



GRANTLEIGH



**DEPARTMENT OF MATHEMATICS
PRELIMINARY EXAMINATION 2014
GRADE 12 PAPER 2**

EXAMINER: L HOFFMANN

MODERATOR: S DZINGWA

TIME: 3 HOURS

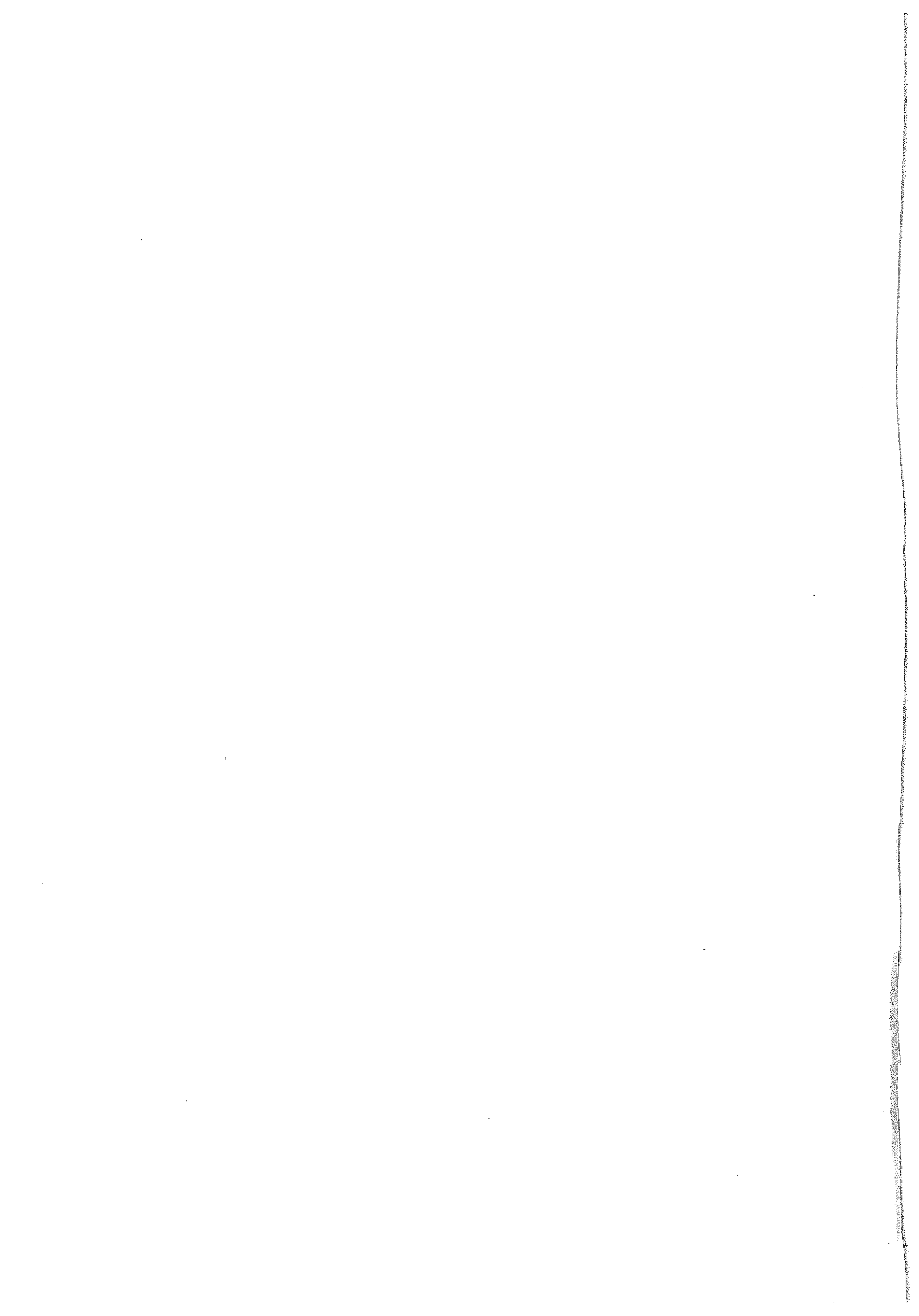
MARKS: 150

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. Write your name on top of this front page.
2. This question paper consists of 29 pages including this cover page. A formulae sheet and diagram sheet is also provided.
3. Answer ALL the questions on this question paper.
4. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
5. Round off your answers to two decimal digits where necessary.
6. All the necessary working details must be clearly shown.
7. It is in your own interest to write legibly and to present your work neatly.
8. Give reason for all geometry questions unless otherwise stated.

