Herzlia APM Prelim 2014 memo.

1.1 $\lim_{x \to 6} \frac{\sqrt{4x+1} - \sqrt{3x+7}}{x-6}$

$= \lim_{x \to 6} \frac{\sqrt{4x+1} - \sqrt{3x+7}}{x-6} \cdot \frac{\sqrt{4x+1} + \sqrt{3x+7}}{\sqrt{4x+1} + \sqrt{3x+7}}$

$= \lim_{x \to 6} \frac{(4x+1) - (3x+7)}{(x-6)(\sqrt{4x+1} + \sqrt{3x+7})}$

$= \lim_{x - 6} \frac{x-6}{\cancel{(x-6)}(\sqrt{4x+1} + \sqrt{3x+7})} = \lim_{x \to 6} \frac{1}{\sqrt{4x+1} + \sqrt{3x+7}}$

$= \frac{1}{\sqrt{25} + \sqrt{25}} = \frac{1}{10}$

1.2. $\lim_{\theta \to \frac{\pi}{4}} \frac{\cos 2\theta}{\cos\theta - \sin\theta} = \lim_{\theta \to \frac{\pi}{4}} \frac{\cos^2\theta - \sin^2\theta}{\cos\theta - \sin\theta}$

$= \lim_{\theta \to \frac{\pi}{4}} \frac{\cancel{(\cos\theta - \sin\theta)}(\cos\theta + \sin\theta)}{\cancel{\cos\theta - \sin\theta}} = \cos\frac{\pi}{4} + \sin\frac{\pi}{4}$

$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$

2.1 x^3: solve for r in $x^{5-r}(x^{-1})^r = x^3$

$\therefore 5 - 2r = 3 \quad \therefore r = 1 \quad \therefore$ coeff of $x^3 = \binom{5}{1} 2^4 (-3)^1$

$= -240$

x^1: $x^{5-r}(x^{-1})^r = x^1 \quad \therefore 5 - 2r = 1 \quad \therefore r = 2$

$\therefore$ coefficient of $x = \binom{5}{2} 2^3 (-3)^2 = 720$

2.2. $\therefore$ coefficient of x in $\left(1 + \frac{2}{x^2}\right)\left(2x - \frac{3}{x}\right)^5$

$= (1)(720) + 2(-240) = 240$

3. $x = 1 - 2i$ is a root $\therefore$ $x = 1 + 2i$ is also a root (complex conjugate thm)

$\therefore [x-(1-2i)][x-(1+2i)]$ is a factor

$= [(x-1)+2i][(x-1)-2i]$

$= (x-1)^2 - 4i^2 \quad = x^2 - 2x + 1 + 4$

$= x^2 - 2x + 5$

$$
\begin{array}{r|l}
 & x^3 - 5x^2 + 9x - 5 \\
\hline
x^2 - 2x + 5 & x^5 - 7x^4 + 24x^3 - 48x^2 + 55x - 25 \\
 & \underline{x^5 - 2x^4 + 5x^3} \\
 & -5x^4 + 19x^3 - 48x^2 \\
 & \underline{-5x^4 + 10x^3 - 25x^2} \\
 & 9x^3 - 23x^2 + 55x \\
 & \underline{9x^3 - 18x^2 + 45x} \\
 & -5x^2 + 10x - 25 \\
 & \underline{-5x^2 + 10x - 25}
\end{array}
$$

$x = 1$ is a factor of $x^3 - 5x^2 + 9x - 5$

$\therefore x^3 - 5x^2 + 9x - 5 = (x-1)(x^2 - 4x + 5)$ (by inspection)

$x^2 - 4x + 5 = 0$ when $x = \frac{4 \pm \sqrt{16 - 20}}{2} = 2 \pm i$

$\therefore$ the roots of $f(x)$ are $x = 1 \pm 2i\,;\ 2 \pm i\,;\ 1$

4.1 $A\hat{O}C = \frac{\pi}{2} - 0{,}4 = 1{,}17$ rad.

4.2 Area of shaded region, $ACB = \triangle AOB$ − sector AOC

$= \frac{1}{2} 7\left(\frac{7}{\tan 0{,}4}\right) - \frac{1}{2} 7^2\left(\frac{\pi}{2} - 0{,}4\right)$

$= 29{,}26 \text{ cm}^2$

5.1 $t=0 \quad \therefore \quad \frac{4}{3} - \frac{P}{60000} = 1 \qquad (\ln 1 = 0)$

$$\therefore \quad \frac{P}{60000} = \frac{4}{3} - 1 = \frac{1}{3}$$

$$\therefore \quad \underline{P = 20000 \text{ people}}$$

5.2 $P = 0 \quad \therefore \quad t = 100 \ln\left(\frac{4}{3}\right)$

$$= 28{,}77 \text{ yrs}$$

$$= 28 \text{ yrs } 280{,}40 \text{ days}$$

$$= 28 \text{ yrs } 280 \text{ days } 9{,}50 \text{ hrs}$$

(ignoring leap year possibility) 273 days till end of Sept.

