HILTON COLLEGE

TRIAL EXAMINATION
AUGUST 2014

MATHEMATICS: PAPER II

Time: 3 hours 150 marks

GENERAL INSTRUCTIONS

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This question paper consists of 14 pages. You are provided with a separate **Answer Booklet for Geometry questions** and an **Information Sheet**. Please check that your paper is complete.

2. Read the questions carefully.

3. This question paper consists of 16 questions. Answer all questions.

4. Number your answers exactly as the questions are numbered.

5. You may use an approved non-programmable and non-graphical calculator, unless a specific question prohibits the use of a calculator.

6. Round off your answers to **one decimal** digit where necessary, unless otherwise stated.

7. All necessary working details must be shown.

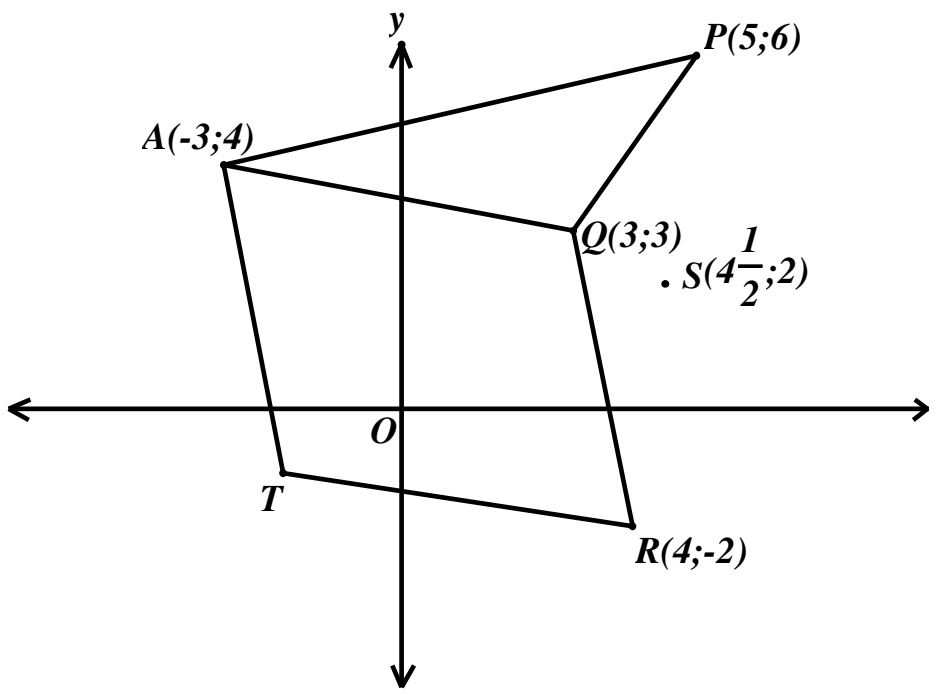
8. It is in your own interest to write legibly and to present your work neatly.

9. Please note that the diagrams are **NOT** necessarily drawn to scale.

Please do not turn over this page until you are asked to do so

SECTION A

QUESTION 1



In the figure, A(-3;4), P(5;6), Q(3;3), R(4;-2) and $S\left(4\frac{1}{2};2\right)$ are given.

AQRT is a parallelogram.

(a) Determine the perimeter of Δ AQP. Give your answer correct to one decimal digit. (4)

(b) Show that P, S and R are collinear. (3)

(c) Determine the midpoint of AR and hence or otherwise determine the coordinates of T. (5)

12 MARKS

QUESTION 2

The following information represents the amount of beef imported to South Africa over 11 years in 1000 tons.

78	54	78	93	68	82	91	42	62	39	48

(a) Calculate the mean amount of beef imported over 11 years. (1)

(b) Calculate the standard deviation and use this to comment on beef imports over the last 11 years in South Africa. (2)

(c) Give the 5 number summary of the data set. (5)

(d) Comment on the skewness of the data by making reference to mean and median. (2)

(e) If an outlier is a value of greater than $Q_3 + 1,5 \times (Inter - quartile\ range)$ or less than $Q_1 - 1,5 \times (Inter - quartile\ range)$, show that there are no outliers in the data set. (3)

13 MARKS

QUESTION 3

Gross Domestic Product (GDP) is the monetary value of all the finished goods and services produced within a country in a specific time period.

The following table shows a comparison of the percentage contribution that Agriculture and Manufacturing respectively made to the country's GDP, over an 11 year period.