DO NOT WRITE IN THIS GRID

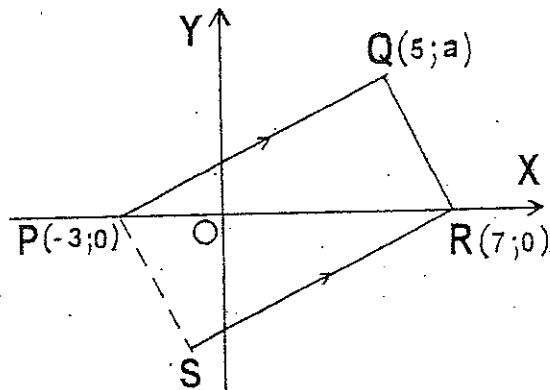
QUE 1	QUE 2	QUE 3	QUE 4	QUE 5	QUE 6	QUE 7
QUE 8	QUE 9	QUE 10	QUE 11	QUE 12	QUE 13	TOTAL



SECTION A

Question 1

In the diagram below, triangle PQR is right-angled at Q with coordinates $P(-3;0)$; $Q(5;a)$; $R(7;0)$ and $PQ \parallel SR$.



- (a) Show clearly by calculation that $a = 4$. (2)

- (b) Write down the gradient of SR. (2)

(c) Determine the equation of SR. (2)

(d) If it is further given that PQRS is a rectangle, calculate the coordinates of S. (4)

[10]

Question 2

The heights (h) of the learners at Grantleigh College in Grade 10, 11 and 12 were recorded in the table below.

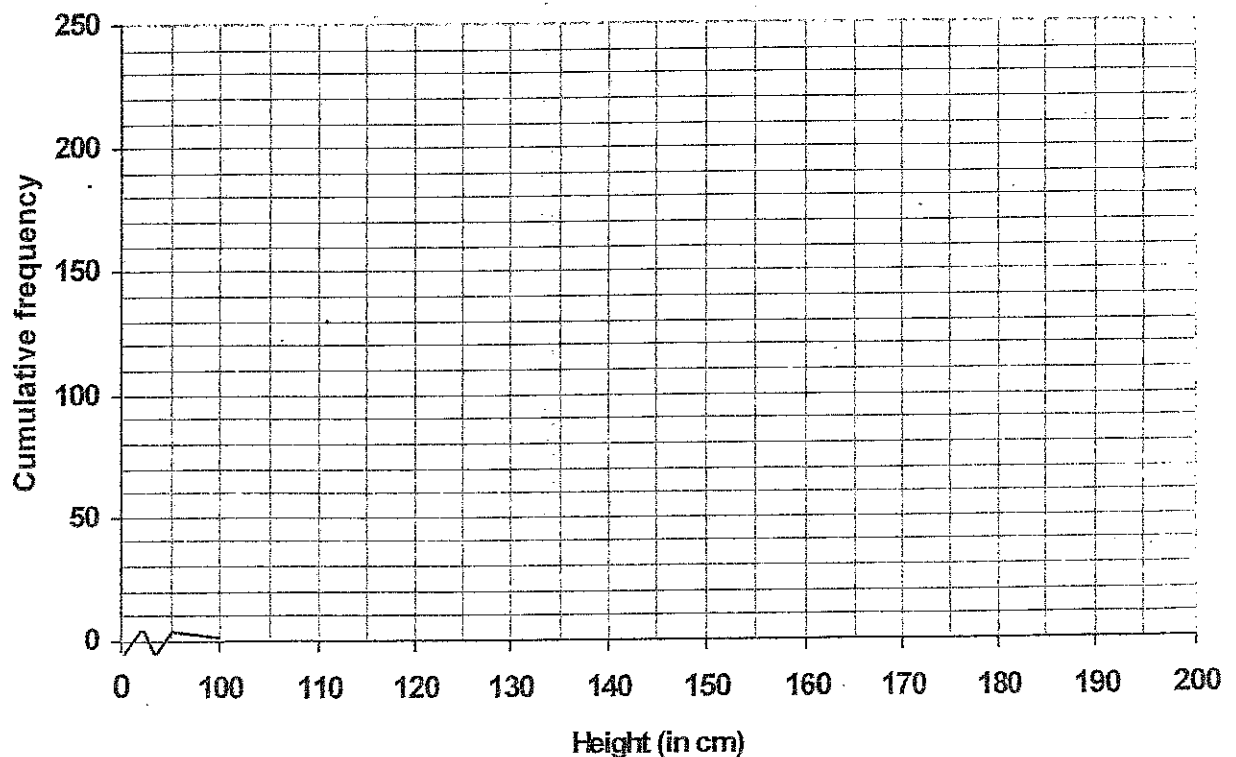
- (a) Complete the table by filling in the columns which record the midpoint of each interval and the cumulative frequency of the given data.

