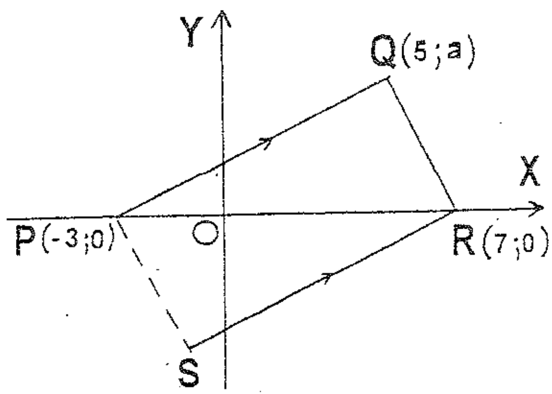
## SECTION A

### Question 1

In the diagram below, triangle PQR is right-angled at Q with coordinates

$P(-3\,;0)\,;\ Q(5\,;a)\,;\,R(7\,;0)$ and $PQ \| SR$.




(a) Show clearly by calculation that $a = 4$. (2)

$m_{PQ} \times m_{QR} = -1$

$\frac{a}{8} \times \frac{a}{-2} = -1$

$a^2 = 16$

$a = 4$

(b) Write down the gradient of SR (2)

$m_{SR} = m_{QR} = \frac{4}{5+3} = \frac{1}{2}$

(c) Determine the equation of SR. (2)

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{1}{2}(x - 7)$

$y = \frac{1}{2}x - \frac{7}{2}$

(d) If it is further given that PQRS is a rectangle, calculate the coordinates of S. (4)

Eq PS $\quad m_{PS} = m_{QR} = -2$

$y - 0 = -2(x + 3)$

$y = -2x - 6$

Sub eq ① ~~into~~ eq ②

$-2x - 6 = \frac{1}{2}x - \frac{7}{2}$

$-4x - 12 = x - 7$

$-5x = 5$

$x = -1$

$y = -2(-1) - 6 = -4$

$S(-1; -4)$

OR $\quad m_{PS} = \frac{y - 0}{x + 3} = \frac{0 - 4}{7 - 5}$ [10]

$= \frac{y}{x + 3} = \frac{-4}{2}$

$y = -4 \quad x = -1$

$S(-1; -4)$

OR $\quad P(x + 2; y - 4) \rightarrow S(-3 + 2; 0 - 4) = S(-1; -4)$

## Question 2

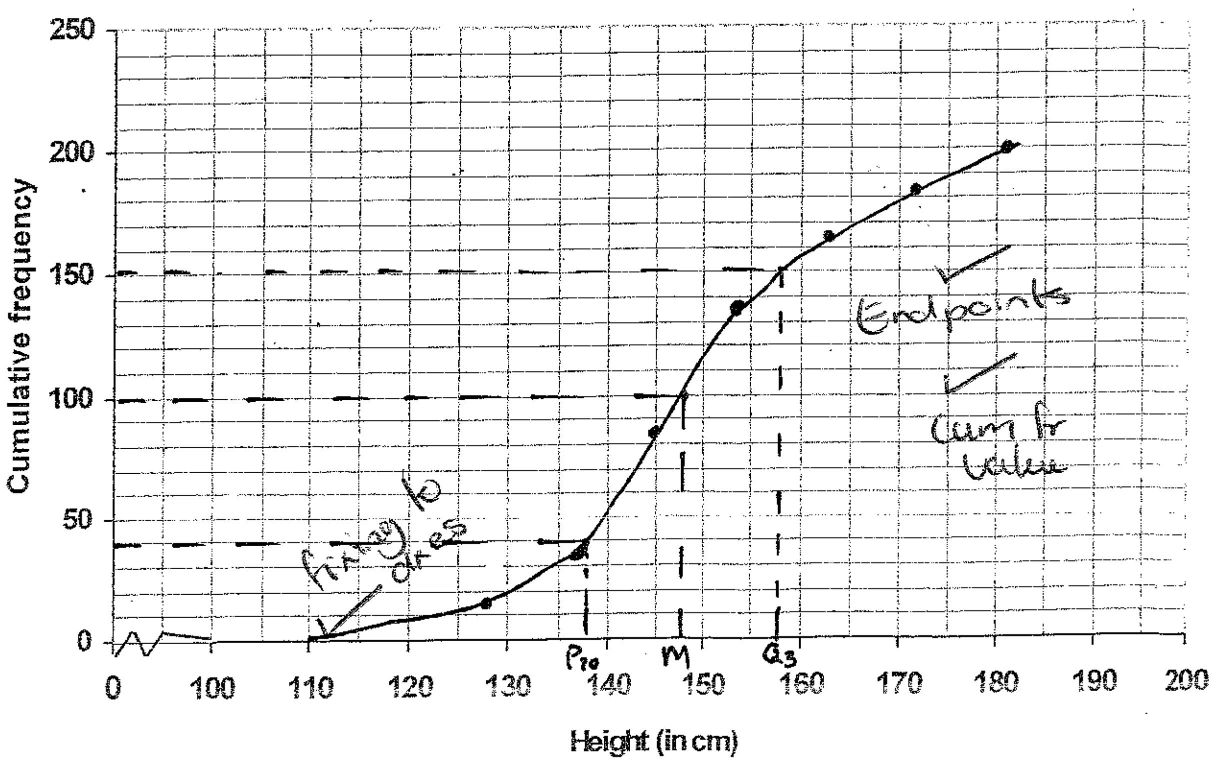
The heights ($h$) of the learners at Grantleigh College in Grade 10, 11 and 12 were recorded in the table below.

