HILTON COLLEGE

TRIAL EXAMINATION
AUGUST 2014

CORE MATHEMATICS PAPER I

Time: 3 hours 150 marks

GENERAL INSTRUCTIONS

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This question paper consists of 10 pages. There is also a **separate** yellow formula sheet. Please check that your paper is complete.
2. Read the questions carefully.
3. This question paper consists of 12 questions. Answer all questions.
4. Number your answers exactly as the questions are numbered.
5. You may use an approved non-programmable and non-graphical calculator, unless a specific question prohibits the use of a calculator.
6. Round off your answers to one decimal digit where necessary, unless otherwise stated.
7. All necessary working details must be shown.
8. It is in your own interest to write legibly and to present your work neatly.
9. Please note that the diagrams are **NOT** necessarily drawn to scale.

Please do not turn over this page until you are asked to do so

SECTION A

QUESTION 1

(a) Simplify

(1) $\dfrac{x^3-8}{x^2+2x+4}$ (3)

$=\dfrac{(x-2)(x^2+2x+4)}{x^2+2x+4}$

$=x-2$ ✓ca

(2) $\dfrac{3^{x+2}-3^x}{3^{x+1}}$ (3)

$=\dfrac{3^x(3^2-1)}{3^x\times 3}$ ✓m ✓a

$=\dfrac{8}{3}$

(b) Solve:

(1) $x^2-8x-3=0$ by completing the square, give your answers to 2 d.p. (4)

$\therefore (x-4)^2=3+16$ ✓m ✓a

$\therefore x-4=\pm\sqrt{19}$

$\therefore x=4\pm\sqrt{19}$ ✓ca

$\therefore x=8.36 \quad or \quad -0.36$ ✓ca

(2) $3^{x+1}+3^x=19$ to 3 d.p. (3)

$\therefore 3^x(3+1)=19$ ✓m

$\therefore 3^x=\dfrac{19}{4}$ ✓a

$\therefore x=\log_3\dfrac{19}{4}=1.418$ ✓ca

(3) $6x^2-7x-3\le 0$ (3)

$\therefore (2x-3)(3x+1)\le 0$ ✓m ✓a

$\therefore -\dfrac{1}{3}\le x\le\dfrac{3}{2}$ ✓ca

(c) If $f(x) = 3x - 2$ *and* $g(x) = x^2$ then determine:

(1) $f(5)$ (1)

13 ✓a

(2) $f(g(-2))$ (2)

$= f(4)$ ✓a

$= 10$ ✓ca

(3) p if $f(p) = 97$ (2)

$3p - 2 = 97$ ✓a

$\therefore p = 33$ ✓a

[21]

QUESTION 2

(a) Give the next term in each of the following sequences:

(1) 12 ; 7 ; 2 ; −3 ; $\underline{-8}$ ✓a (1)

(2) 128 ; −64 ; 32 ; −16 ; $\underline{8}$ ✓a for sign ✓a for magnitude (1)

(3) 2 ; 5 ; 10 ; 17 ; $\underline{26}$ ✓a ✓a (2)

(b) Consider the sequence: 3 ; 7 ; 11 ; 15 ; .....

(1) Determine a formula for the n^{th} term of the sequence (2)

$T_n = a + (n-1)d$

$\therefore T_n = 3 + 4(n-1)$ ✓m

$\therefore T_n = 4n - 1$ ✓a need not be simplified

(2) Determine, by means of a formula, which will be the first term to exceed 10 000 (3)

$4n - 1 = 10000$ ✓ca

$\therefore n = \frac{10001}{4} = 2500.25$ ✓ca

$so, \quad T_{2501} \quad will \quad be \quad first \quad to \quad exceed \quad 10000$ ✓ca

(c) Showing working, determine n if $\sum_{i=1}^{n}(2i-1) = 1089$. (4)

$\therefore 1 + 3 + 5 + ... = 1089$ ✓m ✓a

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$\therefore 1089 = \frac{n}{2}\left[2 + (n-1)2\right]$