$\therefore$ Oct 8, 9:30 am, 2042

[IF using 12 months instead of 365 days : 13:30, Oct 7]

5.3 $\ln\left(\frac{4}{3} - \frac{P}{60000}\right) = \frac{t}{100}$

$$\therefore \left(\frac{4}{3} - \frac{P}{60000}\right) = e^{\frac{t}{100}}$$

$$\therefore \frac{P}{60000} = \frac{4}{3} - e^{\frac{t}{100}}$$

$$\therefore \underline{P = 60000\left(\frac{4}{3} - e^{\frac{t}{100}}\right)}$$

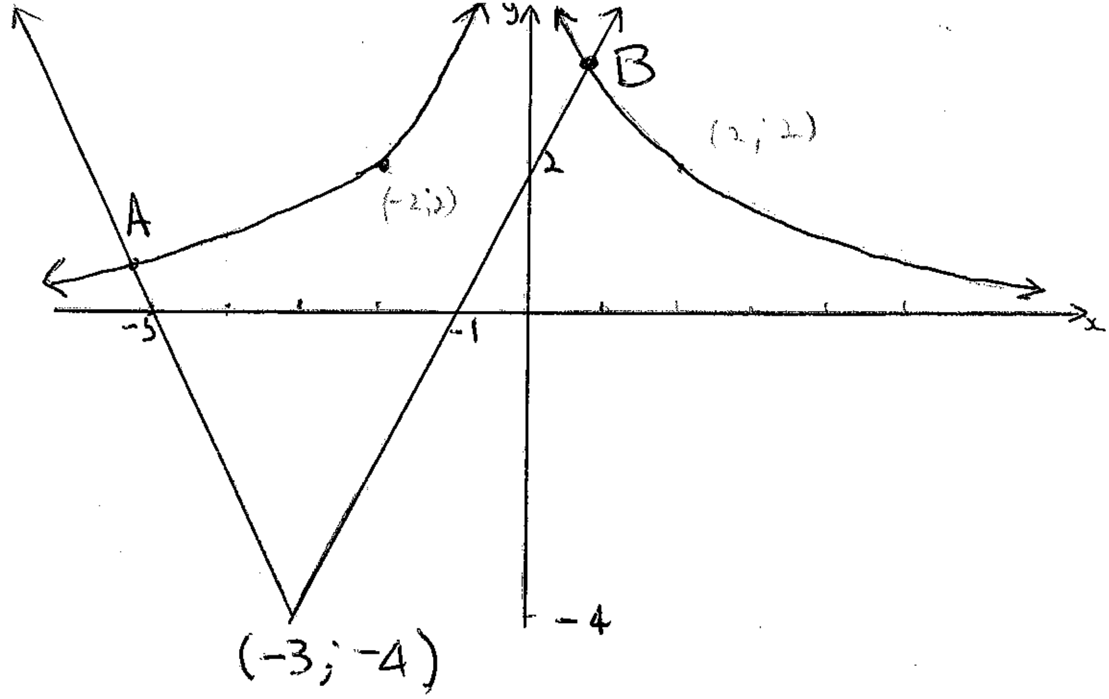
5.4 $\frac{dP}{dt} = -60000\left(\frac{1}{100}\right)e^{\frac{t}{100}}$

$$= -600\, e^{\frac{t}{100}}$$

$$\therefore P'(0) = -600\, e^{\frac{0}{100}}$$

$$= \underline{-600 \text{ people/year}}$$

6.1



6.2 3 regions/domains to consider:

(i) $\underline{x < -3}$: $-2(x+3) - 4 = -\frac{4}{x}$

$\therefore 2x^2 + 10x - 4 = 0$

$\therefore x^2 + 5x - 2 = 0$

$\therefore x = \frac{-5 \pm \sqrt{25+8}}{2}$

$\therefore x = \frac{-5-\sqrt{33}}{2} \quad (\because \frac{-5+\sqrt{33}}{2} > -3)$

$= -5{,}37$ (A)

(ii) $\underline{-3 \leq x < 0}$: $2(x+3) - 4 = \frac{-4}{x}$

$\therefore 2x^2 + 2x + 4 = 0$

$\Delta < 0 \therefore$ no real solutions in this domain

(iii) $\underline{x > 0}$: $2(x+3) - 4 = \frac{4}{x}$

$\therefore 2x^2 + 2x - 4 = 0$

$\therefore x^2 + x - 2 = 0$

$\therefore (x+2)(x-1) = 0$

$\therefore x = 1 \quad (\because x > 0)$ (B)