YEAR	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
Agriculture (x)	4,2	3,4	3,1	2,7	2,9	3,0	2,9	3,0	2,6	2,5	2,6
Manufacturing (y)	19,2	19,4	19,2	18,5	17,5	17,0	16,8	15,2	14,2	12,8	12,4

(a) Determine the least squares regression line for the data set. Give your answer correct to 4 decimal digits. (3)

(b) By the process of extrapolation, use your answer 3(a) to determine the percentage (%) that manufacturing will contribute to GDP if Agriculture drops to 1,2% in 2016. (2)

(c) Comment on the reliability of making a prediction through extrapolation in this context. (2)

(d) Determine the correlation coefficient of the data set and comment on what it tells you. (2)

9 MARKS

QUESTION 4

Given: $\sin\beta = \frac{8}{17}$ and $90° \leq \beta \leq 270°$

With the aid of a sketch and without the use of a calculator, calculate:

(a) $\tan\beta$ (3)

(b) $\sin(90° + \beta)$ (2)

(c) $\cos 2\beta$ (3)

8 MARKS

QUESTION 5

Simplify without using a calculator:

(a) $\frac{\sin(180° + \alpha)}{\tan(\alpha - 180°).\cos(180° - \alpha)} - \sin(90° - \alpha).\cos(-\alpha)$ (6)

(b) $\frac{\sin 190° \cos 225° \tan 390°}{\cos 100° \sin 135°}$ (6)

12 MARKS

QUESTION 6

Given the equation: $\tan(5\theta) = \tan\theta$

(a) Write down the general solution . (3)

(b) Write down the value(s) of $\theta \in [-90°; 90°]$ for which $\tan\theta$ is undefined. (1)

(c) Hence or otherwise write down the values of $\theta \in [-90°; 90°]$ which satisfy the equation. (3)

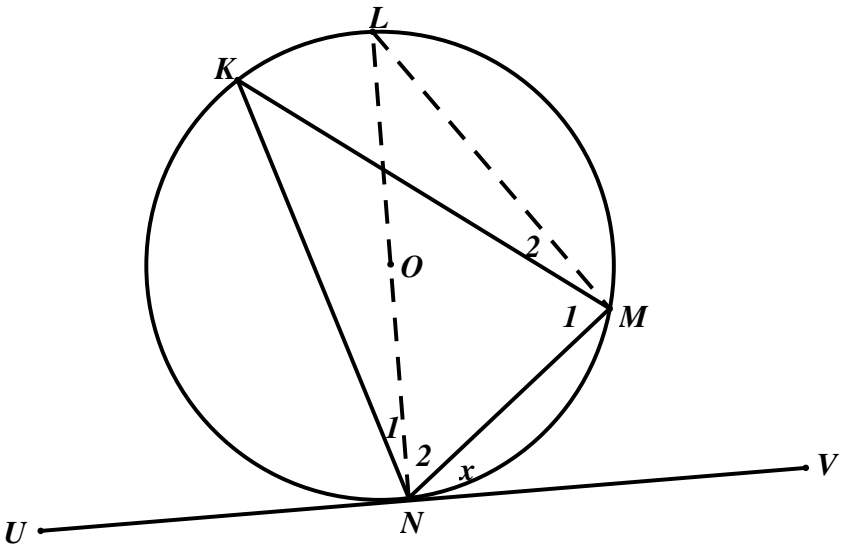
7 MARKS

QUESTION 7 ANSWER THIS QUESTION IN THE GEOMETRY BOOKLET

In the figure, O is the centre of the circle with diameter LN.

$V\hat{N}M = x$
UV is a tangent to the circle at *N*.

Prove that $\hat{K} = x$



Fill in the missing parts of the statements/reasons.

(a) $\hat{N}_2 =$_____ ; _____ to tangent.

(b) $\hat{M}_1 + \hat{M}_2 =$ _____ ; _____

(c) $\hat{L} =$ _____ ; _____

(d) Therefore $\hat{K} = x$; _____

(e) The angle formed between a tangent and chord _____. (4)

