HILTON COLLEGE

TRIAL EXAMINATION
AUGUST 2014

CORE MATHEMATICS PAPER I

Time: 3 hours   150 marks

GENERAL INSTRUCTIONS

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1.   This question paper consists of 10 pages. There is also a **separate** yellow information sheet. Please check that your paper is complete.

2.   Read the questions carefully.

3.   This question paper consists of 12 questions. Answer all questions.

4.   Number your answers exactly as the questions are numbered.

5.   You may use an approved non-programmable and non-graphical calculator, unless a specific question prohibits the use of a calculator.

6.   Round off your answers to one decimal digit where necessary, unless otherwise stated.

7.   All necessary working details must be shown.

8.   It is in your own interest to write legibly and to present your work neatly.

9.   Please note that the diagrams are **NOT** necessarily drawn to scale.

Please do not turn over this page until you are asked to do so

SECTION A

QUESTION 1

(a)   Simplify

   (1) $\dfrac{x^3 - 8}{x^2 + 2x + 4}$ (3)

   (2) $\dfrac{3^{x+2} - 3^x}{3^{x+1}}$ (3)

(b)   Solve:

   (1) $x^2 - 8x - 3 = 0$ by completing the square, give your answers to
          2 decimal places.   (4)

   (2) $3^{x+1} + 3^x = 19$ to 2 decimal places.   (3)

   (3) $6x^2 - 7x - 3 \le 0$ (3)

(c)   If $f(x) = 3x - 2$ *and* $g(x) = x^2$ then determine:

   (1) $f(5)$ (1)

   (2) $f(g(-2))$ (2)

   (3) p if $f(p) = 97$ (2)

$$\boxed{21}$$

QUESTION 2

(a)   Give the next term in each of the following sequences:

   (1) $12 \; ; \; 7 \; ; \; 2 \; ; \; -3 \; ; \; \underline{\qquad}$ (1)

   (2) $128 \; ; \; -64 \; ; \; 32 \; ; \; -16 \; ; \; \underline{\qquad}$ (1)

   (3) $2 \; ; \; 5 \; ; \; 10 \; ; \; 17 \; ; \; \underline{\qquad}$ (2)
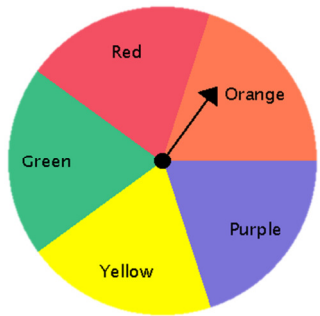
(b)   Consider the sequence: 3 ; 7 ; 11 ; 15 ; .....

      (1)   Determine a formula for the nth term of the sequence   (2)

      (2)   Determine, by means of a formula, which will be the first term   (3)
            to exceed 10 000

(c)   Showing working, determine n if $\displaystyle\sum_{i=1}^{n}(2i-1)=1089$.   (4)

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QUESTION 3

(a)   John has a spinner and a dodecahedral die (12 sided) as shown:

He spins the spinner and tosses the die.

Calculate, giving your answers as simplified fractions in simplest form,
the probability that he:

      (1)   Gets orange on the spinner **and** 8 on the die?   (2)

      (2)   Gets green on the spinner **or** a prime number on the die?   (2)

      (3)   Doesn't get red on the spinner **and** doesn't get 12 on the die?   (2)

(b)   How many different number plates can I make for Gauteng Province (GP) if
they have three letters followed by three numbers followed by GP.
Letters may **not** be repeated but numbers may be repeated.   (3)

> 9

QUESTION 4

(a)   Determine $f'(x)$ by first principles if $f(x) = 2x^2 + 3x + 1$ (5)

(b)   Differentiate the following functions expressing your answers
       with positive exponents where necessary.
       Pay careful attention to notation.
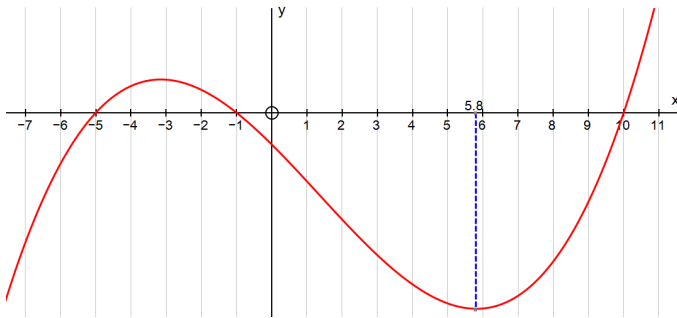
   (1) $g(x) = (2x + 1)(5x - 3)$ (2)