(a) Complete the table by filling in the columns which record the midpoint of each interval and the cumulative frequency of the given data.

| Height (in cm) | Frequency ( f ) | Midpoint | Cumulative frequency |
|---|---|---|---|
| $118 \leq h < 127$ | 16 | 122,5 | 16 |
| $127 \leq h < 136$ | 26 | 131,5 | 42 |
| $136 \leq h < 145$ | 42 | 140,5 ✓ | 84 |
| $145 \leq h < 154$ | 54 | 149,5 ✓ | 138 ✓ |
| $154 \leq h < 163$ | 26 | 158,5 | 164 |
| $163 \leq h < 172$ | 22 | 167,5 | 186 |
| $172 \leq h < 181$ | 14 | 176,5 | 200 |

(3)

(b) Draw a cumulative frequency (ogive) for this data on the grid provided below. (3)



c) Estimate, from your graph, the median, upper quartile, and twentieth percentile of the heights of the students in grade 10, 11 and 12 at Grantleigh College. Show, on your graph, where you obtained your readings. (6)

Median : $n = \frac{1}{2}(201) = 100{,}5$ Median = 147 ✓ca

read at M ✓M

$Q_3$ : $n = \frac{3}{4}(201) = 150{,}75$ $Q_3 = 157$ ✓ca read at $Q_3$ ✓M

$P_{20}$ : $n = 0{,}2(201) = 40{,}2$ $P_{20} = 137$ ✓ca read at $P_{20}$ ✓M

d) Determine the modal interval for the heights of the learners. (1)

$145 \leq h < 154$ ✓a

e) Calculate the estimated mean of the heights of the students in the sample. (3)

mean = $(122{,}5 \times 16 + 131{,}5 \times 26 + 140{,}5 \times 42 + 149{,}5 \times 54 + 158{,}5 \times 26 +$
$167{,}5 \times 22 + 176{,}5 \times 14) \div 200$
$= 28680$ ✓a $\div 200$ ✓a $= 143{,}4$ cm ✓ca

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**QUESTION 3**

(a) (1) Prove that : $\tan\theta + \dfrac{\cos\theta}{\sin\theta} = \dfrac{2}{\sin 2\theta}$ (4)

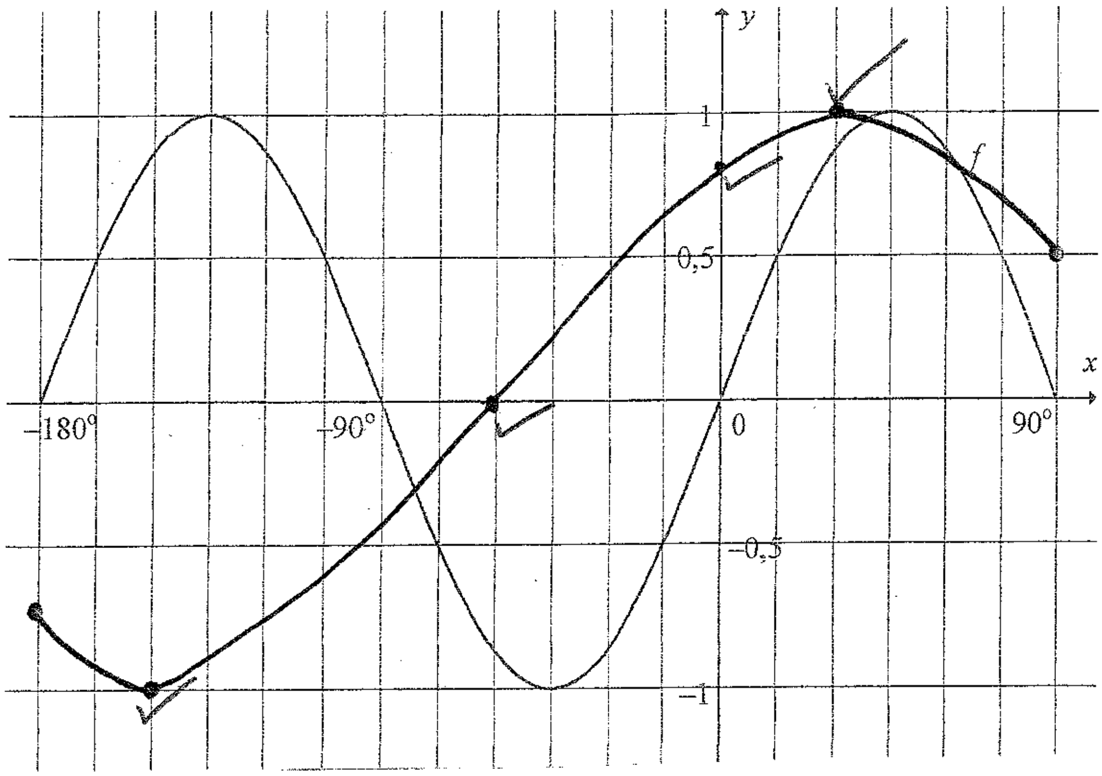
$$\text{LHS} = \tan\theta + \frac{\cos\theta}{\sin\theta}$$
$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$
$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$
$$= \frac{1}{\sin\theta\cos\theta}$$
$$= \frac{2}{2\sin\theta\cos\theta}$$
$$= \frac{2}{2\sin\theta\cos\theta} = \frac{2}{\sin 2\theta} = \text{RHS}$$