Height (in cm)	Frequency (f)	Midpoint	Cumulative frequency
$118 \leq h < 127$	16		
$127 \leq h < 136$	26		
$136 \leq h < 145$	42		
$145 \leq h < 154$	54		
$154 \leq h < 163$	26		
$163 \leq h < 172$	22		
$172 \leq h < 181$	14		

(3)

- (b) Draw a cumulative frequency (ogive) for this data on the grid provided below.

(3)



- c) Estimate, from your graph, the median, upper quartile, and twentieth percentile of the heights of the students in grade 10, 11 and 12 at Grantleigh College. Show, on your graph, where you obtained your readings. (6)

- d) Determine the modal interval for the heights of the learners. (1)

- e) Calculate the estimated mean of the heights of the students in the sample. (3)

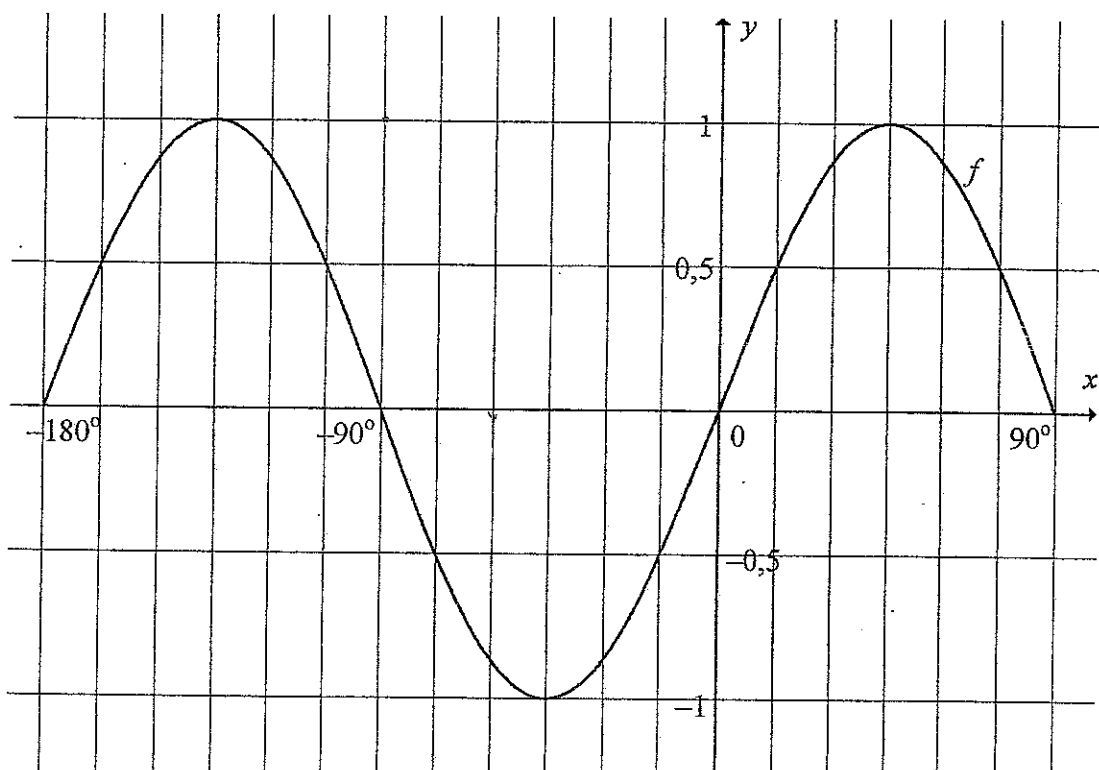
QUESTION 3

(a) (1) Prove that: $\tan \theta + \frac{\cos \theta}{\sin \theta} = \frac{2}{\sin 2\theta}$ (4)

