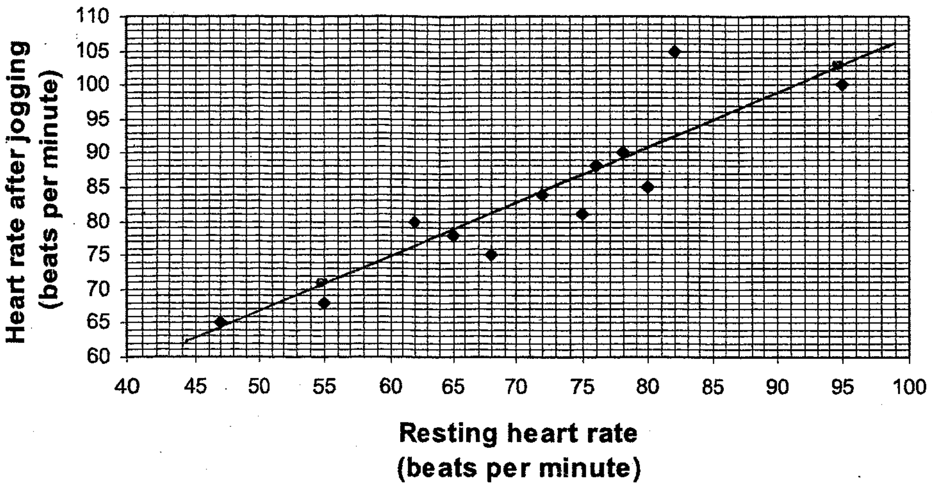
MATHEMATICS (Paper 2) Memo 1 Sept 2014

GR 12

Q1

1.1	**Scatter plot showing resting heart rate vs heart rate after exercising** 	✓✓✓ all 12 points plotted correctly ✓✓ 7 – 11 points plotted correctly ✓ 2 – 6 points plotted correctly (3)
1.2	$a = 25{,}23$ $(25{,}22587269\ldots)$ $b = 0{,}81$ $(0{,}8143737166\ldots)$ $\hat{y} = a + bx$ $\hat{y} = 25{,}23 + 0{,}81x$ **Note:** If the line of best fit is drawn and its equation then calculated: 0 / 4 marks **If using pen and paper method:** $\bar{x} = 71{,}25$ $\bar{y} = 83{,}25$ $a = 25{,}23$ $(25{,}22587269\ldots)$ $b = 0{,}81$ $(0{,}8143737166\ldots)$ $\hat{y} = a + bx$ $\hat{y} = 25{,}23 + 0{,}81x$	✓✓ a or b ✓ b or a ✓ $\hat{y} = 25{,}23 + 0{,}81x$ (4) ✓ $\bar{x}, \bar{y}$ ✓ a ✓ b ✓ $\hat{y} = 25{,}23 + 0{,}81x$ (4)
* 1.4	$r = 0{,}898$ $= 0{,}90$ $(0{,}8979098935\ldots)$	✓ answer (1)
1.5	It is a very strong positive relationship.	✓ strong positive (1)
1.6	$\hat{y} = 25{,}23 + 0{,}81x$ $86 = 25{,}23 + 0{,}81x$ $x = 75{,}024\ldots$ Resting heart rate could be 75 beats per minute. If a and b are not rounded off in the calculation, $x = 74{,}626\ldots$ $x = 74{,}63$ If candidate draws in the least square regression line and reads of x-value where $y = 86$: full marks	substitute $\hat{y} = 86$ ✓ answer (1) Accept $x = 74{,}63$ [12]