4 MARKS

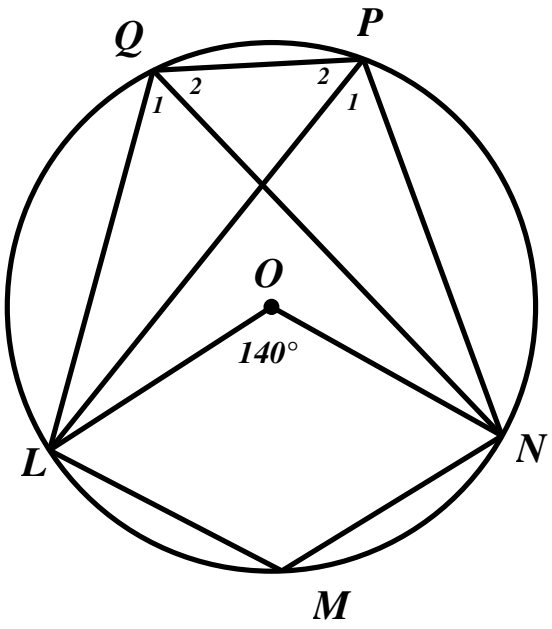
QUESTION 8 ANSWER THIS QUESTION IN THE GEOMETRY BOOKLET

(a) Complete the following statements:

(1) The opposite interior angles of a cyclic quadrilateral are …. (1)

(2) A line from the centre of a circle to the midpoint of a chord, is …… to the chord. (1)

(b) Refer to the diagram: O is the centre of the circle. $L\hat{O}N = 140°$



State whether the following statements are TRUE or FALSE:

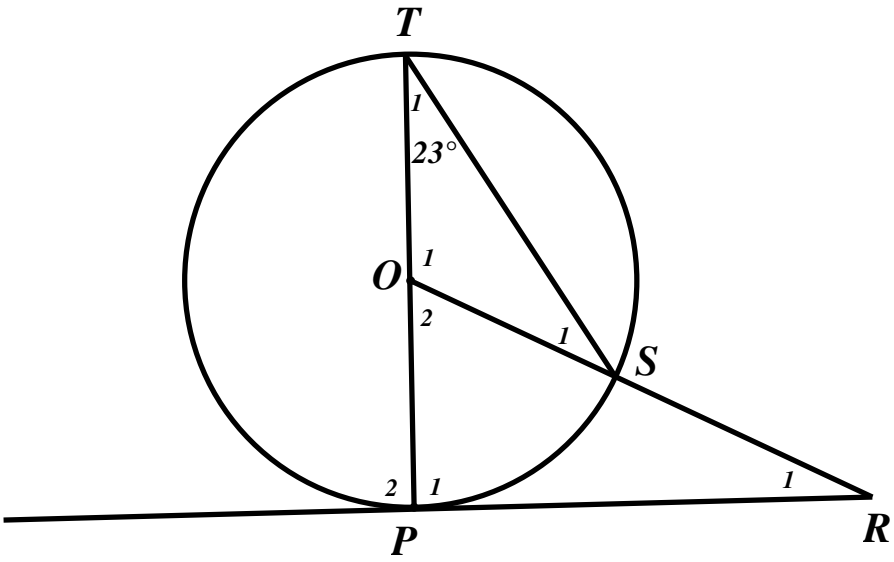
(1) $\hat{M} = 40°$ (1)

(2) $\hat{Q}_1 = 70°$ (1)

(3) PNML is a cyclic quadrilateral. (1)

(4) $\hat{P}_2 = \hat{M}$ (1)

(c) In the diagram below, O is the centre of the circle and TP is the diameter. PR is a tangent to the circle at P and $\hat{T}_1 = 23°$



Find the size of $\hat{R}_1$, giving all reasons. (4)

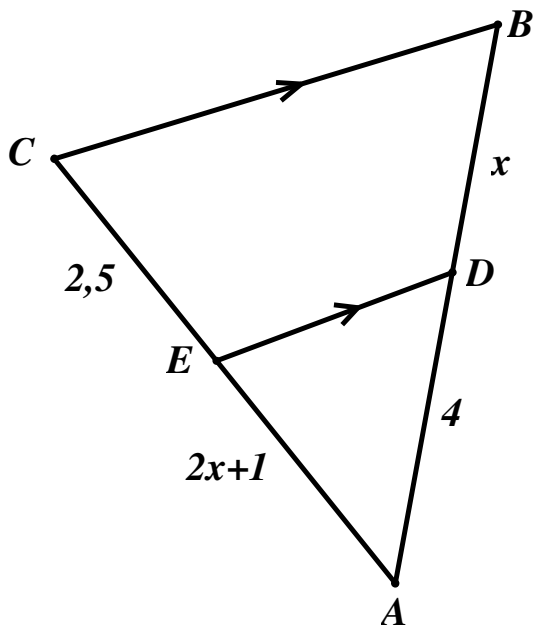
10 MARKS

TOTAL FOR SECTION A: 75 MARKS

SECTION B

QUESTION 9 **ANSWER THIS QUESTION IN THE GEOMETRY BOOKLET**

Refer to the diagram below to determine, with reasons, the value of x.