(2) Simplify the following without the use of a calculator:

$$\frac{\sin(-\beta) + \sin(360^\circ - \beta)}{\sin(180^\circ - \beta) + \sin 180^\circ} \quad (3)$$

- (b) The graph of $f(x) = \sin 2x$ for $-180^\circ \leq x \leq 90^\circ$ is shown in the sketch below:



- (1) Write down the range of f . (1)

- (2) Determine the period of $f\left(\frac{3}{2}x\right)$. (2)

- (3) Draw the graph of $g(x) = \cos(x - 30^\circ)$ for $-180^\circ \leq x \leq 90^\circ$ on the above system of axes with the graph of $f(x) = \sin 2x$. Show the turning points and the x-intercepts clearly on the graph.

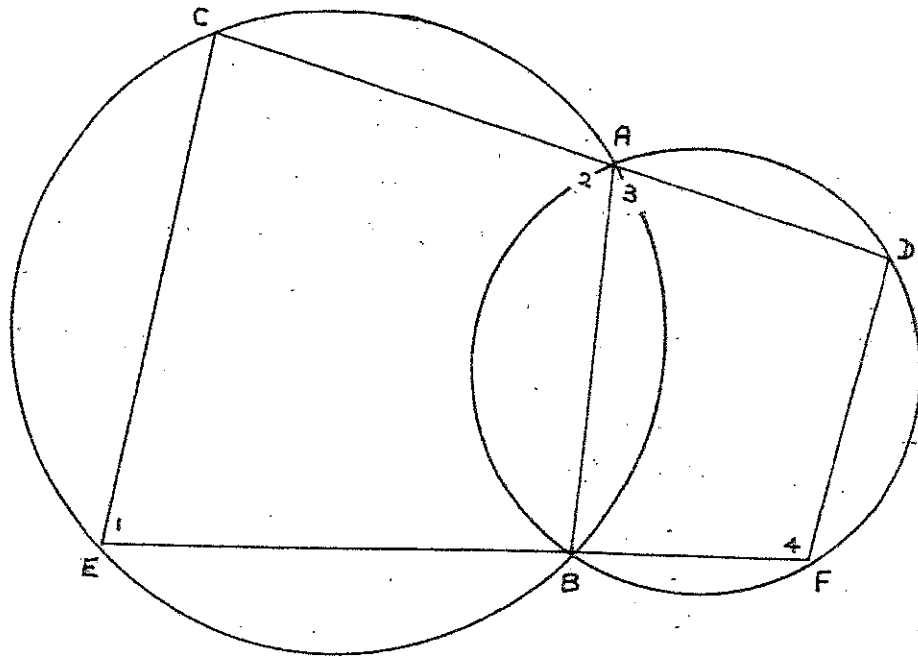
(4)

- (4) Hence, or otherwise, determine the values of x in the interval $-180^\circ \leq x \leq 90^\circ$ for which $f(x).g(x) < 0$. (2)

- (5) Describe the transformation that the graph f has to undergo to form $y = \sin(2x - 60^\circ)$. (2)