$\therefore 1089 = \frac{n}{2}[2n]$ ✓a

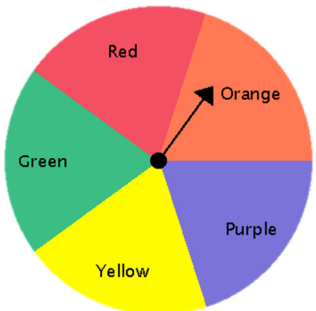
$\therefore n^2 = 1089$

$\therefore n = 33$ ✓a

13

QUESTION 3

(a) John has a spinner and a dodecahedral die (12 sided) as shown:



He spins the spinner and tosses the die.

Calculate, giving your answers as simplified fractions in simplest form, the probability that he:

(1) Gets orange on the spinner **and** 8 on the die? (2)

$P(orange \cap 8) = \frac{1}{5} \times \frac{1}{12} = \frac{1}{60}$ ✓m✓a

(2) Gets green on the spinner **or** a prime number on the die? (2)

✓m

$P(Green \cup prime) = P(Green) + P(prime) - P(Green \cap prime)$

$= \frac{1}{5} + \frac{5}{12} - \frac{5}{60} = \frac{22}{60} = \frac{11}{30}$ ✓a

(3) Doesn't get red on the spinner **and** doesn't get 12 on the die? (2)

$P(not \quad red \cap not \quad 12) = \frac{4}{5} \times \frac{11}{12} = \frac{44}{60}$ ✓m✓a

(b) How many different number plates can I make for Gauteng Province (GP) if they have three letters followed by three numbers followed by GP. Letters may **not** be repeated but numbers may be repeated. (3)

$26 \times 25 \times 24 \times 10 \times 10 \times 10$ ✓a for letters
✓a for numbers

$= 15\,600\,000$ ✓ca

9

QUESTION 4

(a) Determine $f'(x)$ by first principles if $f(x) = 2x^2 + 3x + 1$ (5)

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{2(x+h)^2 + 3(x+h) + 1 - (2x^2 + 3x + 1)}{h} \quad \checkmark\text{m}\checkmark\text{a}$$

$$= \lim_{h \to 0} \frac{2x^2 + 4xh + 2h^2 + 3x + 3h + 1 - 2x^2 - 3x - 1}{h} \quad \checkmark\text{a}$$

$$= \lim_{h \to 0} \frac{4xh + 2h^2 + 3h}{h} \quad \checkmark\text{a}$$

$$= \lim_{h \to 0} \frac{h(4x + 2h + 3)}{h} \quad \checkmark\text{a}$$

$$= 4x + 3 \quad \checkmark\text{a}$$

Penalize up to 3 marks for incorrect notation

(b) Differentiate the following functions expressing your answers with positive exponents where necessary.
Pay careful attention to notation.

(1) $g(x) = (2x+1)(5x-3)$ (2)

$$\therefore g(x) = 10x^2 - x - 3 \quad \checkmark\text{m}$$

$$\therefore g'(x) = 20x - 1 \quad \checkmark\text{a}$$

Penalize notation to the value of 1 mark in total across both (1) and (2)

(2) $y = \dfrac{\sqrt{x} + 3x}{x}$ (4)

$$\therefore y = \frac{x^{\frac{1}{2}}}{x} + 3 \quad \checkmark\text{m}$$

$$\therefore y = x^{-\frac{1}{2}} + 3 \quad \checkmark\text{a}$$

$$\therefore \frac{dy}{dx} = -\frac{1}{2}x^{-\frac{3}{2}} \; \checkmark\text{ca} = -\frac{1}{2x^{\frac{3}{2}}} \quad \checkmark\text{ca}$$

(c) Find the equation of the tangent to the curve $y = x^2 + 4x - 1$ which is perpendicular to the line $2y + x = 4$ (5)

$2y + x = 4 \quad has \quad a \quad slope \quad of \quad -\frac{1}{2}$ ✓m

$so, \quad perpendicular \quad line \quad must \quad have \quad a \quad slope \quad of \quad 2$ ✓ca