7 $\quad -2x^2y + 2xy^2 - 5x - 3y^3 = 12$

differentiate implicitly ($\frac{d}{dx}$ both sides) :

$\therefore -4xy - 2x^2y' + 2y^2 + 4xyy' - 5 - 9y^2y' = 0$

$\therefore y'(-2x^2 + 4xy - 9y^2) = 4xy - 2y^2 + 5$

$\therefore y' = \dfrac{4xy - 2y^2 + 5}{-2x^2 + 4xy - 9y^2}$

$\therefore y'(+3;-1) = \dfrac{4(3)(-1) - 2(-1)^2 + 5}{-2(3)^2 + 4(3)(-1) - 9(-1)^2}$

$= \dfrac{3}{13}$

$\therefore$ tangent at $(3;-1)$ has gradient $\frac{3}{13}$

$\therefore -1 = \frac{3}{13}(3) + c \quad \therefore c = -\frac{22}{13}$

$\therefore y_{tangent} = \frac{3}{13}x - \frac{22}{13}$

8.1 $\quad z = a + bi \quad \therefore \bar{z} = a - bi$

8.2 $\quad |z|^2 = (a+bi)(a-bi) = a^2 + b^2$

8.3 $\quad \dfrac{1}{z} = \left(\dfrac{1}{z}\right)\left(\dfrac{\bar{z}}{\bar{z}}\right) = \dfrac{\bar{z}}{z \cdot \bar{z}} = \dfrac{\bar{z}}{|z|^2}$

8.4 $\therefore$ if $z = 3 - 5i$

$z^{-1} = \dfrac{3 + 5i}{3^2 + 5^2}$

$= \dfrac{3}{34} + \dfrac{5}{34}i$

9.1 $(\cos\theta + i\sin\theta)^2 = \cos^2\theta + i2\sin\theta\cos\theta - \sin^2\theta$

$= (\cos^2\theta - \sin^2\theta) + i(2\sin\theta\cos\theta)$

$= \cos 2\theta + i\sin 2\theta$

$\therefore$ true for $n=2$.

9.2 Assume true for some $k \in \mathbb{N}$

i.e. $(\cos\theta + i\sin\theta)^k = \cos k\theta + i\sin k\theta$

$(\cos\theta + i\sin\theta)^{k+1} = (\cos\theta + i\sin\theta)(\cos\theta + i\sin\theta)^k$

$= (\cos\theta + i\sin\theta)(\cos k\theta + i\sin k\theta)$ (by assumption)

$= (\cos\theta\cos k\theta - \sin\theta\sin k\theta) + i(\sin\theta\cos k\theta + \cos\theta\sin k\theta)$

$= \cos(\theta + k\theta) + i\sin(\theta + k\theta)$

$= \cos(k+1)\theta + i\sin(k+1)\theta$

$\therefore$ If it is true for some natural number k, then it is true for $k+1$. 9.1 shows it is true for $n=1$ and $n=2$, $\therefore$ it is true for $\forall n \in \mathbb{N}$.

9.3 for $m \in \mathbb{N}$, $(\cos\theta + i\sin\theta)^{-m} = [(\cos\theta + i\sin\theta)^m]^{-1}$

$= (\cos m\theta + i\sin m\theta)^{-1}$ (from 9.2)

$= \dfrac{\cos m\theta - i\sin m\theta}{\cos^2 m\theta + \sin^2 m\theta}$ (from 8.3)

$= \cos m\theta - i\sin m\theta$ $(\cos^2\alpha + \sin^2\alpha = 1)$

$= \cos(-m)\theta + i\sin(-m)\theta$

(9.2 + 9.3) $\therefore$ $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$; $\forall n \in \mathbb{Z}$

10 x-values at A and B:

$(x-2)^2 = -2x+7$

$\therefore x^2-2x-3=0$

$\therefore (x+1)(x-3)=0$

$\therefore x=-1$ ("A") or $x=3$ ("B")

$\therefore$ shaded area $= \int_{-1}^{3} [(-2x+7)-(x-2)^2]\,dx$

$= \int_{-1}^{3} (-x^2+2x+3)\,dx$

$= \left[-\frac{1}{3}x^3+x^2+3x\right]_{-1}^{3}$

$= (-9+9+9)-(\frac{1}{3}+1-3)$

$= \frac{32}{3}$ units^2

11.1 major arc $ABC = R\theta$

11.2 base of cone = major arc ABC

$\therefore 2\pi r = R\theta$

$\therefore r = \frac{R\theta}{2\pi}$

11.3 $R^2 = r^2+h^2$ (pythag in right-circular cone)