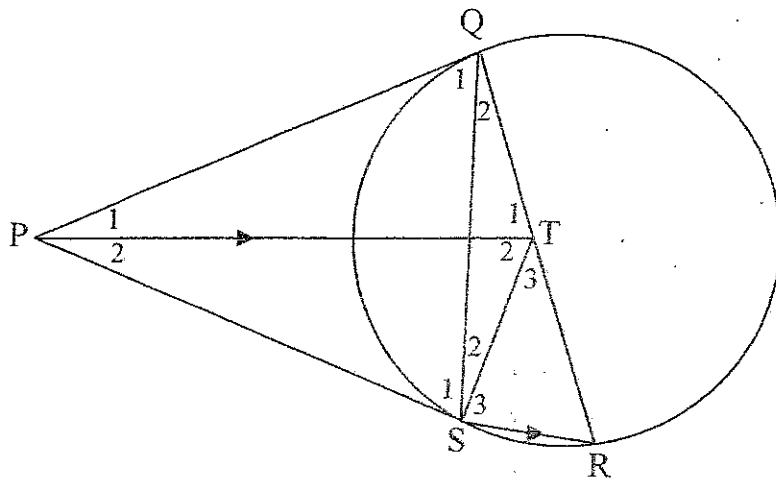
QUESTION 4

Two circles intersect at A and B. CAD and EBF are straight lines. A and B are joined.
 $\hat{A}_3 = 70^\circ$ Prove $EC \parallel FD$.



QUESTION 5

- (a) In the figure, PQ and PS are tangents to the circle through the points Q, S and R. $PT \parallel SR$ with T on QR and $\widehat{S_1} = 67^\circ$

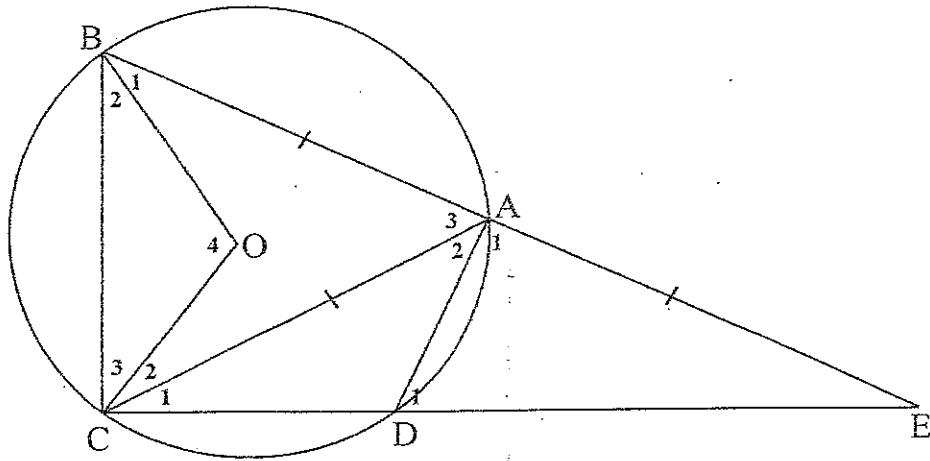


- (1) Name, with reasons, THREE other angles in the given figure each equal to 67° . (3)

(2) If T is the centre of the circle, determine the size of \widehat{T}_2 . (3)

(3) Why is PQTS a cyclic quadrilateral? (1)

- (b) In the figure below, ABCD is a cyclic quadrilateral of the circle with centre O and $BA = CA$. BA and CD produced intersect at E. $BA = AE$ and $\widehat{O_4} = 4x$.



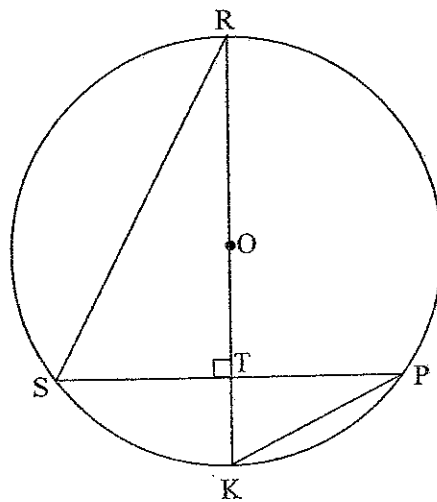
- (1) Determine, with reasons, the size of \widehat{E} in terms of x . (3)

- (2) Hence, prove that ED is a diameter of circle AED. (4)

QUESTION 6

(a) Complete the following statement: A line drawn from the centre of a circle to the midpoint of a chord is _____ to the chord. (1)

(b) In the diagram below, O is the centre of the circle with diameter RK. $PS \perp RK$. RK intersects PS at T.