$if \quad y = x^2 + 4x - 1 \quad then \quad \frac{dy}{dx} = 2x + 4$ ✓m

$so, \quad 2x + 4 = 2, \quad so \quad x = -1 \quad and \quad y = -4$ ✓a

$\therefore y = 2x + c$

$\therefore -4 = -1(2) + c$

$\therefore c = -2$

$\therefore y = 2x - 2$ ✓ca

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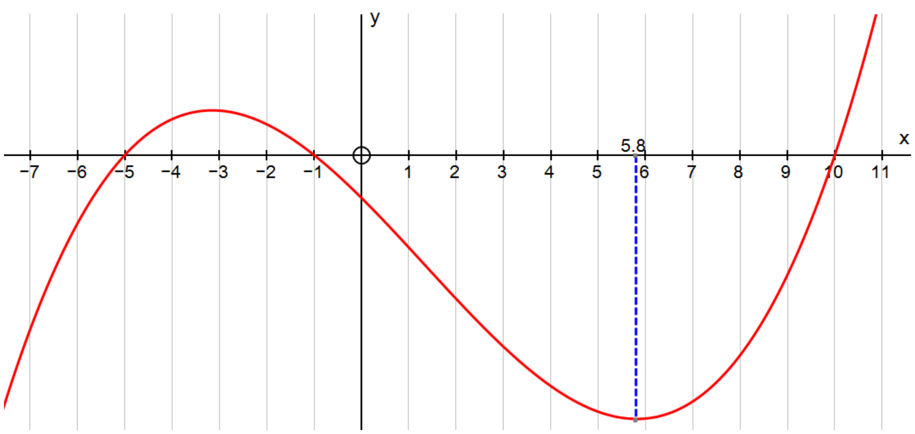
QUESTION 5

(a) The function $y = f(x)$ is drawn below



Say whether each of the following is positive, negative or zero:

(1) $f(-5) = 0$ ✓a (1)

(2) $f'(5.8) = 0$ ✓a (1)

(3) $f''(8) > 0$ ✓a (1)

(4) $f(1)f'(1) > 0$ ✓a (1)

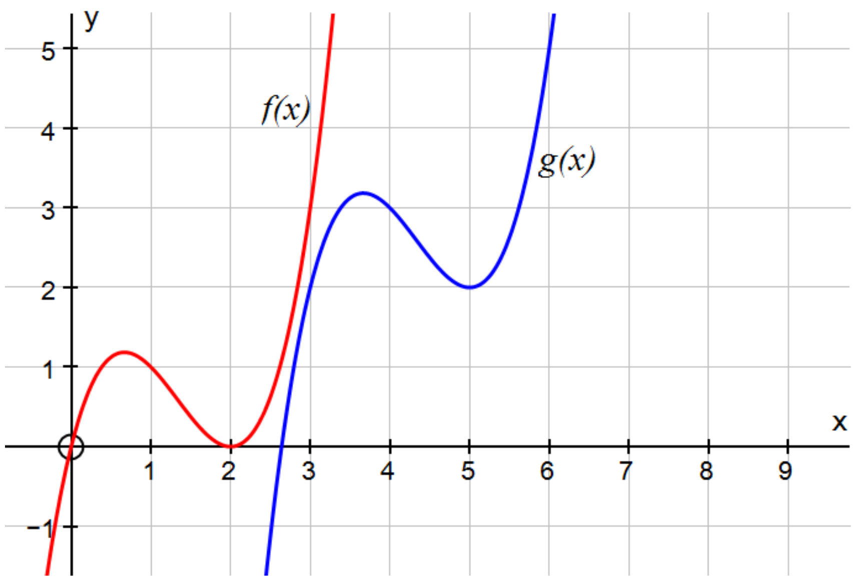
(b) A linear function *f* satisfies the following:

$f(-2)=7 \quad and \quad f(2)=-1$

Determine the equation of $f^{-1}(x)$ in the form $f^{-1}(x)=...$ (4)

we want a line which goes through
$(7;-2) \quad and \quad (-1;2)$ ✓a
$m=\frac{-2-2}{7--1}=-\frac{1}{2}$ ✓m
$\therefore y=-\frac{1}{2}x+c$
$-2=-\frac{1}{2}(7)+c$ ✓a
$c=\frac{3}{2}$
$\therefore y=-\frac{1}{2}x+\frac{3}{2}$ ✓ca

(c) The function $g(x)=f(x+p)+q$. What are the values of *p* and *q*? (2)



$p=-3 \quad and \quad q=2$
✓a ✓a

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QUESTION 6

(a) What annual interest rate, compounded monthly is equivalent to an effective annual rate of 14%? Give your answer as a percentage to 2 d.p. (3)

$$consider \quad R1 \quad for \quad 1 \quad year$$

$$1\left(1+\frac{i}{12}\right)^{12}=1(1+0.14)^{1} \quad ✓\text{a}$$

$$\therefore 1+\frac{i}{12}=1.14^{\frac{1}{12}} \quad ✓\text{m}$$

$$\therefore \frac{i}{12}=1.14^{\frac{1}{12}}-1$$

$$\therefore i=12\left(1.14^{\frac{1}{12}}-1\right)=13.17\% \quad ✓\text{ca}$$

(b) A man invests some money at 12% p.a. compounded monthly. (3)
Determine how long, to the nearest month, before his money doubles in value?

$$A=P(1+i)^{n}$$

$$\therefore 2x=x\left(1+\frac{0.12}{12}\right)^{n} \quad ✓\text{m} ✓\text{a}$$

$$\therefore 2=1.01^{n}$$

$$\therefore n=\log_{1.01}2=70 \quad months \quad ✓\text{ca}$$

6

TOTAL FOR SECTION A : 75 MARKS

SECTION B

QUESTION 7

A man has 5 books, 3 by Grisham, 1 by Smith and 1 by Bryson.

He arranges them on a shelf.

(a) In how many ways can he do this if there are no restrictions? (1)

$5! = 120$ ✓a

(b) What is the probability that the 3 books by Grisham are next to one another? (5)

if we put the three books by Grisham in a rubber band then there are effectively 3 books ✓m

These can be arranged in $3!3! = 36$ *ways* ✓a ✓a

because of the different possible orderings of the Grisham books within the rubber band

so, the probability that the Grisham books will be together $= \frac{36}{120} = \frac{3}{10}$ *or* 30% ✓m ✓ca

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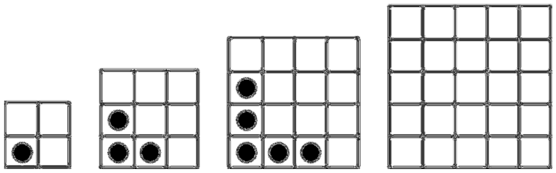
QUESTION 8

(a) Consider the following pattern:



In which figure will there be 1371 squares **without** dots? (5)

Number of squares without dots follows the pattern:
3 ; 6 ; 11 ; 18 ; 27 ; …..
Quadratic sequence with second difference of 2 so ✓m

$T_n = an^2 + bn + c$

$and \quad a = 1$ ✓a

$c = T_0 = 2$

$T_1 = 1 + b + 2 = 3 \quad so \quad b = 0$ ✓a

$\therefore T_n = n^2 + 2$ ✓a

$\therefore 1371 = n^2 + 2$

$\therefore n = 37$ ✓ca

(b) Consider the series $\sum_{i=1}^{\infty} (2x+3)^i$

(1) Determine the value(s) of x for which the series converges. (3)

