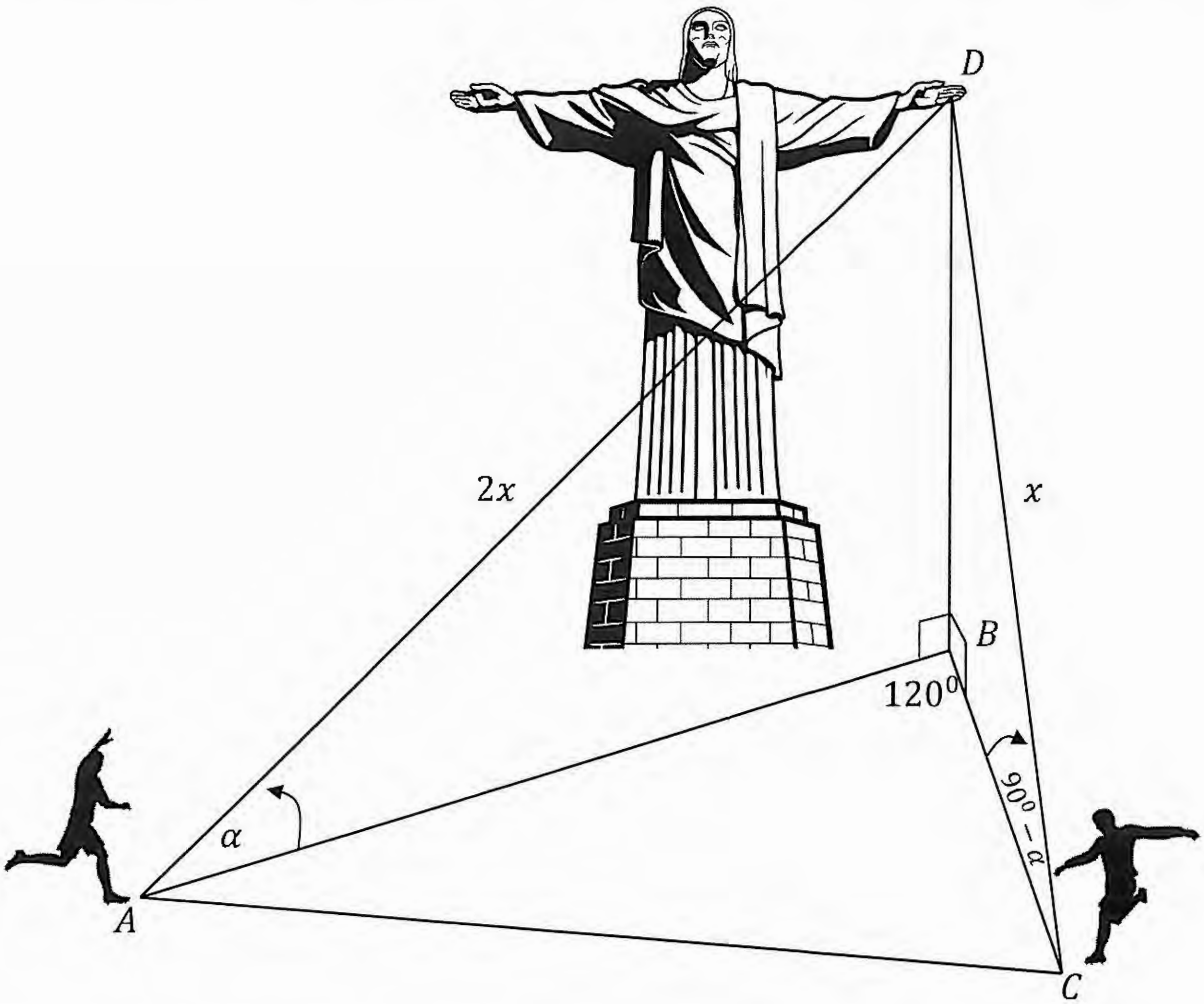
$= 4\cos x\left(2\cos^2 x - 1\right)$ ✓A

$= 8\cos^3 x - 4\cos x$ ✓A

(d) Christ the Redeemer is a statue of Jesus Christ in Rio de Janeiro, Brazil; and was considered the largest Art Deco statue in the world from 1931 until 2010.

It is located at the peak of the Corcovado mountain overlooking the city. A symbol of Brazilian Christianity, the statue has become an icon for Rio de Janeiro and Brazil.

As part of the build up to the soccer world cup, Messi and Ronaldo decided to have a competition to see who could kick a soccer ball into the hands of the statue. Both players got to choose where they would kick the ball from. The diagram below is an illustration of their positions.



The base of the statue, Messi (A) and Ronaldo (C) are all in the same horizontal plane. The angle of elevation from Messi (A) to the left hand (D) of the statue is α. The angle of elevation from Ronaldo (C) to the left hand (D) of the statue is $(90^0 - \alpha)$. The distance the ball will travel from Messi (A) to the left hand is $2x$ metres and from Ronaldo (C) to the left hand is x metres. $A\hat{B}C = 120^0$.