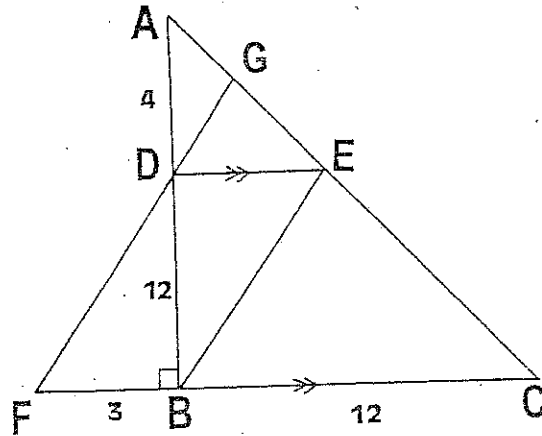


(1) If $PS = 4x$, write down the length of ST in terms of x . (1)

(2) Prove that $\triangle RST \cong \triangle PKT$. (4)

(3) If it is further given that $TK = x$ and $RT = 320 \text{ mm}$, calculate the value of x . (3)

- (b) In the diagram below, $\triangle ABC$ is right-angled at B. The point D lies on AB and the point E lies on AC such that $DE \parallel BC$. CB is produced to F. FD is drawn and produced to G, a point on AC. $BC = 12$ units, $AD = 4$ units, $DB = 12$ units and $FB = 3$ units.



- (1) Show that $AC = 20$ units. (2)

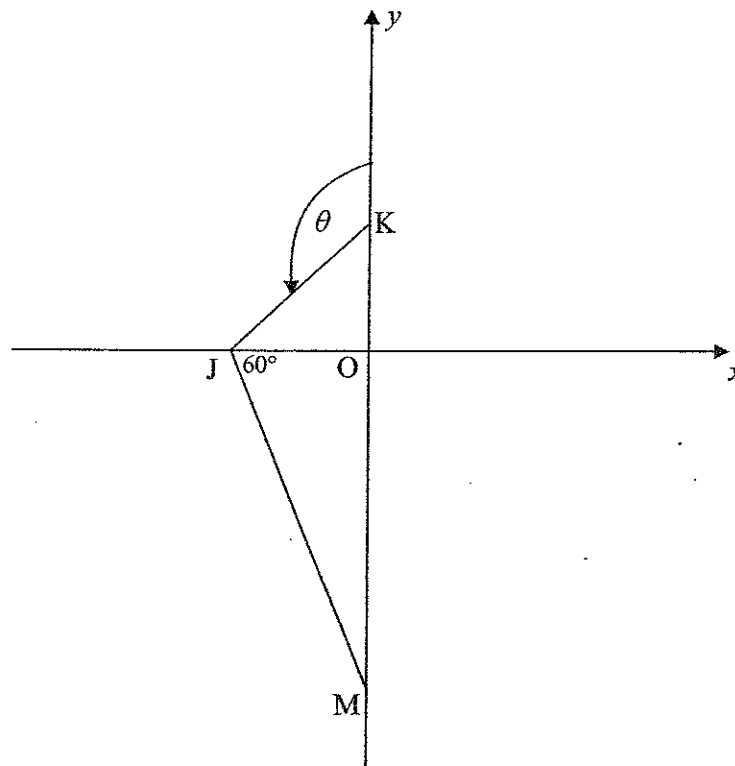
- (2) Calculate, stating reasons, the lengths of AE and EC. (4)

- (3) If it is now further given that $GE = 3\frac{3}{4}$ units, prove that DEBF is a parallelogram. (5)