$\sum_{i=1}^{\infty} (2x+3)^i = (2x+3) + (2x+3)^2 + (2x+3)^3 + ...$ ✓m

$r = 2x + 3$

$convergence \quad when \quad -1 < r < 1$ ✓a

$\therefore -1 < 2x + 3 < 1$

$\therefore -4 < 2x < -2$

$\therefore -2 < x < -1$ ✓ca

(2) Determine the value of the series if $x = -1.6$. (2)

$$S_\infty = \frac{a}{1-r} \quad \checkmark\text{m}$$
$$= \frac{2x+3}{1-(2x+3)}$$
$$= \frac{2x+3}{-2x-2}$$
$$= \frac{2(-1.6)+3}{-2(-1.6)-2}$$
$$= \frac{-0.2}{1.2}$$
$$= -\frac{1}{6} \quad or \quad -0.1\dot{6} \quad \checkmark\text{ca}$$

(c) Solve for x if $\sum_{i=1}^{4} \log\left(x^i\right) = 30$ (5)

$$\sum_{i=1}^{4} \log\left(x^i\right) = \log x + \log x^2 + \log x^3 + \log x^4 = 30 \quad \checkmark\text{m}\checkmark\text{a}$$
$$\therefore \log x + 2\log x + 3\log x + 4\log x = 30$$
$$\therefore 10\log x = 30 \quad \checkmark\text{ca}$$
$$\therefore \log x = 3$$
$$\therefore x = 1000 \quad \checkmark\text{ca}$$

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QUESTION 9

Mr de Wet wishes to buy a new Hilux as pictured.

The price of the vehicle is R400 000.

ABSA bank is prepared to offer Mr de Wet the following terms on a loan:

- He will be required to pay in a 10% deposit
- The balance will be financed over 54 months with equal monthly payments starting one month from the date of purchase
- Interest will be charged at 10% per annum compounded monthly

(a) Calculate his monthly repayments (4)

$$P_v = x\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$\therefore 360\,000 = x\left[\frac{1-\left(1+\frac{0.1}{12}\right)^{-54}}{\frac{0.1}{12}}\right] \quad \checkmark a \; \checkmark a \; \checkmark m$$

$$\therefore 360\,000 = 43.34181446x$$

$$\therefore x = R8\,306.07 \quad \checkmark ca$$

(b) How much will Mr de Wet still owe immediately after making his 40th payment (3)

$$\text{Present value of 14 payments} \quad \checkmark m$$

$$= 8306.07\left[\frac{1-\left(1+\frac{0.1}{12}\right)^{-14}}{\frac{0.1}{12}}\right] \quad \checkmark a$$

$$= R109\,329.08 \quad (note: \; R109\,329.03 \; with \; full \; accuracy) \quad \checkmark ca$$

(c) What percentage of his 41st payment will go to repaying interest? (2)
Express your answer as a percentage to 2 d.p.

$$\frac{\frac{0.1}{12}\times 109329.08}{8306.07}\times 100 \quad \checkmark ca$$

$$= \frac{911.08}{8306.07}\times 100$$

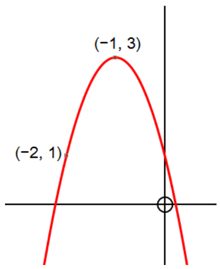
$$= 10.97\% \quad \checkmark ca$$

9

QUESTION 10

Give the equations of the following graphs **in standard form**:

(a) (4)



$y = a(x+1)^2 + 3$ ✓m✓a

$\therefore 1 = a(-2+1)^2 + 3$

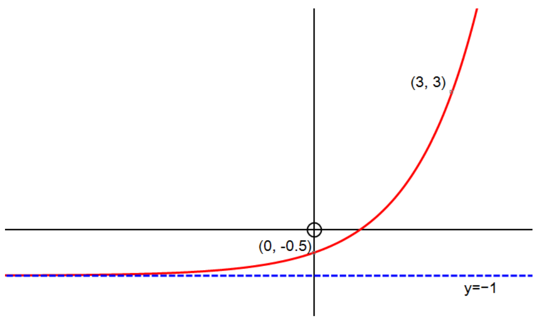
$\therefore 1 = a + 3$

$\therefore a = -2$ ✓a

$\therefore y = -2(x+1)^2 + 3$

$\therefore y = -2x^2 - 4x + 1$ ✓ca

(b) (4)