4 MARKS

QUESTION 10

Prove the following identity:

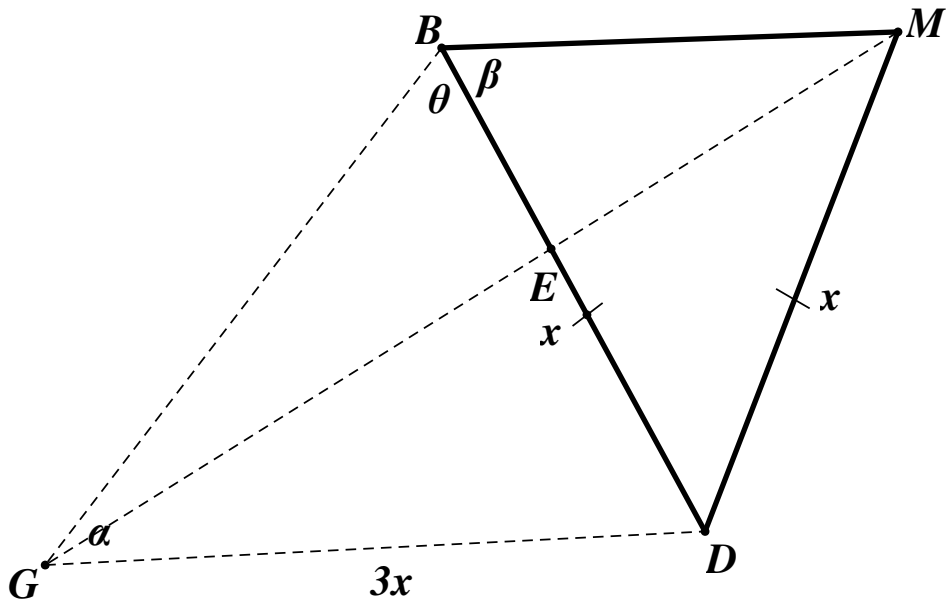
$$\frac{\cos 2\beta}{(\cos\beta+\sin\beta)^3}=\frac{\cos\beta-\sin\beta}{1+\sin 2\beta}$$

5 MARKS

QUESTION 11

A section of this picture of an irrigation system is represented in the figure below. (Not drawn to scale)

$BD = DM = x$
$GD = 3x$
$B\hat{G}D = \alpha$
$G\hat{B}D = \theta$
$D\hat{B}M = \beta$



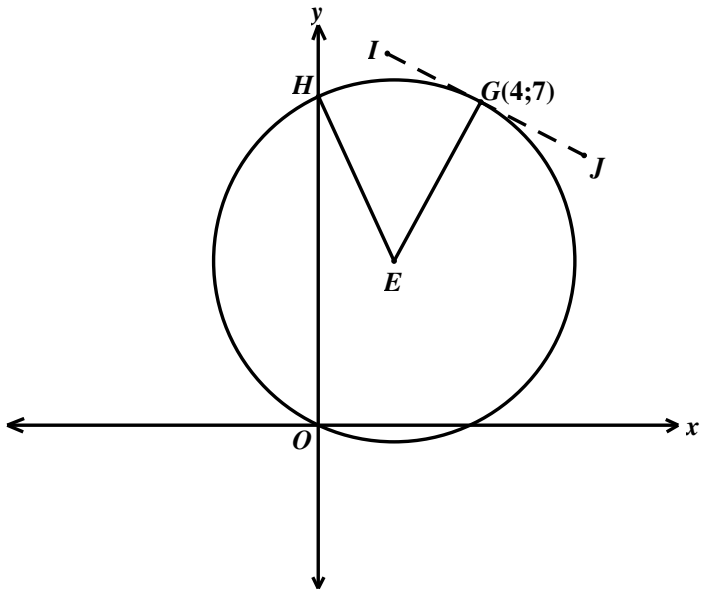
(a) Show that the area of $\Delta BMD = \frac{1}{2}x^2 \sin 2\beta$ (4)

(b) Show that $BG^2 = 2x^2[5 + 3\cos(\alpha + \theta)]$ (4)

8 MARKS

QUESTION 12

The following picture is represented on the figure below.
The centre is represented by the letter E, H is the y-intercept of the circle and the x-intercept is at the origin, O.