SECTION B

QUESTION 8

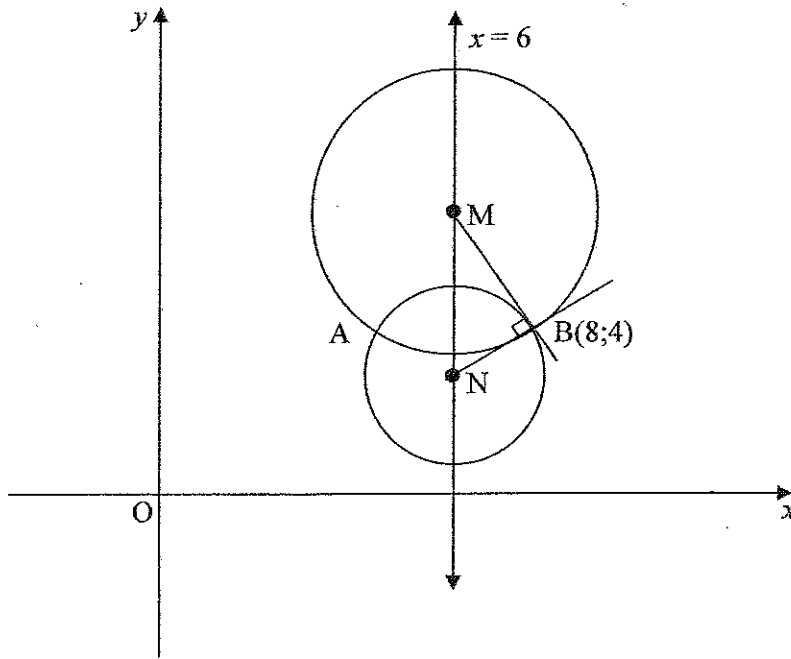
- (a) In the diagram below, triangle JKM is shown. J lies on the x-axis. K and M lie on the y-axis. The equation of JK is given by $y = \frac{1}{\sqrt{3}}x + 6$. Line JK makes an angle of θ with the y-axis as is shown and $M\hat{J}O = 60^\circ$.



- 1) Determine the size of θ . (3)

(b) In the diagram, two circles intersect at A and $B(8; 4)$ as shown.

The centres of the circles lie on the line $x = 6$. The tangents at B to the circles are perpendicular to one another and pass through the centres of the circles.



If the equation of the smaller circle is given by

$$x^2 + y^2 - 12x - 6y = -40, \text{ determine:}$$

(1) the coordinates of N.

(3)

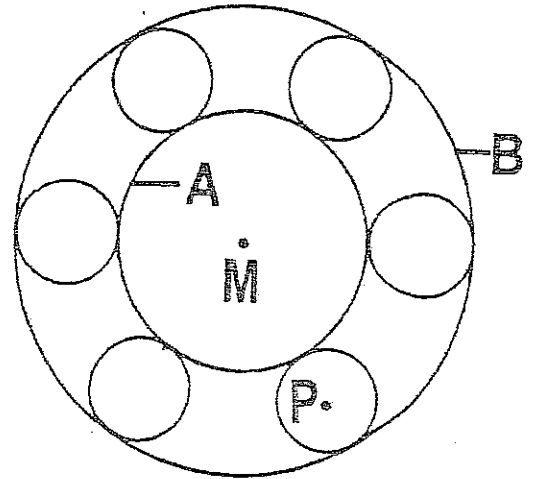
(2) the equation of the circle M.

(7)

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QUESTION 9

The figure shows a cross-section of a wheel bearing. The smaller circles represent ball bearings which roll between the two larger circles A and B which both have centre M.



The equation of circle A is given by:

$$x^2 + y^2 + 3x - 6y = 9$$

P is the centre of one of the ball bearings, each of which has a radius of 2 units.

- (a) Determine the coordinates of M, the centre of circle A. (4)

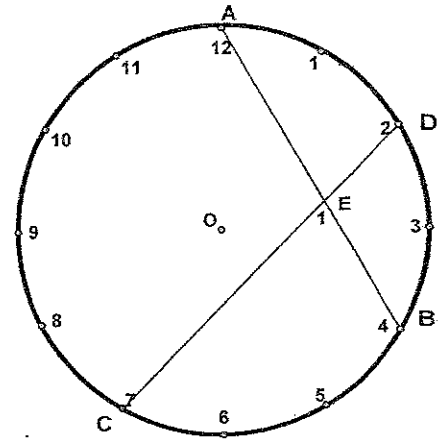
- (b) Find the equation which best describes the path of the centre point P as the small bearing rolls between the two larger circles. Give the answer in the form: $(x - a)^2 + (y - b)^2 = r^2$.