(2) Simplify the following without the use of a calculator:

$$\frac{\sin(-\beta) + \sin(360°-\beta)}{\sin(180°-\beta) + \sin 180°}$$ (3)

$$\frac{-\sin\beta + (-\sin\beta)}{\sin\beta + 0}$$
$$= \frac{-2\sin\beta}{\sin\beta} = -2$$

(b) The graph of $f(x) = \sin 2x$ for $-180° \leq x \leq 90°$ is shown in the sketch below:




(1) Write down the range of $f$. (1)

$y \in [-1; 1]$ a

(2) Determine the period of $f\left(\frac{3}{2}x\right)$. (2)

$f\left(\frac{3}{2}x\right) = \sin 2\left(\frac{3}{2}x\right) = \sin 3x$

$\text{Period} = \frac{360°}{3} = 120°$

(3) Draw the graph of $g(x) = cos(x - 30°)$ for $-180° \leq x \leq 90°$ on the above system of axes with the graph of $f(x) = sin2x$. Show the turning points and the x-intercepts clearly on the graph. (4)

(4) Hence, or otherwise, determine the values of $x$ in the interval $-180° \le x \le 90°$ for which $f(x).g(x) < 0$. (2)

ca ca

$x \in (-180°; -90°) \cup (-60°; 0°)$

$-180° < x < -90°$ OR $-60° < x < 0°$

(5) Describe the transformation that the graph $f$ has to undergo to form $y = \sin(2x - 60°)$. (2)

$y = \sin(2x - 60°)$

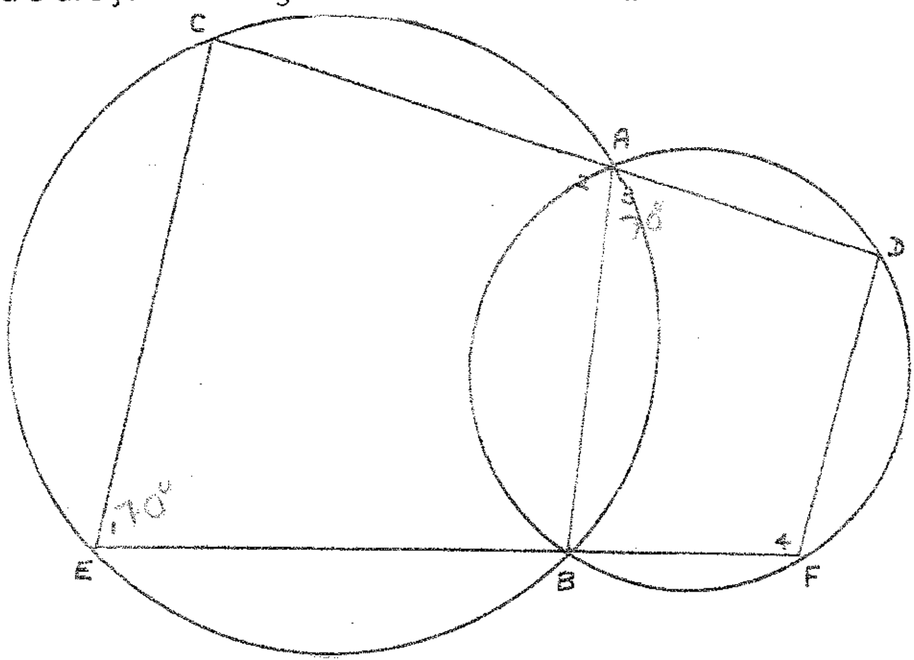
$= \sin 2(x - 30°)$

horizontal shift (translation) to the right of 30° ca ca

**[18]**

## QUESTION 4

(a) Two circles intersect at A and B. CAD and EBF are straight lines. A and B are joined. $\hat{A}_3 = 70°$ Prove $EC \| FD$. (5)




$\hat{A}_3 = \hat{E}_1 = 70°$ (Ext ∠ of cyclic quad ABEC)

$\hat{A}_3 + \hat{F}_4 = 180°$ (int opp ∠ of cyclic quad ADFB)