$y = ab^x + c$

$\therefore y = ab^x - 1 \quad now \quad sub \quad (0;-0.5)$ ✓m✓a

$\therefore -0.5 = a - 1$

$\therefore a = 0.5$ ✓a

$\therefore y = 0.5b^x - 1 \quad now \quad sub \quad (3;3)$

$\therefore 3 = 0.5b^3 - 1$

$\therefore 4 = 0.5b^3$

$\therefore 8 = b^3$

$\therefore b = 2$

$\therefore y = \frac{1}{2} \times 2^x - 1$ ✓ca

(c) The hyperbola $y=\dfrac{12}{x}$ is translated 2 units to the left and 5 units down

(1) Give the equation of the translated graph (2)

$y=\dfrac{12}{x+2}-5$ ✓a for magnitude, ✓a for direction

(2) Give the equation of the axes of symmetry of the translated graph (4)

$$\begin{aligned}&\textit{before translation}\\&y=x \quad \textit{and} \quad y=-x \quad \checkmark\text{a}, \checkmark\\&\textit{after translation}\\&y=x+2+5 \quad \textit{and} \quad y=-(x+2)-5\\&y=x+7 \quad \textit{and} \quad y=-x-7 \quad \checkmark\text{a}, \checkmark\text{a}\end{aligned}$$

(d) The parabola $y=x^2+2x-8$ is shifted 2 units to the right and 1 unit up.
The resulting graph is then reflected in the *y*-axis.

$$\begin{aligned}&y=x^2+2x-8\\&\textit{shifted 2 right and 1 up is}\\&y=(x-2)^2+2(x-2)-8+1 \quad \checkmark\text{a}, \checkmark\text{a}\\&\therefore y=x^2-4x+4+2x-4-7\\&\therefore y=x^2-2x-7\\&\textit{reflected in } y-\textit{axis}\\&y=(-x)^2-2(-x)-7 \quad \checkmark\text{ca}\\&\therefore y=x^2+2x-7 \quad \checkmark\text{ca}\end{aligned}$$

Give, **in standard form**, the equation of the parabola which results. (4)

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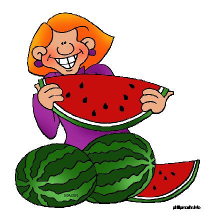
QUESTION 11

(a) The function $f(x) = ax^3 + x^2 + bx - 4$ has a local minimum at the point $(1;-9)$. (5)
Determine the values of *a* and *b*.

$$f(x) = ax^3 + x^2 + bx - 4$$
$$f'(x) = 3ax^2 + 2x + b \quad \checkmark \text{m}$$
$$f(1) = -9 \quad and \quad f'(1) = 0$$
$$a + 1 + b - 4 = -9 \quad and \quad 3a + 2 + b = 0 \quad \checkmark \text{a} \checkmark \text{a}$$
$$\therefore a + b = -6 \quad and \quad 3a + b = -2 \quad \checkmark \text{m}$$
$$subtracting \ the \ left \ hand \ equation \ from \ the \ right \ hand \ one \ gives:$$
$$2a = 4$$
$$\therefore a = 2 \quad and \quad b = -8 \quad \checkmark \text{ca}$$

(b) A garden has 200 kg of watermelons growing in it. Every day, the total mass of watermelon increases by 5 kg. However, every day the price per kg of watermelon goes down by 1c.