(a) Determine the centre (E) and radius correct to one decimal digit of the circle, if its equation is: $x^2 - 4x + y^2 - 7y = 0$ (4)

(b) Determine the coordinate H. (3)

(c) Calculate $E\hat{H}O$ correct to one decimal digit. (4)

(d) If IJ is the tangent to the circle at $G(4;7)$, determine the equation of this tangent. (4)

15 MARKS

QUESTION 13

Given two circles with equations:
$x^2 + y^2 + 2mx - 6y = 2$ and $(x-5)^2 + (y+n)^2 = p^2$

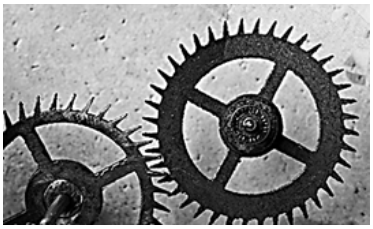
(a) Find m and n if the circles are concentric i.e. they have the same centre. (4)

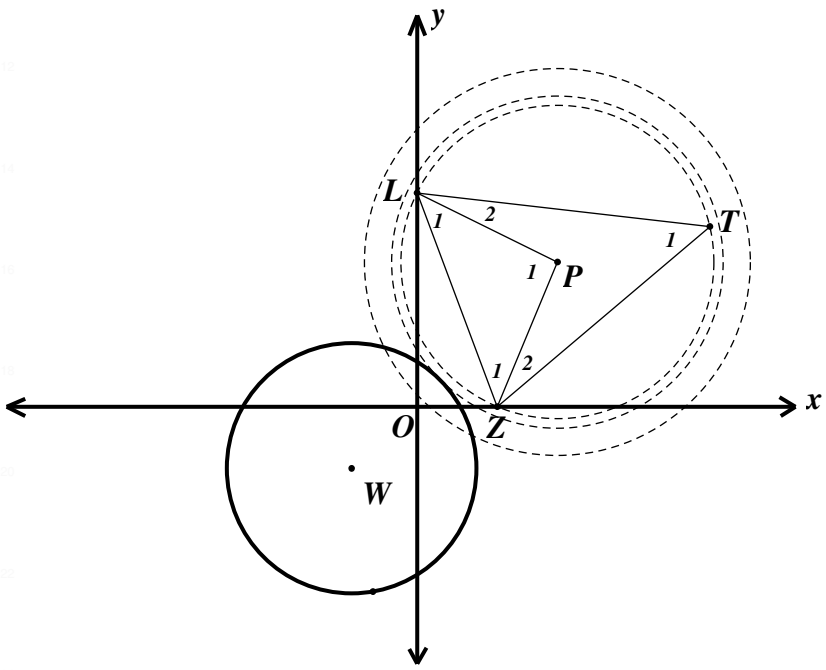
(b) Find two values of p if it is further given that the radii of the circles differ by 2 units. (3)

7 MARKS

QUESTION 14

The picture is represented on the figure below.





The equation of the circle with centre P is: $(x-4)^2+(y-3)^2=21$

The equation of the circle with centre W is: $(x+3)^2+(y+1)^2=10$

L is the y-intercept of circle with centre P and Z is the x-intercept of the same circle.
T is a point on the circle with centre P.
The dashed lines represent the possible positions of circle with centre P.
Circle with centre W is fixed.

(a) Determine whether the two circles intersect, touch at a point or do not intersect. Show all working. (4)