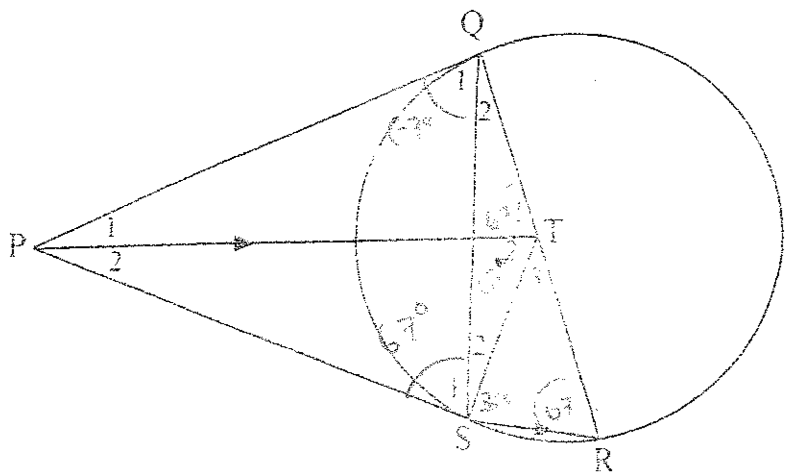
$\therefore \hat{F}_4 = 110°$

$\hat{E}_1 + \hat{F}_4 = 110° + 70° = 180°$

$\therefore$ EC ∥ DF (co-int ∠s supp)

QUESTION 5

a) In the figure, PQ and PS are tangents to the circle through the Points Q, S and R. $PT \parallel SR$ with T on QR and $\hat{S_1} = 67°$




(1) Name, with reasons, THREE other angles in the given figure each equal to 67°. (3)

$\hat{S}_1 = \hat{Q}_1 = 67°$ (tan from common pt)

$\hat{Q}_1 = \hat{R} = 67°$ (tan-chord thm)

$\hat{T}_1 = \hat{R} = 67°$ (corresp ∠'s PT // SR)

(2) If T is the centre of the circle, determine the size of $\widehat{T}_2$. (3)

$\hat{T}_2 = \hat{S}_3$ (alt ∠'s PT ∥ SR)

$\hat{S}_3 = \hat{R} = 67°$ (isos △)

$\therefore \hat{T}_2 = 67°$ ✓

(3) Why is PQTS a cyclic quadrilateral? (1)

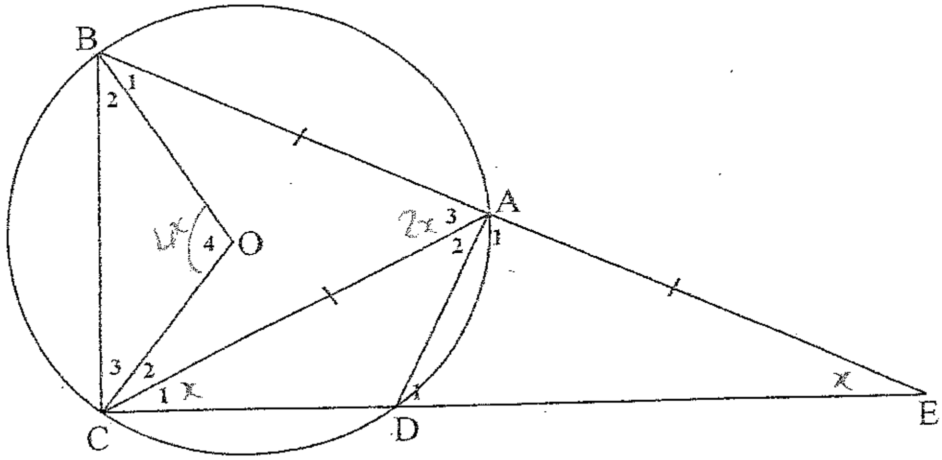
line seg subtends = ∠'s ✓

OR

Conv. of ∠'s in same seg

[12]

(b) In the figure below, ABCD is a cyclic quadrilateral of the circle with centre O and BA = CA. BA and CD produced intersect at E. BA = AE and $\widehat{O_4} = 4x$.




(1) Determine, with reasons, the size of $\hat{E}$ in terms of $x$. (3)

$\hat{A}_3 = \frac{1}{2}\hat{O}_4 = \frac{1}{2}(4x) = 2x$ (∠ at centre = 2x ∠ at circle)

$\hat{C}_1 = \hat{E}$ (isos △)

$\hat{A}_3 = \hat{C}_1 + \hat{E}$ (ext ∠ of △)

$\hat{C}_1 = \hat{E} = x$

(2) Hence, prove that ED is a diameter of circle AED. (4)

$\hat{B} = \hat{C} = \frac{180° - 2x}{2}$ (sum ∠'s of isos △)

∴ $\hat{B} = \hat{C}_{2+3} = 90° - x$

$\hat{A}_1 = \hat{C} = 90° - x + (x)$ (ext ∠ of cyclic quad)

$= 90°$

∴ ED is the diameter (line seg. subtends 90°)

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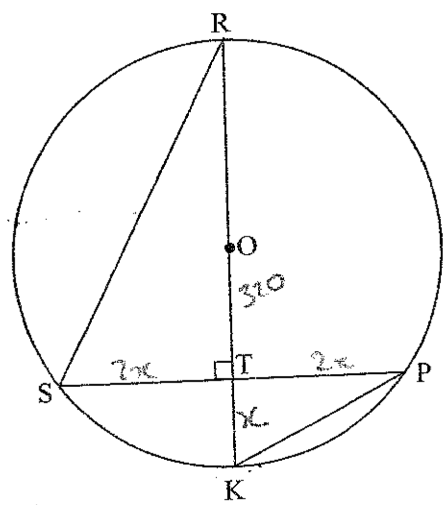
## QUESTION 6

(a) Complete the following statement: A line drawn from the centre of a circle to the midpoint of a chord is perpendicular to the chord . (1)

(b) In the diagram below, O is the centre of the circle with diameter RK. $PS \perp RK$ . RK intersects PS at T.