(1) Show that the distance between Messi (A) and B is given by $AB = 2x\cos\alpha$ and that the distance between Ronaldo (C) and B is given by $BC = x\sin\alpha$. (2)

In $\triangle ABD$

$$\frac{AB}{\sin(90^\circ-\alpha)} = \frac{2x}{\sin 90^\circ}$$

$AB = 2x\cos\alpha$ ✓A

In $\triangle BCD$

$$\frac{BC}{\sin\alpha} = \frac{x}{\sin 90^\circ}$$

$BC = x\sin\alpha$ ✓A

(2) Show that the distance between Messi and Ronaldo is given by $AC = x\sqrt{1 + 3\cos^2\alpha + \sin 2\alpha}$. (6)

✓A Formula ✓ Subs M

$AC^2 = (2x\cos\alpha)^2 + (x\sin\alpha)^2 - 2(2x\cos\alpha)(x\sin\alpha)\cos 120^\circ$

$= 4x^2\cos^2\alpha + x^2\sin^2\alpha - 4x^2\sin\alpha\cos\alpha\,(-\tfrac{1}{2})$

✓A

$= 4x^2\cos^2\alpha + x^2\sin^2\alpha + 2x^2\sin\alpha\cos\alpha$

✓M

$= x^2(4\cos^2\alpha + \sin^2\alpha + 2\sin\alpha\cos\alpha)$

✓A

$= x^2(4\cos^2\alpha + 1 - \cos^2\alpha + \sin 2\alpha)$

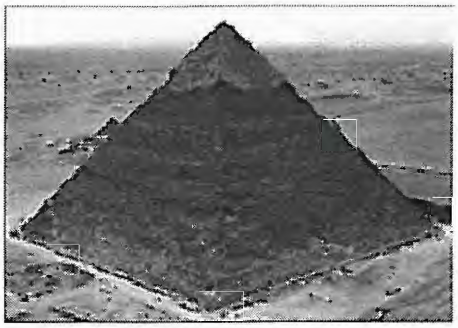
$= x^2(1 + 3\cos^2\alpha + \sin 2\alpha)$

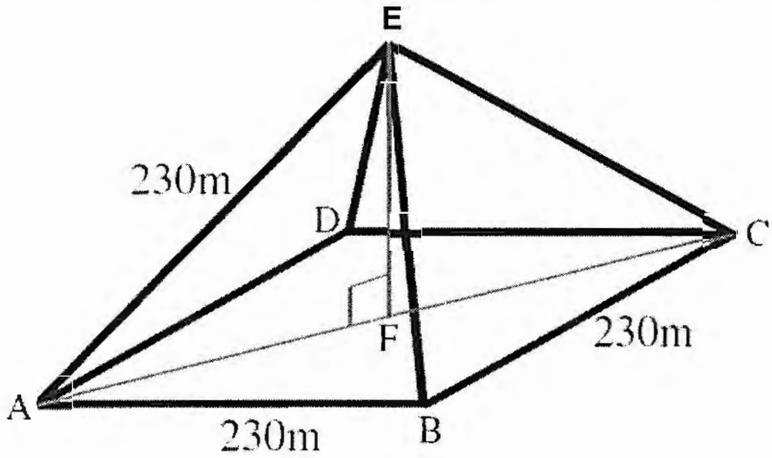
✓M

$AB = x\sqrt{1 + 3\cos^2\alpha + \sin 2\alpha}$

QUESTION 8 **[6]**

Below is the pyramid of The Pyramid of King Khafra, at Giza. It is made out of a square base and 4 equilateral triangles.





(a) Show that the height of the pyramid is 162,6 m. (4)

$AC^2 = 230^2 + 230^2$
$= 105\,800$
$AC = 325,27$ ✓A

$\therefore AF = FC = 162,63$ ✓M

$EF^2 = 230^2 - 162,63^2$
$= 26\,451,48$
$EF = 162,64$ ✓A

$\therefore EF = 162,6$ ✓A

(b) Determine the volume of the pyramid if: $V = \frac{1}{3}(Area\ of\ base) \times Height.$ (2)

✓M

$V = \frac{1}{3}(230 \times 230) \times 162,6$

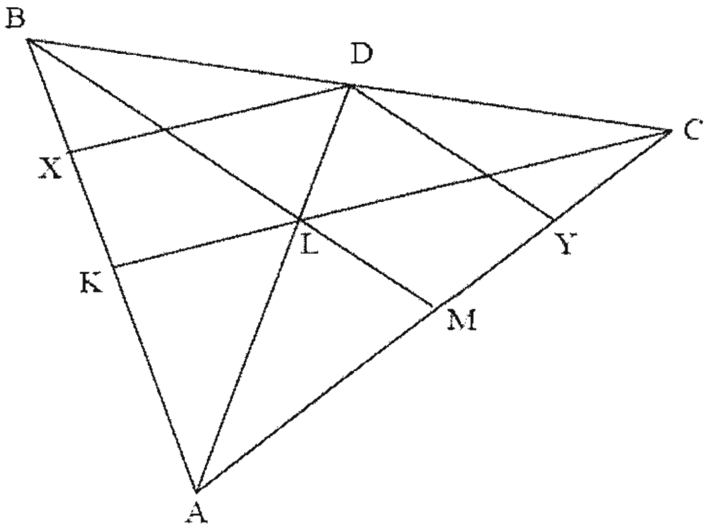
$= 2\,867\,180\ m^3$ ✓A

QUESTION 9 **[6]**

(a) Complete the following sentence:
If a line cuts two sides of a triangle proportionally, then ...... (1)

that line is // to the third side.