* 1.3 Consider the line of best fit ✓✓ (2)
(1) slope ✓
(2) accuracy ✓

Q2

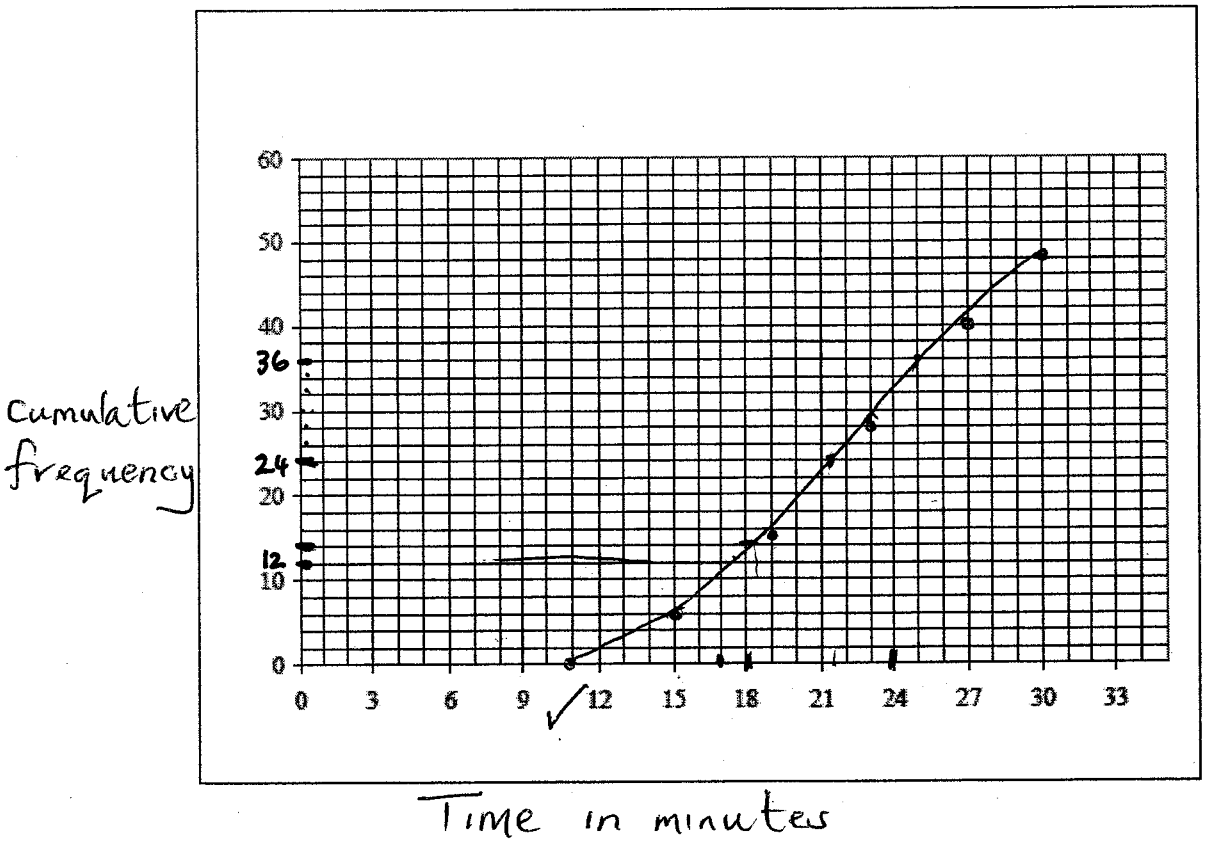
DIAGRAM SHEET 2

QUESTION 2.1

Time (in minutes)	Frequency	Cumulative frequency
$11 \leq t < 15$	6	6
$15 \leq t < 19$	9	15
$19 \leq t < 23$	13	28
$23 \leq t < 27$	12	40
$27 \leq t \leq 30$	8	48

✓✓ (2)

QUESTION 2.2



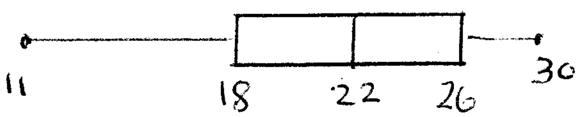
✓✓ (3)

2.3 LQ – (17 → 19) ✓
MED – (±22) ✓
UQ – (24 → 26) ✓

(3)

2.4 Box and whisker Med, LQ, UQ ✓
Min / max ✓

(2)



$\boxed{10}$

Q3 3.1 $m_{BC} = \frac{1}{3}$ ✓✓ (2)

3.2 $\tan\theta = \frac{1}{3}$ ✓ $\therefore \theta = 18,43°$ ✓ (2)