(1) If $PS = 4x$, write down the length of ST in terms of $x$. (1)

ST = 2x (perp from centre to chord)

(2) Prove that $\Delta RST \,|||\, \Delta PKT$ . (4)

In $\triangle RST$ and $\triangle PKT$

1) $\hat{T}_1 = \hat{T}_2 = 90^\circ$ (vert opp ∠s)

2) $\hat{R} = \hat{P}$ (∠'s in same segment)

3) $\hat{S} = \hat{K}$ ( " )

$\therefore \triangle RST \,|||\, \triangle PKT$ (AAA)

(3) If it is further given that $TK = x$ and $RT = 320\,mm$ , calculate the value of $x$. (3)

$\frac{RT}{PT} = \frac{ST}{KT}$ ($\triangle RST \,|||\, \triangle PKT$)

$\frac{320}{2x} = \frac{2x}{x}$

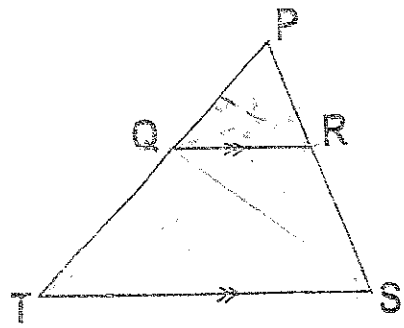
$160 = 2x$

$x = 80\,mm$

[9]

## QUESTION 7

(a) Use the diagram below to prove the theorem that states that a line drawn parallel to one side of a triangle divides the other two sides proportionally. In the diagram, $QR \parallel TS$. Prove that $\frac{PQ}{QT} = \frac{PR}{RS}$ (6)




Given: $\triangle PTS$ with $QR \parallel TS$

RTP: $\frac{PQ}{QT} = \frac{PR}{RS}$

PROOF: Join QS and TR

Draw in height h from R relative to PT

and height k from Q relative to PS

$$\frac{\text{Area } \triangle PQR}{\text{Area } \triangle QRT} = \frac{\frac{1}{2} PQ \times h}{\frac{1}{2} QT \times h} = \frac{PQ}{QT}$$

$$\frac{\text{Area } \triangle PQR}{\text{Area } \triangle QRS} = \frac{\frac{1}{2} \times PR \times k}{\frac{1}{2} \times RS \times k} = \frac{PR}{RS}$$

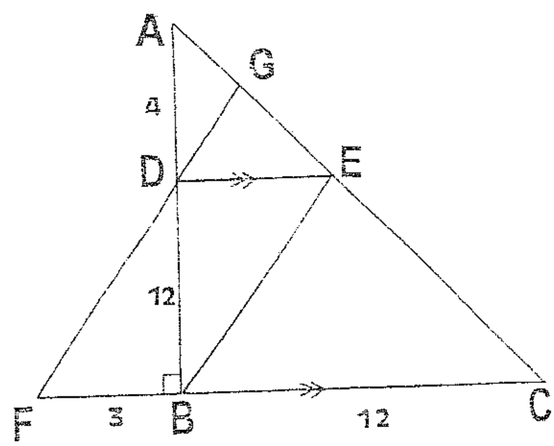
Area of $\triangle PQR$ is common and

Area $\triangle QRT$ = Area $\triangle$ QRS (same base and height)

$$\therefore \frac{\text{Area } \triangle PQR}{\text{Area } \triangle QRT} = \frac{\text{Area } \triangle PQR}{\text{Area } \triangle QRS}$$

$$\therefore \frac{PQ}{QT} = \frac{PR}{RS}$$

(b) In the diagram below, $\Delta ABC$ is right-angled at B. The point D lies on AB and the point E lies on AC such that $DE \| BC$. CB is produced to F. FD is drawn and produced to G, a point on AC. BC = 12 units, AD = 4 units, DB = 12 units and FB = 3 units.




(1) Show that AC = 20 units. (2)

$AC^2 = AB^2 + BC^2$ (Pythagoras)

$AC = \sqrt{16^2 + 144}$

$= 20$

(2) Calculate, stating reasons, the lengths of AE and EC. (4)

$\frac{AE}{EC} = \frac{AD}{DB} = \frac{4}{12} = \frac{1}{3}$ (line ∥ to one side of △)

$AE = \frac{1}{4} \times 20 = 15$ units

∴ $EC = 15$ units

(3) If it is now further given that $GE = 3\,\frac{3}{4}$ units, prove that DEBF is a parallelogram. (5)

$AG = 5 - 3\frac{3}{4} = 1{,}25$ units

$\frac{AG}{GE} = \frac{1{,}25}{3{,}75} = \frac{1}{3}$

$\frac{AD}{DB} = \frac{4}{12} = \frac{1}{3}$

∴ $\frac{AG}{GE} = \frac{AD}{DB}$

∴ DG ∥ BE (line divides sides of △ prop.)