(4)

[8]

QUESTION 10

The diagram shows the face of a clock.
The 12 is joined to the 4 and the 2 to the 7
with straight lines AB and CD respectively.
O is the centre of the clock.



- (a) If AO and DO are joined, show
that $\widehat{AOD} = 60^\circ$.

(No reasons required)

(2)

- (b) If CO and BO are joined, calculate the size of \widehat{COB} . (1)

- (c) Hence, calculate $\widehat{E_1}$, providing reason for your answer. (4)

[7]

QUESTION 11

- (a) If $\cos^2 12^\circ - \sin^2 12^\circ = m$, express, without the use of a calculator, the following in terms of m . Answers may be left in surd form.

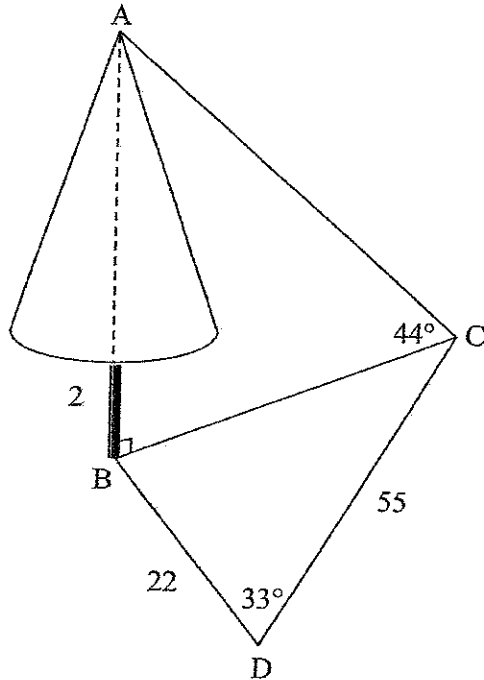
(1) $\cos 24^\circ$ (1)

(2) $\frac{\sqrt{3}}{2} \cdot \cos 6^\circ + \frac{1}{2} \cdot \sin 6^\circ$ (4)

(b) Determine, without the use of a calculator the general solution to:

$$\frac{1}{2} \sin(\theta + 10^\circ) = \sin\theta \cdot \cos\theta \quad (7)$$

- (c) In the diagram below, a tall tree is roughly illustrated by a right cone. The centre of the circular base of the cone is 2 meters above a point ,B, on the horizontal ground.



D and C are points in the same horizontal plane as B so that $BD = 22$ meters, $DC = 55$ meters and $\widehat{BDC} = 33^\circ$. The angle of elevation of the top of the tree, A, from C is 44° .

- (1) Find the height AB of the tree, correct to two decimal places.

(6)

- (2) If the radius of the base of the cone is 3 m, find the volume of the cone depicting the foliage of the tree in the diagram above.

The volume of a cone is given by:

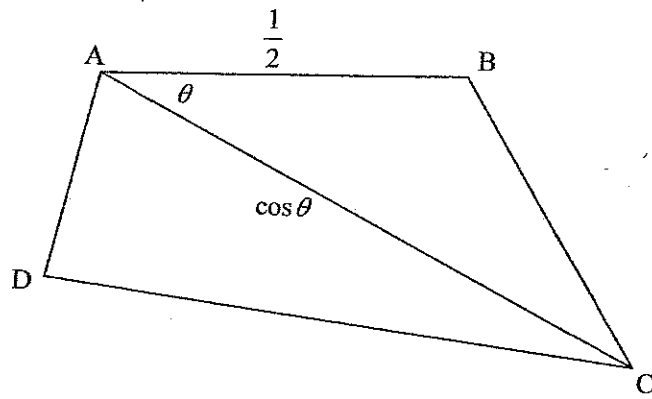
$$Volume = \frac{\text{base area} \times \text{height}}{3} \quad (3)$$

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QUESTION 12

ABCD is a quadrilateral with $AB = \frac{1}{2}$, $\widehat{BAC} = \theta$, $AC = \cos \theta$ and

The area of $\triangle ABD = \frac{1}{8} \sin 2\theta$. Prove that $AB \parallel CD$.



[7]