3.3 $m_{AB} = -3$ ✓

Since $m_{AB} \cdot m_{BC} = -1$ ✓ (2)

3.4 $m_{AD} = m_{BC}$ ✓

$\therefore \frac{t-6}{6} = \frac{1}{3}$ ✓ $\therefore t - 6 = 2$ $\therefore t = 8$ ✓ (3) [9]

$y = \frac{1}{3}x + \frac{17}{3}$ Subs $x = 7$

Q4 4.1.1 $(x^2 - 4x + 4) + (y^2 + 10y + 25)$ ✓ ✓ $= 7 + 4 + 25$

$\therefore (x - 2)^2 + (y + 5)^2 = 6^2$ ✓ ✓

centre $M(2; -5)$ (4)

4.1.2 x-intercepts $(y = 0)$ ✓

$\therefore (x-2)^2 + 5^2 = 36$ ✓

$(x-2)^2 = 11$ ✓

$x = 2 \pm \sqrt{11}$ or $5,32$ and $-1,32$ ✓

OR
$x^2 - 4x - 7 = 0$
$x = \frac{-b \pm \sqrt{}}{2a}$

(If surd form accept) (4)

4.1.3 $r = 6$ ✓ (1)

4.1.4 $M\hat{P}Q = 90°$ ✓ (radius - tangent)

From Pyth: $PQ^2 = QM^2 - PM^2$ ✓

$= [(8-2)^2 + (3+5)^2] - 36$

$= 64$ ✓

$\therefore PQ = 8$ ✓ (4)

Q4.2 4.2.1 $m_{\text{Diamter}} = 2$

$\therefore m_{\text{tang}} = -\frac{1}{2}$ $\qquad y = -\frac{1}{2}x + 6$

Subst $E(12;0)$

$\therefore y = mx + c$

$0 = \left(-\frac{1}{2}\right)(12) + c$ $\qquad \therefore c = 6$ (3)

4.2.2 $2x + 3 = -\frac{1}{2}x + 6$

$\therefore x = 6/5$

$\therefore y = \frac{27}{5}$ $\qquad \therefore E\left(6/5; \frac{27}{5}\right)$ (3)

$\boxed{19}$

Q5 5.1.1 $\hat{G}$ in 4th quad

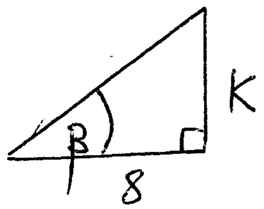
$\therefore \hat{G} = 360° - 43,44°$ (2)

$= 316,6°$

5.1.2 $\tan\left(\frac{2}{3}(316,6°) + 100°\right) = -1,1$ (1)

(or $-1,2$)

5.2.1



$\frac{k}{8} = \frac{1}{4}$ $\qquad \therefore k = 2$ (2)

5.2.2 $OT^2 = 68$ $\therefore OT = \sqrt{68}$

$\therefore \sin\beta = \frac{2}{\sqrt{68}}$ OR $\frac{2}{2\sqrt{17}} = \frac{1}{\sqrt{17}}$ (3)

5.3.1 $= \dfrac{\sin x}{-\sin x \quad -\sin x} = -\frac{1}{2}$ (4)

5.3.2 $= \dfrac{\cos 45° \cos\theta + \sin 45° \sin\theta}{\cos 45° \cos\theta} - \dfrac{\sin\theta}{\cos\theta}$

$= 1 + \dfrac{\sin\theta}{\cos\theta} - \dfrac{\sin\theta}{\cos\theta}$ (not nec to subst. since $\sin 45° = \cos 45°$)

$= 1$

(5)

5.4 $$LHS = \frac{\cos(3A - A) - \cos(3A + A)}{\sin(3A + A) - \sin(3A - A)}$$

$$= \frac{\cos 3A \cos A + \sin 3A \sin A - (\cos 3A \cos A - \sin 3A \sin A)}{\sin 3A \cos A + \cos 3A \sin A - (\sin 3A \cos A - \cos 3A \sin A)}$$

$$= \frac{2 \sin 3A . \sin A}{2 \cos 3A . \sin A}$$

$$= \frac{\sin 3A}{\cos 3A} = RHS$$ (6)