∴ FD ∥ BE

and DE ∥ FB (given)

∴ DEBF is a parm (both prs of opp sides ∥)

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## SECTION B

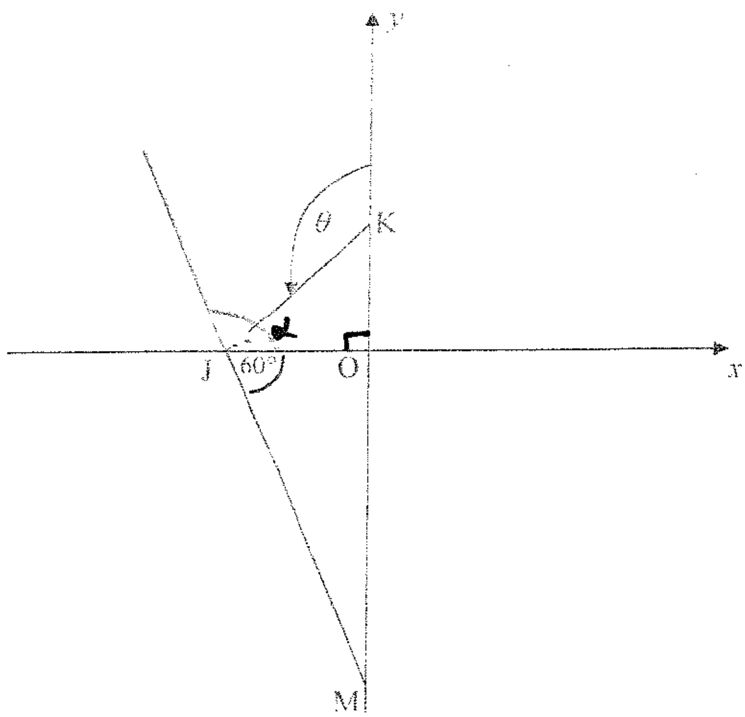
## QUESTION 8

(a) In the diagram below, triangle JKM is shown. J lies on the x-axis.

K and M lie on the y-axis. The equation of JK is given by

$y = \frac{1}{\sqrt{3}}x + 6$ . Line JK makes an angle of $\theta$ with the y-axis as

is shown and $M\hat{J}O = 60°$ .




1) Determine the size of $\theta$. (3)

$\tan \alpha = \frac{1}{\sqrt{3}}$

$\alpha = 30°$

$\theta = 90° + 30° = 120°$ (ext ∠ of △)

(2) Show that the equation of JM is given by $y = -\sqrt{3}\,x - 18$.

(5)

TO FIND J : let $y=0$ $y = \frac{1}{\sqrt{3}}x + 6$

$0 = \frac{1}{\sqrt{3}}x + 6$

$x = -6\sqrt{3}$

$m = \tan 120^\circ = -\sqrt{3}$ OR JK ⊥ JM ∴ $m_{JM} = -\sqrt{3}$

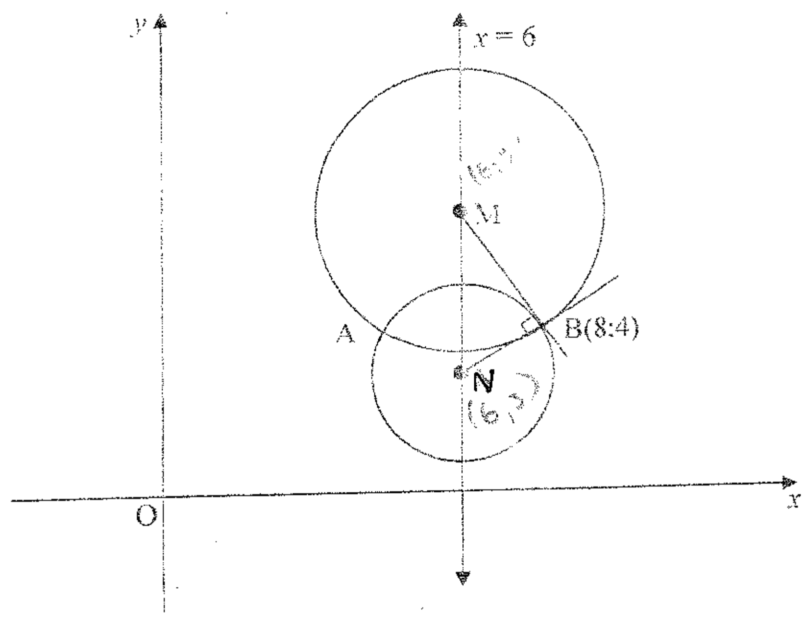
$y - y_1 = m(x - x_1)$

$y - 0 = -\sqrt{3}(x + 6\sqrt{3})$

$y = -\sqrt{3}x - 18$

(b) In the diagram, two circles intersect at $A$ and $B(8 ; 6)$ as shown. 4 4

The centres of the circles lie on the line $x = 6$. The tangents at $B$ to the circles are perpendicular to one another and pass through the centres of the circles.