   (2) $y = \dfrac{\sqrt{x} + 3x}{x}$ (4)

(c)   Find the equation of the tangent to the curve $y = x^2 + 4x - 1$ which is
       perpendicular to the line $2y + x = 4$ (5)

$$\boxed{16}$$

QUESTION 5

(a)   The function $y = f(x)$ is drawn below

       Say whether each of the following is positive, negative or zero:

   (1) $f(-5)$ (1)

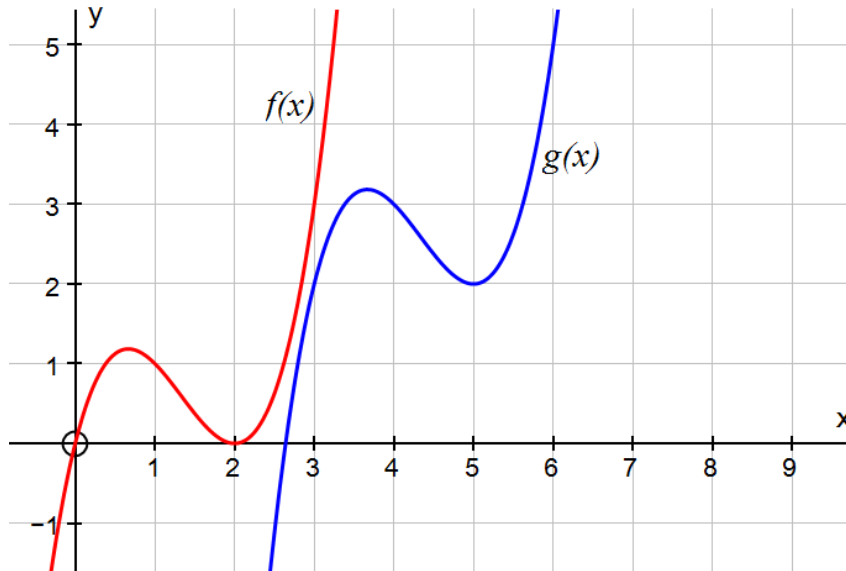   (2) $f'(5.8)$ (1)

   (3) $f''(8)$ (1)

   (4) $f(1)\,f'(1)$ (1)

(b)   A linear function f satisfies the following:

$f(-2)=7$ *and* $f(2)=-1$

Determine the equation of $f^{-1}(x)$ in the form $f^{-1}(x)=...$ (4)

(c)   The function $g(x)=f(x+p)+q$. What are the values of p and q?   (2)

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QUESTION 6

(a)   What annual interest rate, compounded monthly is equivalent to an effective   (3)
annual rate of 14%? Give your answer as a percentage to 2 d.p.

(b)   A man invests some money at 12% p.a. compounded monthly.   (3)
Determine how long, to the nearest month, before his money doubles in value?

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TOTAL FOR SECTION A : 75 MARKS

SECTION B

QUESTION 7

A man has 5 books, 3 by Grisham, 1 by Smith and 1 by Bryson.
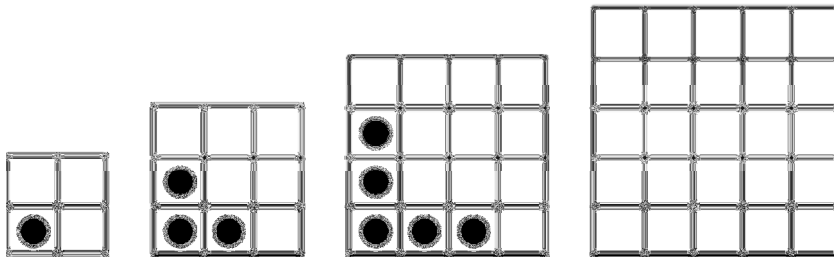
He arranges them on a shelf.

(a)   In how many ways can he do this if there are no restrictions?   (1)

(b)   What is the probability that the 3 books by Grisham are next to one another?   (5)

$$\boxed{6}$$

QUESTION 8

(a)   Consider the following pattern:

In which figure will there be 1371 squares **without** dots?   (5)

(b)   Consider the series $\sum_{i=1}^{\infty}(2x+3)^{i}$

(1)   Determine the value(s) of x for which the series converges.   (3)

(2)   Determine the value of the series if $x=-1.6$.   (2)

(c)   Solve for x if $\sum_{i=1}^{4}\log\left(x^{i}\right)=30$ (5)

$$\boxed{15}$$

QUESTION 9

Mr de Wet wishes to buy a new Hilux as pictured.