5.5 $1 + \sin x = 1 - 2 \sin^2 x$

$\therefore 2 \sin^2 x + \sin x = 0$

$\therefore \sin x (2 \sin x + 1) = 0$

$\therefore \sin x = 0$ or $\sin x = -\frac{1}{2}$

$\therefore x = 0^\circ + K360^\circ$ or $180^\circ + K360^\circ$ or $x = 210^\circ + K360^\circ$ or $330^\circ + K360^\circ$

OR $x = 0^\circ + K180^\circ$

$(K \in \text{Integers})$ (7)

$\boxed{30}$

Q6 6.1 $a = 2$, $b = 0$, $c = -3$ (3)

6.2 $90^\circ < \theta < 270^\circ$ (2)

6.3 $p = 1$, $q = -2$ (3)

$\boxed{8}$

Q7 7.1 Any acute-angled Δ sketch

(3)

Q7 7.2.1

$$\tan\alpha = \frac{AP}{AB} \quad \therefore AP = AB\tan\alpha \quad \text{—— (1)}$$

Also $\dfrac{AB}{\sin\beta} = \dfrac{20}{\sin[180° - (\theta+\beta)]}$

$$\therefore AB = \frac{20\sin\beta}{\sin(\theta+\beta)} \quad \text{—— (2)}$$

Subst (2) → (1) Thus result. (4)

7.2.2 If AB = AC, then $\theta = \beta$

Thus $AP = \dfrac{20\sin\beta\tan\alpha}{\sin(\theta+\beta)}$

$$= \frac{20\sin\beta\tan\alpha}{\sin 2\beta}$$

$$= \frac{20\sin\beta\tan\alpha}{2\sin\beta\cos\beta} \quad (2)$$

$=$ Result

$\boxed{9}$

Q8 8.1 $DC = 13x$ (1)

8.2 radius (OC) $= \frac{13}{2}x$

$\therefore OM = \frac{13}{2}x - 4x = \frac{5}{2}x$ (2)

8.3 $BM = 12$ units ($OM \perp AB$)

From Pyth:

$$OB^2 = OM^2 + BM^2$$

$$\left(\frac{13}{2}x\right)^2 = \left(\frac{5}{2}x\right)^2 + 144$$

$$\therefore \frac{169x^2 - 25x^2}{4} = 144$$

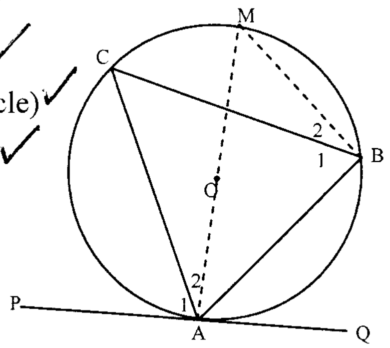
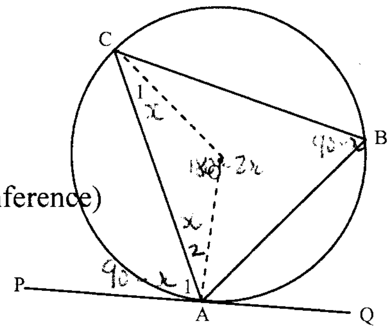
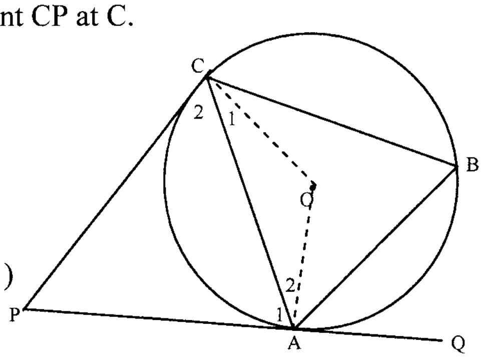
$$\therefore x^2 = 4$$

$$x = 2$$

$\therefore r = 13$ units (4)