If the equation of the smaller circle is given by

$x^2 + y^2 - 12x - 6y = -40$, determine:

(1) the coordinates of N. (3)

$x = 6$ $\quad x^2 - 12x + 36 + y^2 - 6y + 9 = -40 + 36 + 9$

$(x - 6)^2 + (y - 3)^2 = 5$

$N(6 ; 3)$

(2) the equation of the circle M. (7)

$x = 6$

$m_{MB} \times m_{NB} = -1$ M

$\frac{4-3}{8-6} \times \frac{y-4}{6-8} = -1$ a

$\frac{1}{2} \times \frac{y-4}{-2} = -1$

$\frac{y-4}{-4} = -1$

$y - 4 = +4$

$y = 8$

$M(6;8)$

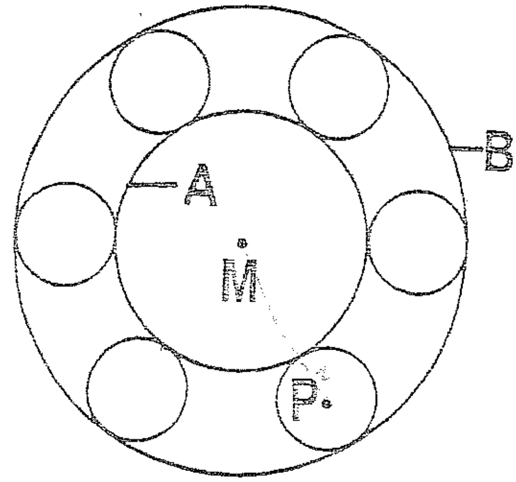
$r = MB = \sqrt{(4)^2 + (-2)^2}$

$= \sqrt{16+4} = \sqrt{20}$

$(x-6)^2 + (y-8)^2 = 20$

[18]

## QUESTION 9




The figure shows a cross-section of a wheel bearing. The smaller circles represent ball bearings which roll between the two larger circles A and B which both have centre M.

The equation of circle A is given by:

$$x^2 + y^2 + 3x - 6y = 9$$

P is the centre of one of the ball bearings , each of which has a radius of 2 units.

(a) Determine the coordinates of M, the centre of circle A. (4)

$x^2 + 3x + (\frac{3}{2})^2 + y^2 - 6y + (3)^2 = 9 + \frac{3}{2}^2 + 3^2$

$(x + \frac{3}{2})^2 + (y - 3)^2 = \frac{81}{4}$ ✓

$M(-\frac{3}{2}; 3)$

(b) Find the equation which best describes the path of the centre point $P$ as the small bearing rolls between the two larger circles. Give the answer in the form: $(x-a)^2 + (y-b)^2 = r^2$.

(4)

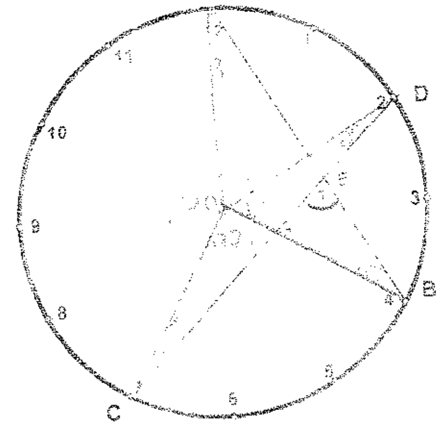
$r = \frac{9}{2} + 2 = \frac{13}{2}$ ✓

$M(-3/2 ; 3)$

$(x + 3/2)^2 + (y - 3)^2 = \frac{169}{4}$ ✓

[8]

## QUESTION 10




The diagram shows the face of a clock.
The 12 is joined to the 4 and the 2 to the 7 with straight lines AB and CD respectively.
O is the centre of the clock.

(a) If AO and DO are joined, show that $A\hat{O}D = 60°$. (2)

(No reasons required)

$A\hat{O}D = \frac{360°}{6} = 60°$

(b) If CO and BO are joined, calculate the size of $C\hat{O}B$. (1)

$C\hat{O}B = \frac{360°}{4} = 90°$

(c) Hence, calculate $\widehat{E_1}$, providing reason for your answer. (4)

Join AC

$\hat{A} = \frac{1}{2}\hat{O}_1 = 45°$ ( ∠ at centre = 2 × ∠ at circle)

$\hat{C} = \frac{1}{2}\hat{Q} = 30°$ ( " )

$\hat{E}_1 = \hat{A} + \hat{C} = 75°$ (ext ∠ of △)

[7]

**QUESTION 11**

(a) If $\boldsymbol{cos^2 12° - sin^2 12° = m}$ , express, without the use of a calculator, the following in terms of $\boldsymbol{m}$ . Answers may be left in surd form.