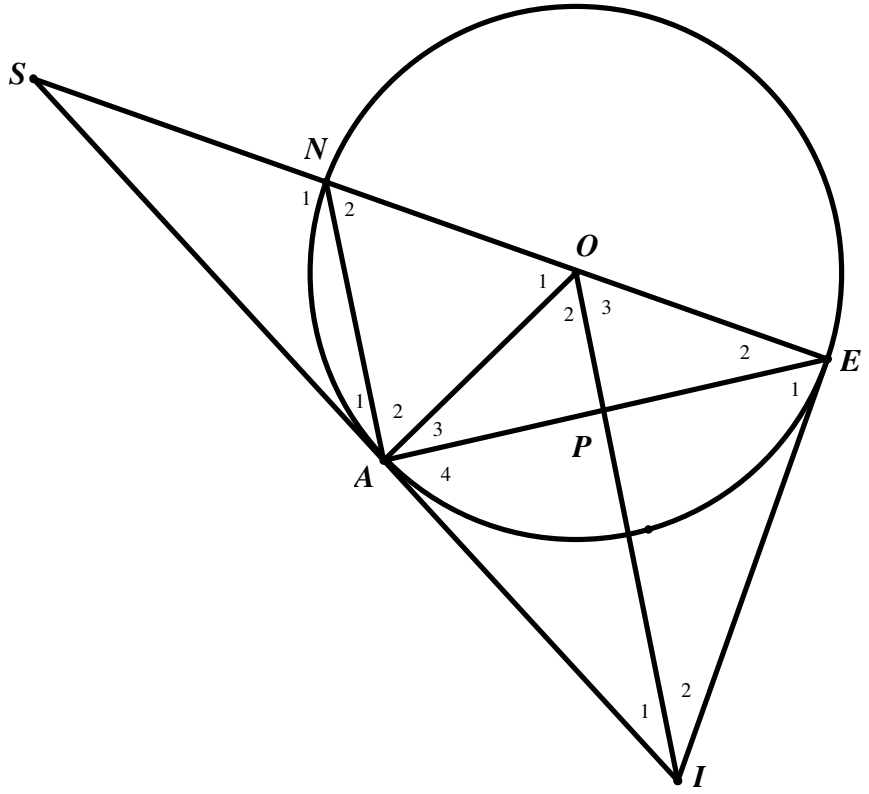
(b) If $\hat{Z}_1 = x$, determine $\hat{T}_1$ in terms of . Give reasons. (3)

(c) If it is further given that $P\hat{Z}O = 135°$ and $Z\left(\frac{1}{2};0\right)$, determine the numerical size of $\hat{T}_1$. Round your answers correct to one decimal digit. (4)

11 MARKS

QUESTION 15 ANSWER THIS QUESTION IN THE GEOMETRY BOOKLET

In the diagram, IA and IE are tangents at the points A and E respectively to the circle with centre O.
The diameter EN is produced to S.
IA is also produced to S.
IO and AE intersect at point P.



Let $\hat{A}_1 = x$

(a) Prove that IEOA is a cyclic quadrilateral. (4)

(b) Name, with reasons, four other angles equal to x . (4)

(c) Express $\hat{S}$ in terms of x . (2)

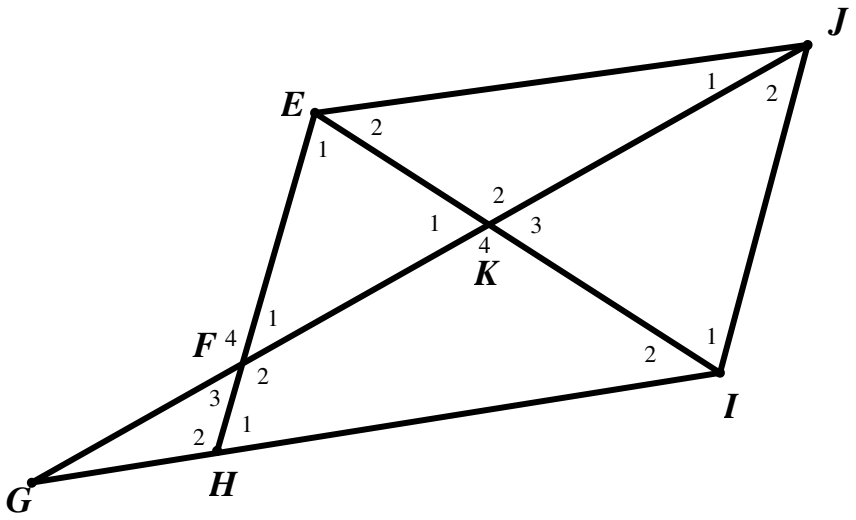
(d) If SA = 100 *units* and $x = 30°$, determine the length of the diameter NE of the circle. (4)

14 MARKS

QUESTION 16 ANSWER THIS QUESTION IN THE GEOMETRY BOOKLET

(a) Complete the statement: If two triangles are equiangular, then _____. (1)

(b) In the diagram below: EHIJ is a parallelogram. F is on EH. JF produced meets IH produced at G. FJ intersects EI at K.



Prove:

(1) $\frac{JK}{KF} = \frac{IK}{KE}$ (3)

(2) $\Delta GKI \,///\, \Delta JKE$ (3)

(3) $JK^2 = KF.KG$ (4)

11 MARKS

TOTAL FOR SECTION B: 75 MARKS