$\boxed{7}$

QUESTION 9.1

	Draw diameter AM and join M to B. $\hat{A}_1 + \hat{A}_2 = 90°$ (rad ⊥ tangent) $\hat{B}_1 + \hat{B}_2 = 90°$ (∠s in a semi circle) $\hat{B}_2 = \hat{A}_2$ (∠s in same seg) $\hat{B}_1 = \hat{A}_1$  **OR**	✓ construction ✓ S/R ✓ $\hat{B}_1 + \hat{B}_2 = 90°$ ✓ ∠s in a semi circle ✓ S/R (5)
	Draw radii OC and OA Let $\hat{A}_2 = x$ $\hat{C}_1 = x$ (∠ opp = radii) $\hat{A}_1 = 90° - x$ (rad ⊥ tan) $A\hat{O}C = 180° - 2x$ (∠ sum Δ) $A\hat{B}C = 90° - x$ (∠ circ cent = 2∠ circumference) $A\hat{B}C = \hat{A}_1$ (= $90° - x$)  **NOTE:** If there is no construction: 0 / 5 marks If candidate changes lettering and states "Similarly": full marks **OR**	✓ construction ✓ $\hat{A}_1 = 90° - x$ ✓ rad ⊥ tan ✓ S/R ✓ S/R (5)
	Draw QA extend to P. Draw tangent CP at C. PC = PA (tan from comm pt) $\hat{C}_2 = \hat{A}_1$ (∠s opp = sides) $C\hat{O}A = 2A\hat{B}C$ (∠ circ cent = 2∠ circumf) $\hat{A}_1 + \hat{A}_2 = 90°$ (tan ⊥ radius) $C\hat{O}A = 180° - (90° - \hat{A}_1 + 90° - \hat{C}_2)$ $= \hat{A}_1 + \hat{C}_2$ $= \hat{A}_1 + \hat{A}_1$ $= 2\hat{A}_1$ $\hat{A}_1 = \frac{1}{2} C\hat{O}A$ $= C\hat{B}A$  **OR**	✓ construction ✓ S/R ✓ S/R ✓ $\hat{A}_1 + \hat{A}_2 = 90°$ ✓ tan ⊥ radius (5)

Q9 (9.1) Theorem: Construction ✓ (1) } (6)
Proof: ✓✓✓✓✓ (5)

(9.2) (9.2.1) $\hat{A}_2 = x$ ✓ (Tan / chord) ✓
$= \hat{A}_5$ ✓ (vert. opp) ✓
$= \hat{P}_2$ ✓ (Tan / chord) (5)

(9.2.2) Let $\hat{P}_1 = y$ ✓
$\therefore \hat{A}_3 = y$ ✓ (base $\angle^s$ of Δ with equal tangents)
$= \hat{A}_6$ ✓ (vert. opp.)
$= \hat{R}_2$ ✓ (Tan / chord)
Since $\hat{P}_1 = \hat{R}_2$ ✓
APTR is a c.q. (ext. $\angle$ = opp. int. ..) ✓ (5)

OR ....

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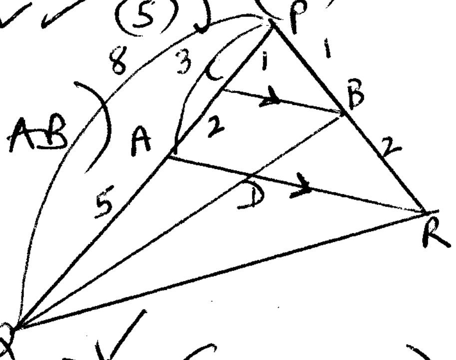
Q10 (10.1) Theorem: Construction ✓ (1) } (6)
Proof: ✓✓✓✓✓ (5)



(10.2) (10.2.1) $\dfrac{PC}{CA} = \dfrac{1}{2}$ ✓ ($BC \parallel AB$)

$\dfrac{PA}{AQ} = \dfrac{3}{5}$ ✓ (given)

Thus $\dfrac{CA}{AQ} = \dfrac{2}{5}$ ✓ $= \dfrac{BD}{DQ}$ ✓ ($AD \parallel CB$)

$\therefore \dfrac{BD}{BQ} = \dfrac{2}{7}$ ✓ (5)