$\therefore h^2 = R^2-r^2$

$\therefore V = \frac{1}{3}\pi r^2\sqrt{R^2-r^2}$

$\therefore V = \frac{1}{3}\pi\left(\frac{R\theta}{2\pi}\right)^2\sqrt{R^2-\left(\frac{R\theta}{2\pi}\right)^2}$

$= \frac{1}{3}\,\frac{R^2\theta^2}{4\pi}\sqrt{\frac{4\pi^2R^2-R^2\theta^2}{4\pi^2}}$

$= \frac{1}{3}\,\frac{R^2\theta^2}{4\pi}\left(\frac{R}{2\pi}\right)\sqrt{4\pi^2-\theta^2}$

$= \frac{R^3\theta^2}{24\pi^2}\sqrt{4\pi^2-\theta^2}$

11.4 $\frac{dV}{d\theta} = \frac{R^3}{24\pi^2}\left\{2\theta(4\pi^2-\theta^2)^{\frac{1}{2}} + \theta^2(4\pi^2-\theta^2)^{-\frac{1}{2}}(-2\theta)\right\}$

for min/max DER = 0

$\therefore\ 2\theta(4\pi^2-\theta^2)^{\frac{1}{2}} - 2\theta^3(4\pi^2-\theta^2)^{-\frac{1}{2}} = 0$

multiply by $(4\pi^2-\theta^2)^{\frac{1}{2}}$

$\therefore\ 2\theta(4\pi^2-\theta^2) - 2\theta^3 = 0$

$\therefore\ \theta = 0$ or $8\pi^2 - 3\theta^2 = 0$

(clearly min) $\therefore\ \underline{\theta = 5,13 \text{ rad}}$

12.1 y-int : $g(0) = \frac{6}{-6} = -1$

x-int : $x^2+6=0$ $\therefore$ no x-int.

12.2 $\lim\limits_{x\to\pm\infty} g(x) = \lim\limits_{x\to\pm\infty} \frac{1+\frac{6}{x^2}}{1+\frac{1}{x}-\frac{6}{x^2}} = \frac{1+0}{1+0-0}$

$= 1$

12.3 Vertical asymptotes when denominator = 0

$\therefore$ when $x^2+x-6=0$ $\therefore (x+3)(x-2)=0$

vert. asymptotes $\therefore$ $\underline{x=-3}$ and $\underline{x=2}$

12.4 $g'(x) = \frac{2x(x^2+x-6)-(x^2+6)(2x+1)}{(x^2+x-6)^2}$

$= \frac{x^2-24x-6}{(x^2+x-6)^2}$

$\therefore$ stationary points when $x^2-24x-6=0$

$\therefore\ x = \frac{24 \pm \sqrt{576+24}}{2} = 12 \pm 5\sqrt{6}$

$\therefore\ x = -0,25$ or $x = 24,25$

$g(-0,25) = -0,98$ $g(24,25) = 0,98$

$$g''(x) = \frac{(2x-24)(x^2+x-6)^2 - 2(x^2-24x-6)(x^2+x-6)(2x+1)}{(x^2+x-6)^4}$$

$g''(-0{,}25) = -0{,}64 < 0 \quad \therefore (-0{,}25\,;\,-0{,}98)$ local max

$g''(24{,}25) = 6{,}67 \times 10^{-5} > 0 \quad \therefore (24{,}25\,;\,0{,}98)$ local min.