The price of the vehicle is R400 000.

ABSA bank is prepared to offer Mr de Wet the following terms on a loan:

- He will be required to pay in a 10% deposit
- The balance will be financed over 54 months with equal monthly payments starting one month from the date of purchase
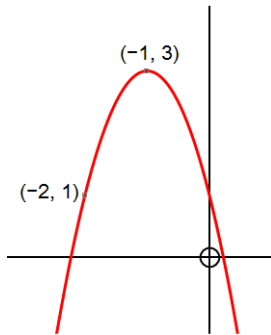- Interest will be charged at 10% per annum compounded monthly

(a)   Calculate his monthly repayments   (4)

(b)   How much will Mr de Wet still owe immediately after making his 40th payment   (3)

(c)   What percentage of his 41st payment will go to repaying interest?   (2)
         Express your answer as a percentage to 2 d.p.

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QUESTION 10

Give the equations of the following graphs **in standard form**:
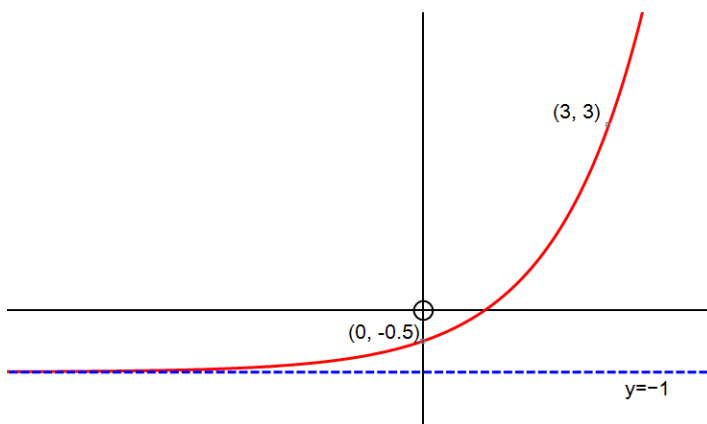
(a)   (4)

(b)   (4)

(c)   The hyperbola $y = \dfrac{12}{x}$ is translated 2 units to the left and 5 units down

     (1)   Give the equation of the translated graph   (2)

   (2)   Give the equation of the axes of symmetry of the translated graph   (4)

(d)   The parabola $y = x^2 + 2x - 8$ is shifted 2 units to the right and 1 unit up.
    The resulting graph is then reflected in the *y*-axis.

    Give, **in standard form**, the equation of the parabola which results.   (4)

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QUESTION 11

(a)   The function $f(x) = ax^3 + x^2 + bx - 4$ has a local minimum at the point $(1; -9)$.   (5)
        Determine the values of a and b.

(b)   A garden has 200 kg of watermelons growing in it. Every day, the
        total mass of watermelon increases by 5 kg. However, every day the
        price per kg of watermelon goes down by 1c.
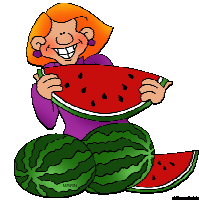
        (1)   If the current price is 90c per kg then what is the crop worth if   (1)
                it is harvested today?

        (2)   What will the crop be worth in t days?   (3)

        (3)   How much longer should the watermelons be left to grow   (3)
                in order to maximize the income?

(c)   A bucket has two pipes entering it. One is filling the tank at a
        variable rate while the other is draining it at a variable rate. The
        volume of water (in litres) in the tank at time t (in hours) with
        $(t \in [0;3])$ is given by $V = -t^3 - 2t^2 + 15t$

        (1)   What is the average rate of flow in litres/hour in the first hour?   (2)

        (2)   Did the volume of water increase or decrease over that time?   (1)

        (3)   What is the instantaneous rate of flow at 2 hours?   (2)

        (4)   At what point in the three hour time interval was the bucket fullest?   (2)
                Give your answer to the nearest minute.

        (5)   What is the maximum volume the bucket contained?   (1)
                Give your answer to the nearest litre.

                                                                                    | 20 |

QUESTION 12

Getting a feel for **BIG** numbers!

The number 7^{2014} is very big! Far too big to display on your calculator.

I wonder how many digits it has?

(a)   How many digits does the number 10^6 have?   (1)

(b)   How many digits would 10^{25} have?   (1)

(c)   Can you solve $7^{2014} = 10^x$?   (4)

(d)   Hence, can you say how many digits 7^{2014} has?   (1)

$$\boxed{7}$$

TOTAL FOR SECTION B : 75 MARKS