(10.2.2) $\dfrac{\text{Area of } \Delta PRA}{\text{Area of } \Delta QRA} = \dfrac{PA}{QA}$ ✓

$= \dfrac{3}{5}$ ✓ (2)

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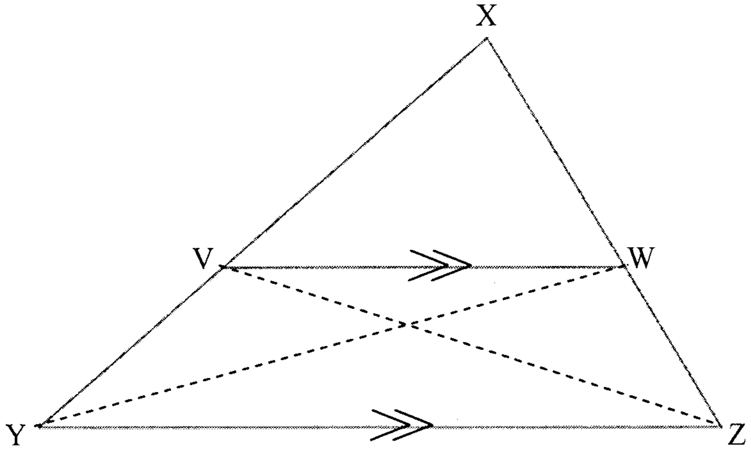
QUESTION 10.1



	Construct VZ and WY ✓ $\frac{\text{area } \Delta XVW}{\text{area } \Delta VWY} = \frac{XV}{VY}$ ✓ (equal altitudes) $\frac{\text{area } \Delta XVW}{\text{area } \Delta WVZ} = \frac{XW}{WZ}$ ✓ (equal altitudes) area ΔYVW = area ΔVWZ ✓ (VW $\parallel$ YZ) ✓ area ΔXVW is common ✓ $\frac{XW}{WZ} = \frac{XV}{VY}$	✓ construction ✓ $\frac{\text{area } \Delta XVW}{\text{area } \Delta VWY} = \frac{XV}{VY}$ ✓ $\frac{\text{area } \Delta XVW}{\text{area } \Delta WVZ} = \frac{XW}{WZ}$ ✓ area ΔYVW = area ΔVWZ ✓ VW $\parallel$ YZ ✓ conclusion (6)

Q 11

11.1.1

$\frac{CO}{OB} = \frac{1}{1}$ (radii)

$= \frac{AE}{BE}$ (OE ∥ CA)

Thus result.

$C\hat{A}B = 90°$ (∠ semi-circle)

$\therefore \hat{E}_1 = 90°$ (corresp ∠s AC ∥ EO)

or midpt. TH.
(line from centre ⊥ to chord) (2)

11.1.2

In △s

(1) $\hat{D} = \hat{B}$ (subt. by CA)

(2) $\hat{E}_4 = \hat{E}_1$ (vert. opp.) $C\hat{E}B$

$\hat{C}_1 = \hat{A}_3$

NOT NEC. for (3) OR

$\therefore$ △AED ||| △CEB (∠; ∠; ∠) (3)

11.1.3

From ||| in (11.1.2)

$\frac{AE}{ED} = \frac{CE}{EB}$ (2)

Since EB = AE, result

11.2.1

In △s

(1) $\hat{B}_1$ is common

(2) $\hat{P}_2 = \hat{D} = 90°$ ($\hat{D}$ subt. by diameter)

Thus △BPE ||| △BDA (∠; ∠; ∠) (3)

11.2.2

Thus From ||| (2)

$\frac{BP}{BD} = \frac{PE}{AD}$ (OR $\frac{BP}{PE} = \frac{BD}{DA}$)

11.2.3

Since $\hat{D} = 90°$

$AB^2 = BD^2 + AD^2$ (Pyth) —— (i)

From (11.2.2) $AD = \frac{BD \cdot PE}{BP}$

$\therefore AD^2 = \frac{BD^2 \cdot PE^2}{BP^2}$ —— (ii)

subst (ii) → (i) Thus Result. (5)

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