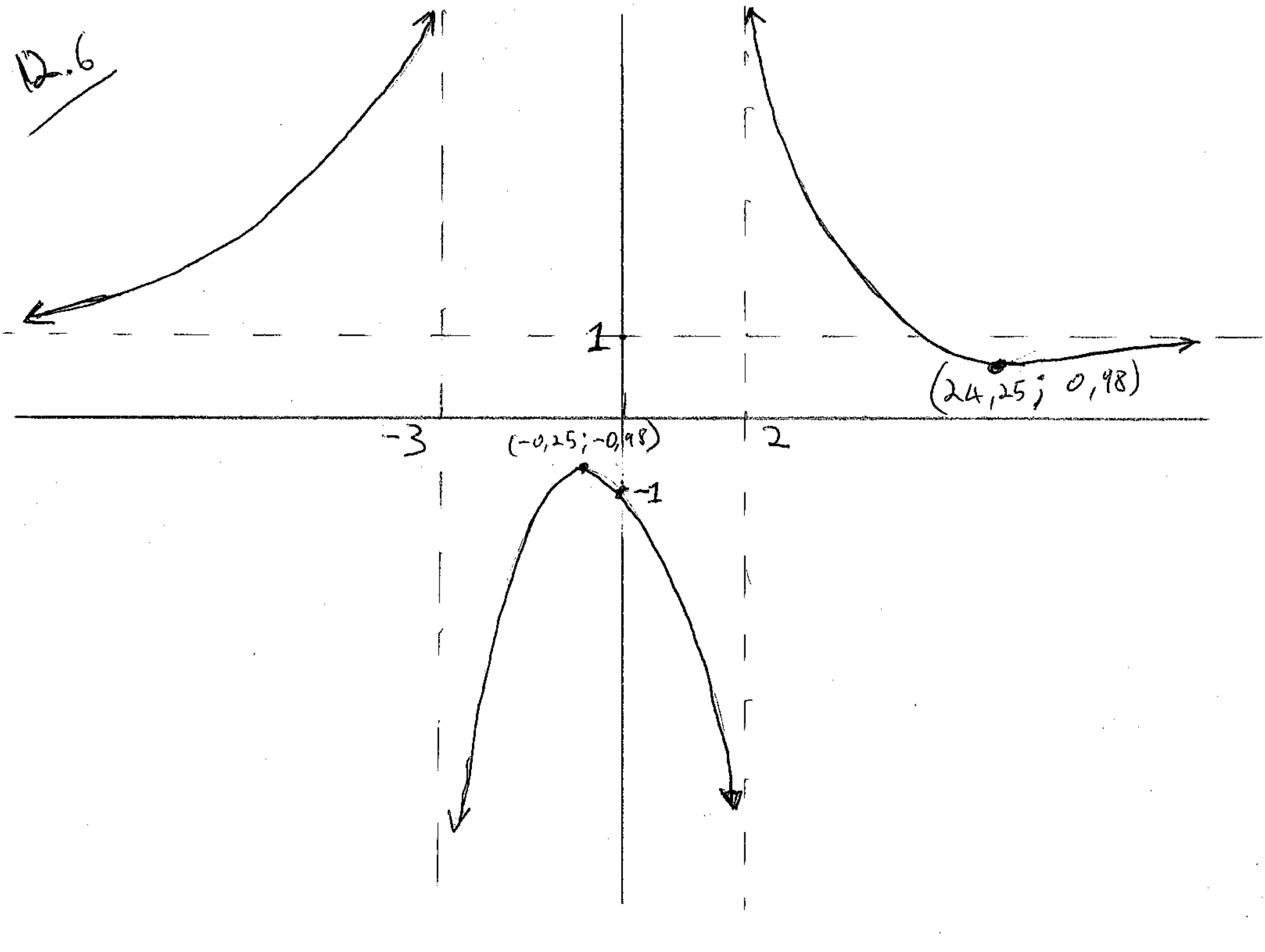
12.5 Domain of $g(x)$: $x \in \mathbb{R} - \{-3\,;\,2\}$

from 12.2 $\lim_{x \to \pm\infty} g(x) = 1 \quad \therefore y = 1$ is horizontal asymptote.

$\lim_{x \to -3^-} g(x) = \frac{+}{0^+} = +\infty \qquad \lim_{x \to -3^+} g(x) = \frac{+}{0^-} = -\infty$

$\lim_{x \to 2^-} g(x) = \frac{+}{0^-} = -\infty \qquad \lim_{x \to 2^+} g(x) = \frac{+}{0^+} = +\infty$

12.6



13.1 equation of graph: $y = e^{-2x} - a$

$\therefore$ y-int $= e^{0} - a = 1 - a$

$\therefore b = 1 - a$

$\therefore a = 1 - b$

13.2 x-int when $x = b$ $\therefore e^{-2b} - a = 0$

$\therefore e^{-2b} - (1-b) = 0$

$\therefore e^{-2b} - 1 + b = 0$

13.3 let $f(b) = e^{-2b} - 1 + b$ $\therefore f'(b) = -2e^{-2b} + 1$

Newton's: $\therefore b_{n+1} = b_n - \dfrac{e^{-2b_n} - 1 + b_n}{-2e^{-2b_n} + 1}$