(1) $\cos 24°$ (1)

$= \cos 2(12°) = m$

(2) $\frac{\sqrt{3}}{2} . \cos 6° + \frac{1}{2} . \sin 6°$ (4)

$\cos 30° \cos 6° + \sin 30° \sin 6°$

$= \cos (30° - 6°)$

$= \cos 24°$

$= m$

(b) Determine, without the use of a calculator the general solution to:

$\frac{1}{2}\sin(\theta + 10°) = sin\theta . cos\theta$ (7)

$\frac{1}{2}\sin(\theta + 10°) = \frac{1}{2}\sin 2\theta$

$\sin(\theta + 10°) = \sin 2\theta$

I $\theta + 10° = 2\theta + k360°$ ; $k \in \mathbb{Z}$

$-\theta = -10° + k360°$

$\theta = 10° + k360°$

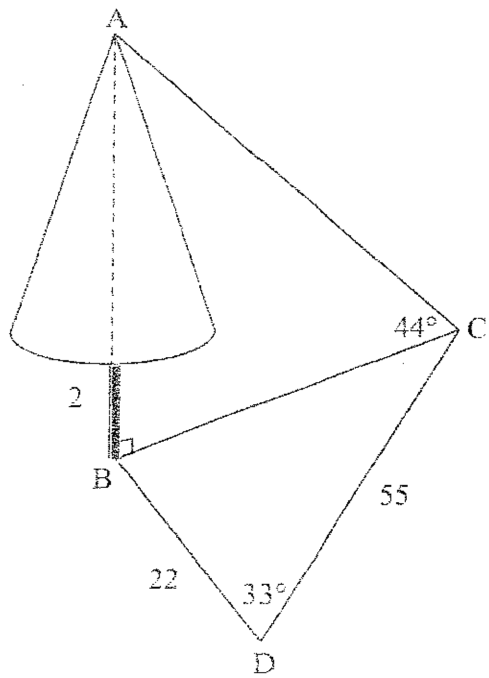
OR

II $\theta + 10° = 180° - 2\theta + k360°$

$3\theta = 170° + k360°$

$\theta = \frac{170°}{3} + k120°$

(c) In the diagram below, a tall tree is roughly illustrated by a right cone. The centre of the circular base of the cone is 2 meters above a point ,B, on the horizontal ground.




D and C are points in the same horizontal plane as B so that $BD = 22$ meters, $DC = 55$ meters and $B\hat{D}C = 33°$. The angle of elevation of the top of the tree, A, from C is $44°$.

(1) Find the height AB of the tree, correct to two decimal places. (6)

$BC^2 = 22^2 + 55^2 - 2(22)(55)\cos 33°$

$BC = 38{,}46$

$\frac{AB}{BC} = \tan 44°$

$AB = 38{,}66 \times \tan 44° = 37{,}14\text{ m}$

(2) If the radius of the base of the cone is 3 m, find the volume of the cone depicting the foliage of the tree in the diagram above.

The volume of a cone is given by:

$$Volume = \frac{base\ area \times height}{3}$$ (3)

$$V = \frac{\pi r^2 \times Ht}{3}$$

$$= \frac{\pi\ (3)^2 (35,14)}{3}$$

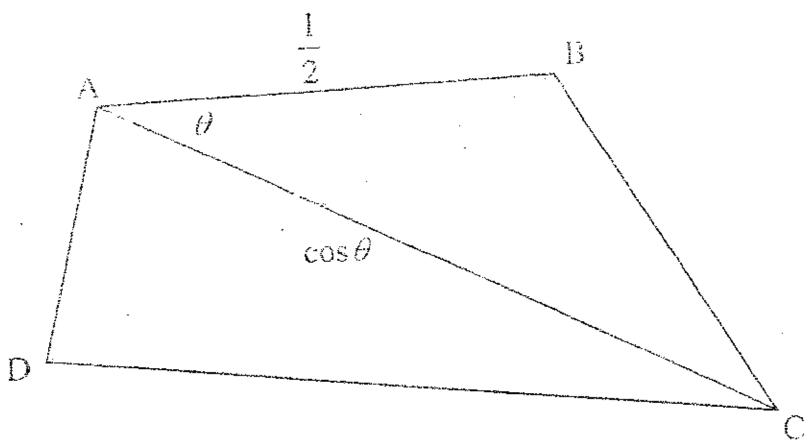
$$= 331,22\ m^3$$

**[21]**

## QUESTION 12

ABCD is a quadrilateral with $AB = \frac{1}{2}$ , $B\widehat{A}C = \theta$ , $AC = \cos\theta$ and

The area of $\Delta ABD = \frac{1}{8}\sin 2\theta$ . Prove that $AB \| CD$ .




m a

Area $\Delta$ ABC $= \frac{1}{2}\left(\frac{1}{2}\right)\cos\theta \sin\theta$

$= \frac{1}{4}\cos\theta\sin\theta$

$= \frac{2}{8}\cos\theta\sin\theta$

$= \frac{1}{8}\sin 2\theta$ = Area $\Delta$ ABD

$\therefore$ AB $\|$ DC (same base and area)

[7]