(1) If the current price is 90c per kg then what is the crop worth if it is harvested today? (1)

$$200 \times 0.9$$
$$= R180 \quad \checkmark \text{a}$$

(2) What will the crop be worth in *t* days? (3)

$$\overset{\checkmark \text{a}}{V} = (\overset{\checkmark \text{a}}{200 + 5t})(\overset{\checkmark \text{a}}{90 - t})$$

(3) How much longer should the watermelons be left to grow in order to maximize the income? (3)

$$V = (200 + 5t)(90 - t)$$
$$V = 18000 + 250t - 5t^2$$
$$\therefore \frac{dV}{dt} = 250 - 10t = 0 \quad \checkmark \text{m} \checkmark \text{a}$$
$$\therefore t = 25 \quad days \quad \checkmark \text{ca}$$

(c) A bucket has two pipes entering it. One is filling the tank at a variable rate while the other is draining it at a variable rate. The volume of water (in litres) in the tank at time *t* (in hours) with $\left(t \in [0;3]\right)$ is given by $V = -t^3 - 2t^2 + 15t$

(1) What is the average rate of flow in litres/hour in the first hour? (2)

$$at \quad t = 0, \quad V = 0 \quad \checkmark a$$
$$at \quad t = 1, \quad V = 12$$
$$average \quad flow = 12 \quad l/hr \quad \checkmark ca$$

(2) Did the volume of water increase or decrease over that time? (1)

Increase ✓a

(3) What is the instantaneous rate of flow at 2 hours? (2)

$$V = -t^3 - 2t^2 + 15t$$
$$\frac{dV}{dt} = -3t^2 - 4t + 15 \quad \checkmark m$$
$$at \quad t = 2, \quad \frac{dV}{dt} = -5 \quad l/hr \quad \checkmark a$$

(4) At what point in the three hour time interval was the bucket fullest? Give your answer to the nearest minute. (2)

$$\frac{dV}{dt} = -3t^2 - 4t + 15 = 0 \quad \checkmark a$$
$$\therefore t = \frac{5}{3} \quad (silver \quad fox)$$
$$\therefore 1 \quad hour \quad 40 \quad minutes \quad \checkmark ca$$

(5) What is the maximum volume the bucket contained? Give your answer to the nearest litre. (1)

$$V = -\left(\frac{5}{3}\right)^3 - 2\left(\frac{5}{3}\right)^2 + 15\left(\frac{5}{3}\right) = 15 \quad litres \quad \checkmark ca$$

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QUESTION 12

Getting a feel for **BIG** numbers!

The number 7^{2014} is very big! Far too big to display on your calculator.

I wonder how many digits it has?

(a) How many digits does the number 10^6 have? (1)

7 ✓a

(b) How many digits would 10^{25} have? (1)

26 ✓a

(c) Can you solve $7^{2014} = 10^x$? (4)

$7^{2014} = 10^x$

$\therefore x = \log 7^{2014}$ ✓m ✓a

$\therefore x = 2014 \log 7$ ✓ca

$\therefore x = 1702.03$ ✓ca

(d) Hence, can you say how many digits 7^{2014} has? (1)

1703 ✓ca

7

Just for fun, here they are! Count them if you wish!

1065252565271409287182039213944812502445146327826217629384881706064828004970995827709805292311З
2904343565085561380874189301886175608694234671283842188861178337051472501482699987409233667209З
4259972004102789576454556051609278295019289751909521713885276886868545098359617108227857146804Ч
20760708689757188451237930447143923829275728702619275843679009769100346130696848686480251571691
95363160082758033700768382179717431632265516816852666431337240100320186637468984287055463569841
7664393912010187038567048887527465780532569243341114915000927680783632462778770883912771770022Ч
3755731818541186218801532526744371139531678135912527048048761530700951044106387760488103195395l
8698353711425038516599240470811863504264495999838111670846248097206663980235246119317932108826Ч
2827553106894414936973016810296386338225940577543170283711673427798255472538019819720639854131O
0599157793512914103224204715859109628636359128274118551954485803559699112363426253775047611693б
1672408660551317812452476015036395903878191698225383979067119142108214425986965499845426633357б
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3680678147756352786835185211078635017461125753534767047208162985517249392380215436922060828329Э
0532832743404074802273528276535824222812368254306127248824873227433630299481755569927162938632Э
1869337993391714475249861559976365829718615933994295286265467511514664302766530161636453009849Э
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TOTAL FOR SECTION B : 75 MARKS