$b_0 = \frac{1}{2}$; $b_1 = 1$; $b_2 = 0,814439$

$b_3 = 0,797015$

$b_4 = 0,797812$; $b_5 = 0,797812$

14.1 $f(x) = 0$ when $\ln(x) = 0$ $\therefore x = 1$

$\therefore A(1;0)$

14.2 $f'(x) = \dfrac{\left(\frac{1}{x}\right)x^2 - \ln x\,(2x)}{x^4}$

$\therefore$ max when $x - 2x\ln x = 0$

$\therefore x(1 - 2\ln x) = 0$

$\therefore \ln x = \frac{1}{2}$ or $x \neq 0$

$\therefore x = \sqrt{e}$

$f(\sqrt{e}) = \dfrac{\ln e^{\frac{1}{2}}}{(e^{\frac{1}{2}})^2} = \dfrac{\frac{1}{2}}{e} = \dfrac{1}{2e}$

$\therefore M\left(\sqrt{e}; \frac{1}{2e}\right) = M(1,65; 0,18)$

14.3 $\displaystyle\int_1^e \frac{\ln x}{x^2}\,dx \qquad \left[\int fg' = fg - \int gf'\right]$

$f = \ln x \quad \therefore f' = \frac{1}{x}$

$g' = \frac{1}{x^2} \quad \therefore g = -\frac{1}{x}$

$$\therefore \int_1^e \frac{\ln x}{x^2}\,dx = \left[-\frac{\ln x}{x}\right]_1^e - \int_1^e \left(-\frac{1}{x}\right)\left(\frac{1}{x}\right)dx$$

$$= \left(-\frac{1}{e} - 0\right) + \left[-\frac{1}{x}\right]_1^e$$

$$= 1 - \frac{2}{e}$$

$$= 0{,}26 \text{ units}^2$$

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15 $$V = \pi \int_0^a \left(\frac{\arctan(x)}{\sqrt{1+x^2}}\right)^2 dx = \frac{\pi^4}{192}$$

$$\therefore \int_0^a \frac{\arctan^2(x)}{1+x^2}\,dx = \frac{\pi^3}{192}$$

$$\therefore \left[\frac{1}{3}\arctan^3(x)\right]_0^a = \frac{\pi^3}{192}$$

$$\therefore \frac{1}{3}\arctan^3(a) - 0 = \frac{\pi^3}{192}$$

$$\therefore a = \tan\sqrt[3]{\frac{\pi^3}{64}}$$

$$\therefore a = \tan\left(\frac{\pi}{4}\right)$$

$$= 1$$

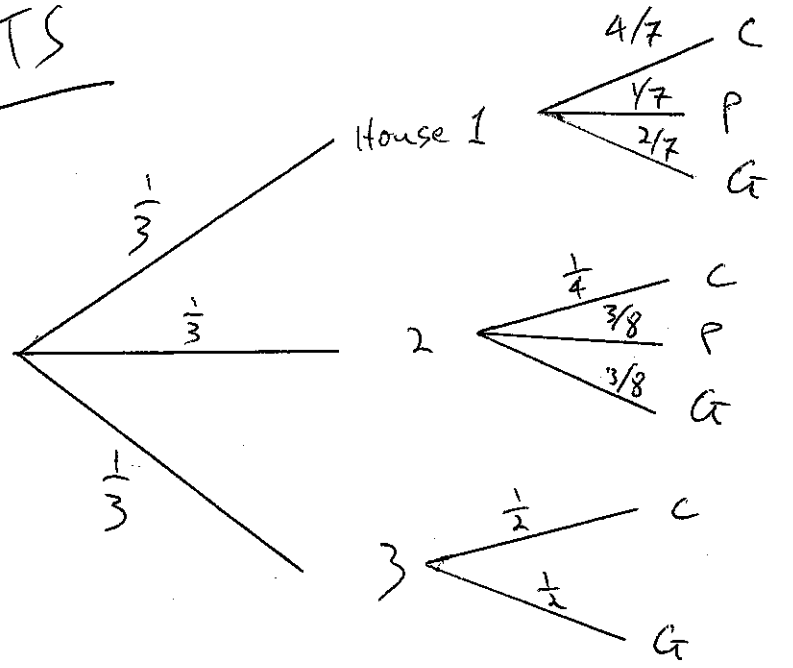
STATS

1.1 All 17 in one room; 6 are grandparents

$\therefore P(G) = \frac{6}{17}$

$= 35,29\%$

1.2



$$P(G) = \frac{1}{3}\left\{\frac{2}{7} + \frac{3}{8} + \frac{1}{2}\right\} = \frac{65}{168} = 38,69\%$$

1.3 $$P(P|G) = \frac{P(G \text{ in a house with } P)}{P(G)}$$

$$= \frac{\frac{1}{3}\left\{\frac{2}{7} + \frac{3}{8}\right\}}{\frac{1}{3}\left\{\frac{2}{7} + \frac{3}{8} + \frac{1}{2}\right\}} = \frac{37}{65} = 56,92\%$$