(b) In the figure K is a point on AB, such that AK : KB = 3 : 2. L is a point on AD such that AL : LD = 3 : 1. KX = XB.



Prove that: $\dfrac{\mathbf{BD}}{\mathbf{BC}} = \dfrac{\mathbf{1}}{\mathbf{2}}$ (5)

$\frac{AK}{KB} = \frac{3}{2}$ given

but $KX = XB$ given

$\therefore \frac{AK}{KX} = \frac{3}{1}$

but $\frac{AL}{LD} = \frac{3}{1}$ given

$\therefore \frac{AK}{KX} = \frac{AL}{LD} = \frac{3}{1}$

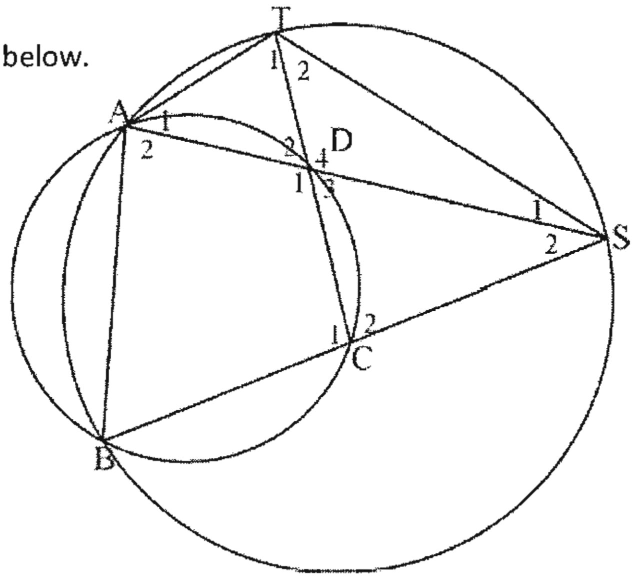
$\therefore KL \parallel XD$ line divides sides in prop.

$\therefore \frac{BD}{DC} = \frac{BX}{XK} = \frac{1}{1}$ line // to one sides of Δ

$\therefore \frac{BD}{BC} = \frac{1}{2}$

QUESTION 10 **[12]**

Refer to the diagram below.



Two circles intersect at A and B. Chords AS and BS of the larger circle meet the smaller circle at D and C respectively. CD produced meets the larger circle at T. AT and TS are joined.

(a) Prove that $\hat{T}_1 + \hat{T}_2 = \hat{D}_4$ (3)

$\hat{T}_1 + \hat{T}_2 + \hat{B} = 180°$ opp ∠'s of cyclic quad.

$\hat{D}_1 + \hat{B} = 180°$ opp ∠'s of cyclic quad.

$\therefore \hat{T}_1 + \hat{T}_2 = \hat{D}_1$

but $\hat{D}_1 = \hat{D}_4$

$\therefore \hat{T}_1 + \hat{T}_2 = \hat{D}_4$

(b) Prove $\Delta STD \mid\mid\mid \Delta SAT$ (3)

In ΔSTD and ΔSAT :

1) $\hat{S}_1 = \hat{S}_1$ ✓ common

2) $\hat{D}_4 = \hat{T}_1 + \hat{T}_2$ ✓ proved

3) $\therefore \hat{T}_2 = \hat{A}_1$ int ∠ of Δ

$\therefore \Delta STD \mid\mid\mid \Delta SAT$ ✓ AAA

(c) Prove $\Delta SDC \mid\mid\mid \Delta SBA$ (3)

In ΔSDC and ΔSBA

1) $\hat{S}_2 = \hat{S}_2$ ✓ common

2) $\hat{D}_3 = \hat{B}$ ✓ ext ∠ of cyclic quad

3) $\hat{C}_2 = \hat{A}_2$ int ∠ of Δ

$\therefore \Delta SDC \mid\mid\mid \Delta SBA$ ✓ AAA

(d) $TS^2 = BS.CS$ (3)

$\triangle STD \parallel\mid \triangle SAT$

$\frac{ST}{SA} = \frac{SD}{ST}$

$\therefore TS^2 = SD \cdot SA$ ✓

$\triangle SDC \parallel\mid \triangle SBA$

$\frac{SD}{SB} = \frac{SC}{SA}$

$\therefore SD \cdot SA = SC \cdot SB$ ✓

$\therefore TS^2 = SC \cdot SB$

$= CS \cdot BS.$ ✓