2

2.1

	Foil	Unwrapped	Total
Choc-covered	7	10	17
Not choc	5	8	13
Total	12	18	30

2.2 $P(\text{Foil}) = \frac{12}{30} = 40\%$

2.3 $P(\text{choc}|\text{unwrapped}) = \frac{10}{18} = 55,56\%$

2.4 $P(\text{unwrapped}|\text{choc}) = \frac{10}{17} = 58,82\%$

2.5 $$P(2 \text{ Foil out of } 4) = \frac{\binom{12}{2}\binom{18}{2}}{\binom{30}{4}} = \frac{374}{1015} = 36,85\%$$

3.1 $\binom{14}{5}\binom{9}{5}\binom{4}{4} = \underline{252\,252}$

3.2.1 more men than women $\therefore$ $\binom{6}{3}\binom{4}{2} + \binom{6}{4}\binom{4}{1} + \binom{6}{5}\binom{4}{0}$

$= \underline{186}$

3.2.2 number of committee of 3 and 2 with particular man and woman together $= \binom{5}{2}\binom{3}{1}$

$\therefore$ number of committees without them together

$$= \binom{6}{3}\binom{4}{2} - \binom{5}{2}\binom{3}{1} = 120 - 0$$

$$= \underline{90}$$

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4 $$b = \frac{n\sum(xy) - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$$

$$= \frac{10(53563) - (218)(2283)}{10(4896) - (218)^2}$$

$$= \underline{26{,}42}$$

line of least squares go through $(\bar{x};\bar{y})$

$\therefore$ sub in $(21{,}8\,;\,228{,}3)$ to $y = bx + a$

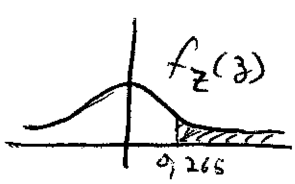
$\therefore$ $228{,}3 = 26{,}42(21{,}8) + a$

$\therefore$ $a = \underline{-347{,}61}$

5.1 $X \sim N(41{,}1\,;\,(3{,}4)^2)$

$$P(X > 42) = P\left(Z > \frac{42 - 41{,}1}{3{,}4}\right)$$

$$= 0{,}5 - H(0{,}2647)$$



$$= 0{,}5 - 0{,}1044$$

$$= 0{,}3956 = \underline{39{,}56\%}$$

using casio: (i) 991ES : in STAT 1-VAR → under DISTR option: R(0,2647)

(ii) 991ZA: MODE→DIST→ 2: Normal CD → Lower 42 → upper 999999 (i.e. ∞) → $\sigma = 3{,}4$ → $\mu = 41{,}1$

5.2 $P(X < 42 \text{ min at least 3 times out of } 5)$

$= P(X > 42 \text{ min at most twice})$

$= \sum_{k=0}^{2} [P(X>42)]^k [1 - P(X>42)]^{5-k}$

$= \binom{5}{0}(0,3956)^0(1-0,3956)^5 + \binom{5}{1}(0,3956)(0,6044)^4 + \binom{5}{2}(0,3956)^2(0,6044)^3$

$= 0,0807 + 0,2640 + 0,3455$

$= 69,01\,\%$

5.3 let $Y \sim N(\mu ; \sigma^2)$ be times for Beethoven's 5^{th}

given $P(Y < 26,5) = 0,1$ and $P(Y > 34,6) = 0,05$

$\therefore P\left(Z < \frac{26,5-\mu}{\sigma}\right) = 0,1$ $\quad \therefore P\left(Z > \frac{34,6-\mu}{\sigma}\right) = 0,05$

From table:

$\therefore \frac{26,5-\mu}{\sigma} = -1,28$ and $\frac{34,6-\mu}{\sigma} = 1,645$

$\therefore \mu = 26,5 + 1,28\sigma \ldots (1)$ $\quad \therefore \mu = 34,6 - 1,645\sigma \ldots (2)$

$\therefore 26,5 + 1,28\sigma = 34,6 - 1,645\sigma$

$\therefore \sigma = 2,77 \text{ min}$

$\therefore \mu = 30,04 \text{ min}$

6.1 $\bar{X} \sim N\left(2{,}1 \,;\, \left(\frac{0{,}9}{\sqrt{20}}\right)^2\right)$

The probability of a false positive is the probability of the above random variable, $\bar{X}$, being less than 1,7.

$\therefore P(\text{Type I error}) = P(\bar{X} < 1{,}7)$

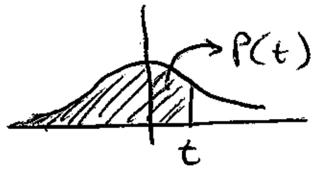
$= P(Z < -1{,}988) \quad \left(\frac{1{,}7 - 2{,}1}{\frac{0{,}9}{\sqrt{20}}}\right)$

$= P(Z > 1{,}988) \quad (\because \text{ of symmetry})$

$= 0{,}5 - 0{,}4767 \quad$ (rounding "Z" to 1,99)

$= 2{,}33\%$

[using STAT mode → DISTR 1) P(t) =



with $t = \dfrac{1{,}7 - 2{,}1}{\left(\frac{0{,}9}{\sqrt{20}}\right)}$: $P(-1{,}9876) = 2{,}34\%$]

6.2 $\bar{Y} \sim N\left(1{,}5 \,;\, \left(\frac{0{,}9}{\sqrt{20}}\right)^2\right)$ (assuming same σ)

$P(\text{false negative}) = P(\bar{Y} > 1{,}7)$

$= P\left(Z > \dfrac{1{,}7 - 1{,}5}{\left(\frac{0{,}9}{\sqrt{20}}\right)}\right)$

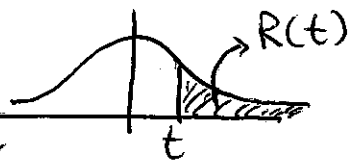
$= P(Z > 0{,}9938)$

$= 0{,}5 - \{H(0{,}99) + 0{,}38[H(1{,}00) - H(0{,}99)]\}$ (interpolation of table values)

$= 0{,}5 - 0{,}3398$

$= 16{,}02\%$

[using STAT → DISTR 3) R(t)



$R\left(\dfrac{1{,}7 - 1{,}5}{\left(\frac{0{,}9}{\sqrt{20}}\right)}\right) = 16{,}02\%$]

7 $H(z) = 0{,}475 \quad \therefore \quad z = 1{,}96$

$$z\sqrt{\frac{p(1-p)}{n}} \leq 0{,}01$$

$$\therefore \quad 1{,}96\sqrt{\frac{0{,}47(1-0{,}47)}{n}} \leq 0{,}01$$

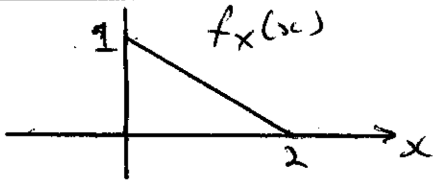
$$\sqrt{\frac{n}{0{,}47(1-0{,}47)}} \geq \frac{1{,}96}{0{,}01}$$

$$\therefore \quad n \geq 0{,}47(1-0{,}47)\left(\frac{1{,}96}{0{,}01}\right)^2$$

$$\therefore \quad n \geq 9569{,}43$$

$\therefore$ minimum of 9570 people required

8.1



clearly $0 \leq f_X(x) \leq 1$

now, RTP (required to prove) $\int_{-\infty}^{\infty} f_X(x)\,dx = 1$ (i.e. total prob = 1)

$$\int_{-\infty}^{\infty} f_X(x)\,dx = \int_0^2 (1 - \tfrac{1}{2}x)\,dx$$

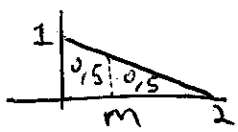
$$= \left[x - \tfrac{1}{4}x^2\right]_0^2 = \left(2 - \tfrac{1}{4}2^2\right) - 0 = 1$$

8.2 $P(X > 1{,}5) = \int_{1{,}5}^{2} (1 - \frac{1}{2}x)\,dx = \left[x - \frac{1}{4}x^2\right]_{1{,}5}^{2}$

$$= \left(2 - \tfrac{1}{4}2^2\right) - \left(1{,}5 - \tfrac{1}{4}1{,}5^2\right)$$

$$= \frac{1}{16} = 6{,}25\%$$

8.3 $\int_0^m (1 - \frac{1}{2}x)\,dx = \frac{1}{2} \quad \therefore \quad m - \frac{1}{4}m^2 = \frac{1}{2}$



$\therefore \quad m^2 - 4m + 2 = 0$

$\therefore \quad m = 2 \pm \sqrt{2}$

but $0 \leq m \leq 2 \quad \therefore \quad m = 2 - \sqrt{2}$

$= 0{,}59$

8.4

$$\mu = \int_{-\infty}^{\infty} x \cdot f_X(x)\,dx$$

$$= \int_0^2 x\left(1 - \tfrac{1}{2}x\right)dx$$

$$= \int_0^2 \left(x - \tfrac{1}{2}x^2\right)dx$$

$$= \left[\tfrac{1}{2}x^2 - \tfrac{1}{6}x^3\right]_0^2$$

$$= \frac{4}{2} - \frac{8}{6}$$

$$= \frac{2}{3}$$